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VOLUME II

ALTERNATING CURRENTS

AND

ALTERNATING CURRENT MACHINERY

The
Co

ALTERNATING CURRENTS

AND

ALTERNATING CURRENT MACHINERY

BEING VOLUME II OF THE

TEXT-BOOK ON ELECTRO-MAGNETISM AND THE CONSTRUCTION OF DYNAMOS

BY

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PREFACE.

THE matter of this book consists, in essence, of the lectures which have been delivered for two or three years past by Professor D. C. Jackson to the senior and graduate students in Electrical Engineering at the University of Wisconsin, but Professor J. P. Jackson, of the Pennsylvania State College, carefully revised and extended the manuscript before it was sent to the printers. The method carried out in the book is based on the self-evident but little recognized principle, that methods which have proved best in teaching other branches of Engineering must be equally advantageous in treating Electrical subjects. A treatise on electro-magnetism or on alternating currents should therefore deal with its subject in much the same way that thermodynamics and the steam engine, hydraulics and hydraulic machinery, or the theory of structures are respectively presented in the best works on those subjects. The startlingly rapid advances which have been made in our knowledge of the phenomena relating to electro-magnetism and the electric current, tend to confuse the best teachers, and doubtless account for the rather superficial methods used in many colleges, in which results only have been presented to the student with little reference to reasons. This error is generally admitted, and it is hoped that this work may, by furnishing a satisfactory text-book, aid those teachers who desire to improve their methods.

PREFACE.

The book treats of the fundamental phenomena of alternating currents as met with in engineering practice, and points out their controlling principles and applications. Descriptions and illustrations of commercial machinery are not included *per se*, since they would be little more than repetitions of matter which is available in the current technical journals, and would crowd important material from the book. This does not mean that practical data are excluded; on the contrary, where they may be useful in illustrating deductions in the text, they are copiously used, selections having been made for the purpose from an extensive mas[illegible] which is original. A large number of referer[illegible] given in foot-notes for the fuller information [illegible]ese cover, in general, those articles which ma[illegible]riginal by the student with most profit; exc[illegible]rs on polyphase currents. Here the list of [illegible]h less complete, since the subject has not [illegible]ught out, and material of overshadowing in[illegible] constantly published, so that only a few of [illegible]ost substantial importance can be advantageously cited. A number of excellent articles have lately been published upon polyphase currents but at too late a date to be put into the plates. Where articles of importance have been published in a number of prominent American and foreign technical periodicals, the references have usually been made to the *Electrical World* and the *London Electrician*.

Throughout the book, occupation of space by the description of classical experiments which have come to be of historical interest only, has been carefully avoided, except where it seems desirable to trace the natural development of knowledge in a particularly important subject: as, for instance, the subjects relating to the tracing of alternating-current curves, methods of measuring inductances, or practice in the parallel running of alternators. The last is a subject of much importance at the present time, on account of its influence on the uniformity of the service given from alternating-current central stations, and its rapidly increasing adoption either for regular operation or for the purpose of transferring the load from generator to generator.

Much very important matter which heretofore has been found only in technical periodicals, and sometimes has been unavailable for use, is to be found in this book. In a number of cases original methods have been introduced to gain simple paths to results, every effort being made to present a full physical conception of phenomena to the reader's mind. The mathematics used are merely a means to the end, and are by no means to be considered from any other standpoint. In this respect it has been sought to avoid either the error of presenting unnecessary formulas or on the other hand of giving results without reasons, both of which are fatal to the reader's true progress as they leave him with no true physical conception of the phenomena studied. Numerous original demonstrations of the standard formulas, which it is thought have some merit, have been introduced, and a few additions have been made to the nomenclature. The most important of the latter is the introduction of the term *active* to represent the component of pressure or electromotive force in phase with current, and to represent the working component of current. The introduction of this term removes the inconvenience, of which Professor S. P. Thompson bitterly complains, caused by the use of the term *effective* in the formally adopted meaning of $\sqrt{\text{mean}^2}$.

Where the volume is used as a text-book, and time for completing the full course is not available, the following chapters may be omitted without interfering with the continuity of the subject: Chapters IV., X., XII., XIII., XIV., XV.,

indebted to the delightfully lucid expositio
ench writers.

of of the entire book has received, to its great a
areful reading of Professor George D. Shepa
iversity of Minnesota, and several chapters
by Professor C. S. Slichter, of the Univers
to both of whom we are indebted for many va

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LIST OF IMPORTANT SYMBOLS.

A,	area, constants.	
B,	magnetic induction per sq. cm., or magnetic density.	
B_m,	maximum magnetic density.	
B_a,	magnetic density in armature core.	
B_f,	magnetic density in field core.	
C,	electric current, effective value of alternating current.	
c,	instantaneous current.	
c_m,	maximum value of alternating current.	
C',	primary current.	In chapters dealing with transformers and induction motors.
C'',	secondary current.	
C_1',	exciting current.	
C_μ,	magnetizing current.	
D, d,	readings of instruments.	
E,	electric pressure or electromotive force, effective value of alternating pressure.	
e,	instantaneous electric pressure.	
e_m,	maximum value of alternating pressure.	
E_a,	active pressure (effective value).	
E_i,	impressed pressure (effective value).	
E_r,	reactive pressure (effective value).	
F,	friction loss in watts.	
f,	frequency.	
G,	galvanometer resistance.	
H,	magnetic force, hysteresis loss in watts.	
I,	impedance.	

self-inductance. tion motors.
ngth.
utual inductance, magneto-motive force.
imber of phases in polyphase circuits.
imber of lines of force,
or total magnetism.
mature magnetism.
ld magnetism.
akage magnetism.
Maximum magi
transformers a
tion motors.
imber of turns in a coil.
imber of primary turns.
imber of secondary turns.
Transformers anc
tion motors.
npere-turns.
agnetic reluctance.
imber of pairs of poles.
iantity of electricity.
stantaneous quantity of electricity.
sistance.
imber of armature conductors.
imber of primary conductors.
imber of secondary conductors.
In induction

, frequency of current in armature of induction motor.
V, power.
, instantaneous power.
, foucault current loss in watts.
leakage coefficient.
angles, phase of alternating current.
angles or deflections.
deflections.
base of Napierian logarithms.
commercial efficiency.
angles or deflections.
magnetic permeability.
ratio of circumference to radius.
time constant.
angle of lag.
angle between currents C' and C_1' in transformers.
angular velocity.

OTE. — Students who are acquainted with the methods of solving rential equations may pass over the solution of the equation

$$dc + \frac{R}{L} c\,dt = \frac{e_m}{L} \sin \alpha \, dt$$

n in Section 24. This is a linear equation of the first order, and its ion may be written down at once.

udents who are not acquainted with the elementary principles of lex quantities may advantageously read Chapter 31 of Van Velzer lichter's *University Algebra* before entering upon Section 56.

ALTERNATING CURRENTS.

CHAPTER I.

THE ELECTRIC PRESSURE DEVELOPED BY ALTERNATORS.

In dealing with alternating currents, several variables enter into the problem which make it impossible to use, without modification, the results gained in the study of continuous currents. But the same fundamental laws control the phenomena of both, and if care is taken to apply these with due regard to the limiting conditions, it will be found that the subject may be clearly grasped and be reduced almost to the simplicity of continuous-current work. In many particulars, the design of alternating-current machinery varies widely from that of continuous-current apparatus, as the requirements are materially different, though in the matter of good workmanship and substantial construction there is no difference in the two classes of machines.

1. Form of the Pressure Curve of an Alternator. — In a multiple-coil continuous-current armature, if S be the number of conductors, N_a the useful magnetic lines of force passing into the armature, and V the speed in revolutions per minute, the pressure developed will be

$$E=\frac{SN_aV}{10^8\times 60}.$$ [*]

In the demonstration in Vol. I. it has been assumed that the magnetic field is uniform, which is by no means the case in commercial dynamos. The assumption, however, simplifies the demonstration without introducing error into the result, as will now be shown.

Suppose the field be perfectly uniform, then each armature condu [illegible] ies of force at the uniform rate of $\frac{dN}{dt}$ [illegible] per second. If the field be not uniform, [illegible] will, at any instant, cut lines of force at [illegible] which will be variable, but the average [illegible] ich is evidently $\frac{2N_aV}{60}$. Since the conductors [illegible] ch other in consecutive order, and the sum of the lines cut by all the conductors at each instant is practically uniform throughout the revolution, the total electric pressure developed must be proportional to the average rate of cutting by each conductor multiplied by the number of conductors in series. Or, as above,

$$E=\frac{SN_aV}{10^8\times 60}.$$

In the case of alternating-current dynamos this averaging does not occur, because the number of lines which are cut at each instant by all the conductors in series is variable. Thus, assume that in Figs. 1 and 2 (reproduced from Fig. 36, Vol. I.), the field is altered so that it is weakest near the centre; then the rate of cutting lines no longer varies as a sine curve, but the curve

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[*] *Textbook*, Vol. I. Chap. 4.

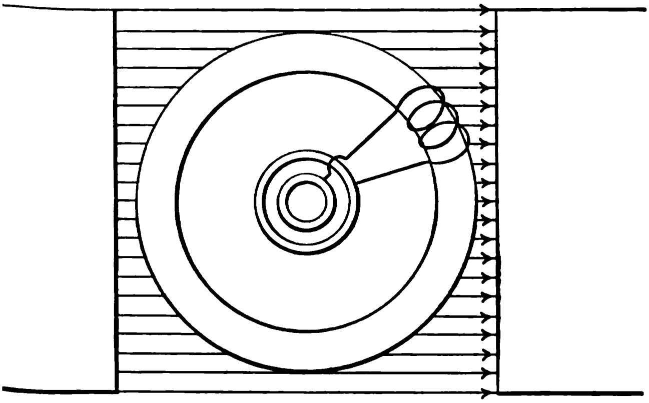


Fig. 1

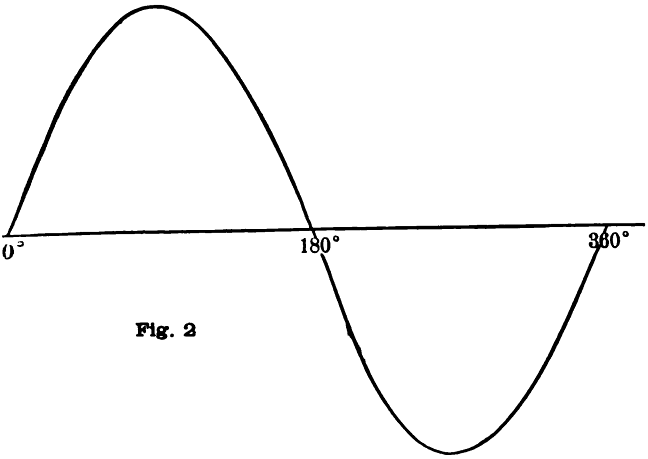


Fig. 2

s more rapidly in its lower portions and does not ch as great a maximum. If the field be concentrated towards the centre, the curve becomes higher at t centre, but does not rise as rapidly in its lower portio Figure 3 shows the curve of electric pressure develop in a coil which revolves in a field of the same to number of lines of force as that of Figs. 1 and 2, but which the induction at the centre is 50 per cent l than the average. Figure 4 shows a similar cu when the induction at the centre is 50 per cent grea than the average. In each case, the induction is sumed to change gradually and uniformly from centre to the edges. In all cases where a certain to number of lines is cut per revolution by a coil revolv at constant speed, the average electric pressure rema constant regardless of the magnetic distribution, but effective pressure ($\sqrt{\text{av.}\, e^2}$) is by no means independ of the distribution. Taking the maximum pressure the sine curve shown in Fig. 2 as the unit, the aver pressure in each figure is $\frac{2}{\pi} = .637$ (see p. 82, Vol. The effective pressure of the sine curve shown in Fig

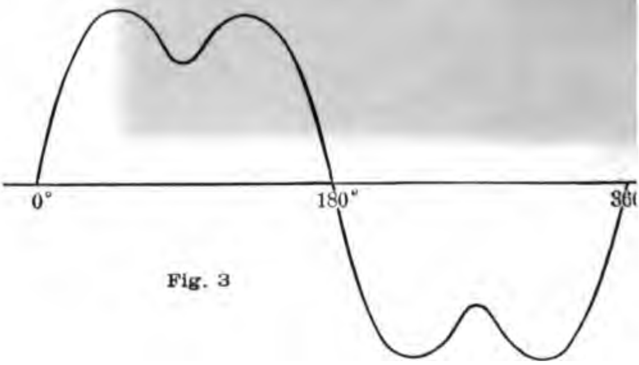


Fig. 3

is $\frac{1}{\sqrt{2}} = .707$. The maximum pressures shown in the curves of Figs. 3 and 4 are .81 and 1.5, and the effective pressures are .58 and .85. If the field were distributed as in Fig. 5 (the total magnetization remaining constant), the maximum, average, and effective pressures would be equal to each other (Fig. 6), and of a numerical value of .637. On the other hand, if the field be greatly concentrated towards the centre, as in Fig. 7, the maximum pressure is very great, and the effective pressure is con-

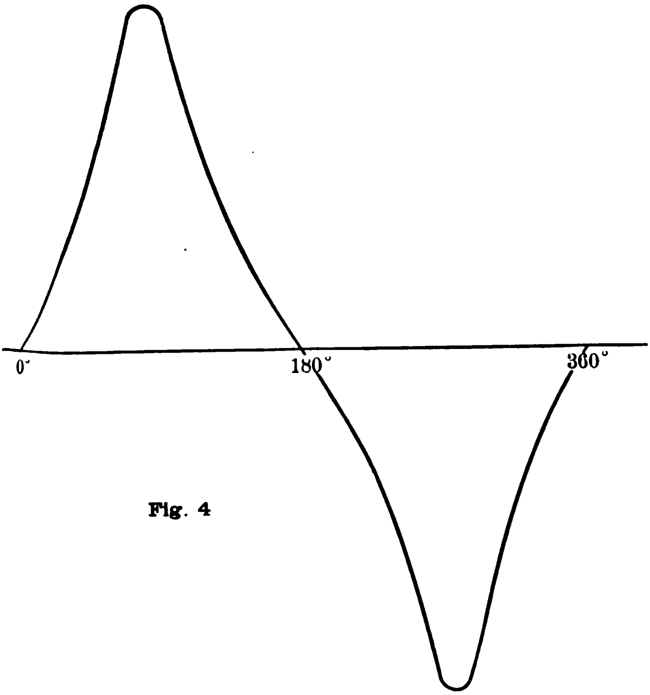


Fig. 4

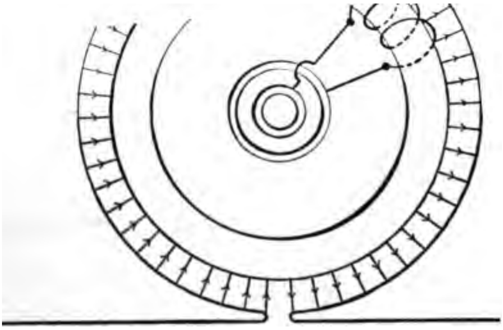
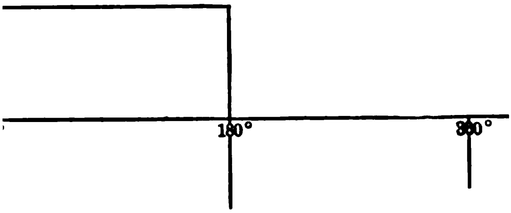


Fig. 5

Period and Frequency of an Alternating C time in seconds, *T*, required to pass th lete cycle or curve is called the **Period** of



Electrical Congress). Usually an alternating current or pressure is designated by its *effective value and frequency.* It is quite common, however, to use the number of half-periods, or the number of **Alternations** in a minute, instead of the frequency. In this case, the number of alternations is equal to 2 × 60 × the frequency. Example: a current with a frequency of 100 makes 12,000 alternations per minute. The term **Periodicity** is sometimes used for frequency.

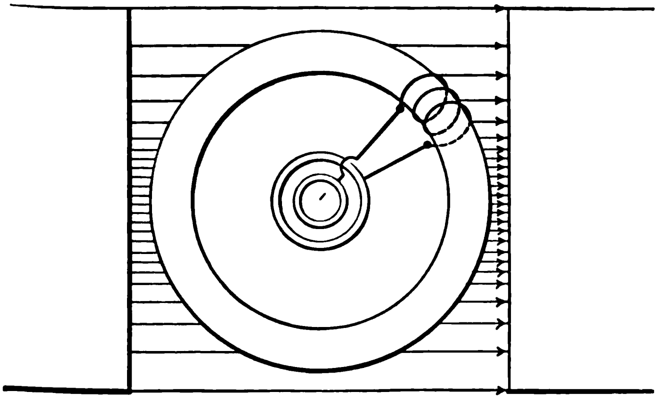


Fig. 7

For the general commercial purposes of the present day the frequency of alternating currents varies widely, but nearly all cases fall within the limits of 40 and 135 (4800 and 16,200 alternations per minute). The majority of American alternating-current dynamos, or **Alternators,** give a frequency of between 120 and 135, but in Europe a somewhat lower frequency is more common, and a frequency of 60 is coming into quite general use in this country. The rotation of a coil in a two-pole field gives one complete period, or two altern tions, for each revolution, and the frequency is therefo equal to the number of revolutions per second. Sin the armatures of two-pole machines would be require to run at an impracticable speed in order to give th ordinary commercial frequencies, alternators are nearl always ma [illegible] able number of poles. Th number of [illegible] on the size of the alternato and other [illegible] may control the speed of th armature, [illegible] may be said to vary fro eight upwa [illegible] any of the alternators bui in the Unit [illegible] een designed to give 16,00 alternations [illegible] a speed of 1600 revolution per minute, and hence [illegible] been built with ten pole The frequency which is produced by an alternator i either equal to one-sixtieth of the product of the nun ber of its pairs of poles and the speed of its armatu in revolutions per minute, or is twice as great. Th number of alternations per minute is equal to the fr quency multiplied by 120, as shown above.

3. Field Excitation of Alternators. — As an alterna ing current will not serve to magnetize the fields dynamos, some arrangement for obtaining a direct cu rent must be made for the excitation of alternator This may be done either by commutating or rectifyin all or a part of the alternating current produced the machine, or a small auxiliary continuous-curre dynamo called an **Exciter** may be supplied for the pu pose. Sometimes the exciter is mounted on the be plate of the alternator (compare Sect. 71).

4. Effective Pressure. — Assuming that the armatu conductors are so arranged that all may be effective, it is seen from what has preceded that the average pressure developed by an alternator is $\frac{2\,S'N_aVp}{10^8 \times 60}$, where S' is the number of conductors in series, N_a is the number of lines of force passing through the armature in each magnetic circuit (emanating from each pole), and p is the number of pairs of poles. If the windings of the armature are all connected up in series, as is common, this becomes $\frac{2\,SN_aVp}{10^8 \times 60}$, where S is the number of conductors on the armature. From p. 278, Vol. I., it is seen that in multipolar continuous-current dynamos with series-path armatures,

$$E = \frac{SN_aVp}{10^8 \times 60}.$$

Now, assuming two machines, one producing continuous currents and the other alternating currents, in which S, N_a, V, p, are equal, the formulas show that the *average* electric pressure developed in the alternator armature with its conductors all in series is twice that developed in the continuous-current armature with halves in parallel. The *effective* alternating pressure of commercial machines has been shown to be usually greater than the average pressure. Calling the ratio of effective to average pressure k, it is seen that the effective pressure of the alternator with armature conductors all in series is $2\,k$ times that of the continuous-current machine. On the other hand, in the continuous-current armature two paths exist for the current against one in the alternator armature. Consequently, the out-

ther as 1 : 2.22, and the outputs
1 : 1.11. The windings on alternato
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allel, instead of in series, in which cas
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this case both the pressures and outpu
uous- and alternating-current machines
1 : 1.11. The value of *k* in commercia
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tre) and upon the relative arrangement
s. It has been shown (Sect. 1) to have
it of unity and a maximum limit which
at. In commercial machines it may be
lly lie between about 1 and 1.25.

. **The Effect of Differential Action by t**
ductors. — It is now well to examine
of the two armatures compared abo
t of the arrangem

uous-current armature, such as a Gramme ring, and connected to armature conductors which are in opposite coils, an alternating current may be taken from the rings. The electric pressure between the rings has its maximum when the conductors connected to the rings are under the points of commutation, and the pressure is zero when the armature has revolved 90° (compare Vol. I., p. 90). (In the case of a narrow coil, the latter point is the point of *maximum pressure.*) The maximum of the alternating pressure must be equal to the continuous pressure for which the armature was designed, and the effective alternating pressure is .707 times the continuous pressure if the field is uniform. In commercial machines the field is not uniform, but it is not likely to be sufficiently irregular to materially disturb the ratio of the continuous and effective alternating pressures when the armature is carrying little current. When the armature carries considerable current, armature reactions may disturb the relations to a greater or less degree. An armature thus arranged, with the conductors uniformly distributed over its surface, gives a sine alternating current. The value of k is therefore 1.11; but the effective alternating pressure is only .707 times that of the continuous pressure developed by the same conductors. The question at once arises as to the cause of this loss of 40 per cent or more. A little consideration shows that the Gramme ring acts like two broad coils in parallel, which cover the whole armature and unite at the points where the collecting rings are connected. When in the position of zero pressure the two halves of each coil are so located in the field as to cut lines in opposite directions, and hence the pressure is zero. Similar differential action is found when the coils cover only a portion of the armature, the extent of the effect depending upon the ratio of the width and pitch of the poles to the width of the coils. It is almost entirely avoided if the coils are never wider than the distance between the pole tips. On account of this differential action, it is necessary to include another constant in the formula for the electric pressure developed by alternating-current armatures. Calling this constant k', and replacing the product $k'k$ by K, the formula becomes $E=\frac{2KSN_aVp}{10^8\times 60}$. Kapp states that K varies from .29 to 1.15,* but in the greater number of commercial cases it is between 1.00 and 1.11.

The output of an alternator is proportional to th product $SN_a = swbw'$, where s is the number of conductors per unit width of coil, b the number of lines o force per unit width of pole face, and w and w' ar respectively the widths of coil and pole face. In orde to economize material, the distance between pole tip may be taken as equal to the width of coil, and thi makes $w + w'$ equal to the pitch of the poles, which i constant; this makes the product ww', and hence th output of the machine, a maximum when w is equal t w', or when the width of coil and pole face are eac equal to half the pitch. The result thus derived mu be modified to suit practical conditions, since fringi tends to increase the width of field, and armature rea tions tend to crowd the field towards the trailing pole tip, thus narrowing the field. Experiment has shown that it is best to have the coils somewhat wider than half the pitch, and it is often advantageous to have the poles of slightly less width.*

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* Kapp's *Dynamos, Alternators, and Transformers*, p. 374.

6. Relative Dimensions of Continuous- and Alternating-Current Dynamos. — From this discussion it is seen that only about one-half the surface of alternator armatures should be covered with wire, so that the construction is quite different from that of continuous-current armatures. With a fixed size of armature and depth of winding it is apparent that an alternating-current armature will carry only about half as much wire as is carried by a similar continuous-current armature. In order to gain an equal output from the machines, it is therefore necessary to increase either the size of the alternator armature or its speed. Both of these devices are used in common practice, and it is not unusual for an alternator armature to be run at a surface speed upwards of 7000 feet per minute. The attached table shows, to some degree, how far the effect of the difference in the lengths of the wire on the armatures is overcome in practice. In the comparison, it must be remembered that the induction in the alternator is usually considerably less than that in continuous-current machines (compare Sect. 61).

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* Compare Elihu Thomson in *Proc. Institution of Civil Engineers*, Vol. 97, p. 101.

ALTERNATORS.

Designation.	Revolutions per Minute.	Diameter rev. part.	Periphery Speed.	Frequency.	Conductor. Inches per Volt.	Capac in K.
1	1200	18″	5700	100	8	15
2	600			60	14.5	30
3	15[illegible]	[illegible]	[illegible]	125	14	35
4	15[illegible]	[illegible]	[illegible]	125	9	35
5	[illegible]	[illegible]	[illegible]		22	40
6	4[illegible]	[illegible]	[illegible]	86		40
7	5[illegible]	[illegible]	[illegible]	83	11.2	50
8	165[illegible]	[illegible]	[illegible]	138		50
9	6[illegible]	[illegible]	[illegible]	70	15	60
10	6[illegible]	[illegible]	[illegible]	100	9.6	120
11	108[illegible]	[illegible]	[illegible]	144	15	140
12	33[illegible]	[illegible]	[illegible]	67	15.2	225

CONTINUOUS-CURRENT DYNAMOS.

Designation.	Revolutions per Minute.	Diameter of Armature.	Periphery Speed.	Conductor. Inches per Volt.	Capacity i K.W.
1	1750	6.25	2850	10	6
2	2750	4.5	3200	11.5	6
3	800	10	2200	24	7.5
4	460	17.5	2250	26	20
5	900	12.5	3000	18	35
6	1650	12	3300	15	35
7	800	10	2100	13	35
8	530	14	1950	18	50
9	700	15.5	2850	18	55
10	900	16	3800	18	60
11	400	15.5	1600	26	70
12	450	17.5	2050	20	80
13	400	25	2600	19	150

CHAPTER II.

ARMATURE WINDINGS FOR ALTERNATORS.

7. **Classification of Armatures.** — Referring to page 93 of Volume I. it will be seen that dynamo armatures are classified under three divisions:

1. Where a wire cuts lines of force by moving across them, as in the case of a slider or of a wire moving around a magnet pole.
2. Where a coil, or set of coils, is moved parallel to itself, or nearly so, between points of different field strength.
3. Where a coil, or set of coils, is wound on a ring or drum and given a rotary motion in a fixed magnetic field.

The first division includes only the so-called unipolar armatures, and requires no treatment here. Alternator armatures are in general classed in the second division. It is not uncommon for the field to move instead of the armature, and in some cases neither the armature nor field coils move, but the induction through the former is varied by the revolution of iron **Inductors.** Where the field rotates, or the variation of the induction is effected by moving inductors, the electric pressure generated in the armature windings is produced in exactly the same manner as if they moved, and such armatures therefore belong to the second division.

The construction of the machines thus enumerated requires an additional classification into:

1. Alternators with moving armatures.
2. Alternators with moving fields.
3. Inductor alternators.

The armat rs belonging to the first and second o nay, for convenience, be divided into f

1. Drum ar
2. Ring ar
3. Disc armatures.
4. Pole armatures.

8. Drum Arm n-wound alternator armatures are always chord-w (see p. 273, Vol. I.). In two-path chord-wound continuous-current drum armatures, the individual turns of each armature section cross each other on the heads in very much the same way as in armatures for two-pole machines (Fig. 8). An exactly similar arrangement may be used in winding alternating-current armatures, but the armature surface is not entirely covered with wire (Fig. 9). Since either the whole or one-half of the armature wire is connected permanently in series and only the ends of the windings are carried to collecting rings, there is no advantage to be gained by permitting the wires to cross each other in alternator armatures. The crossing of the wires, on the other hand, presents a positive disadvantage by increasing the difficulty of satisfactorily insulating them. This is particularly marked on account of the high pressures for which alternators are commonly built. The winding of alternator

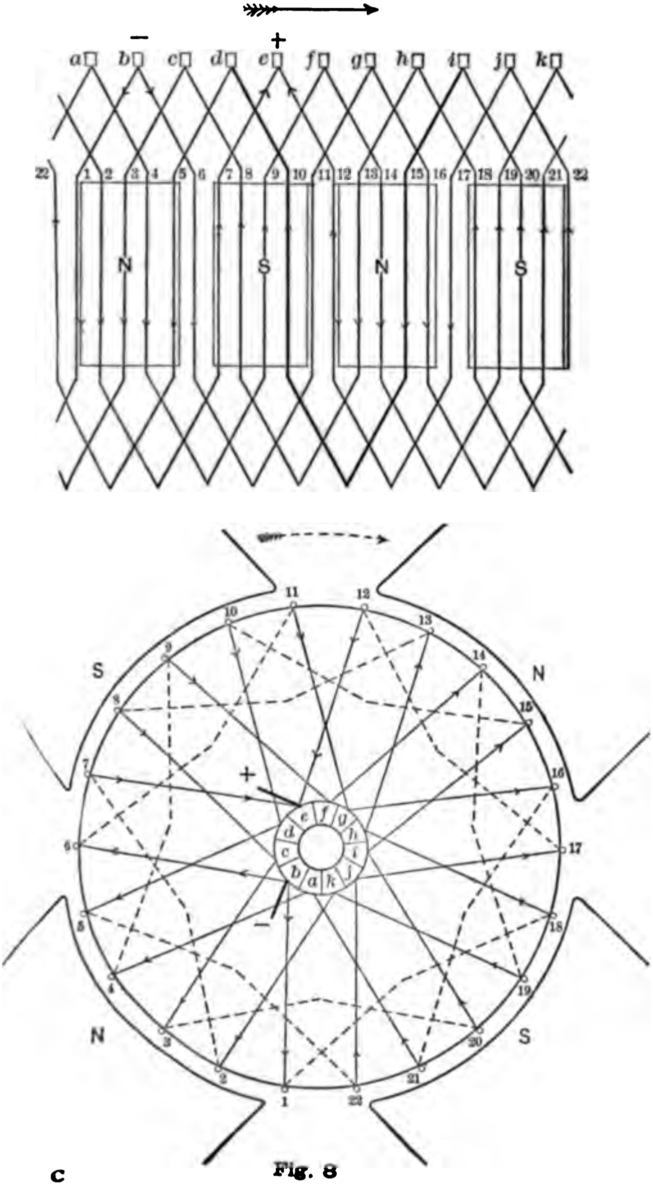
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Fig. 8

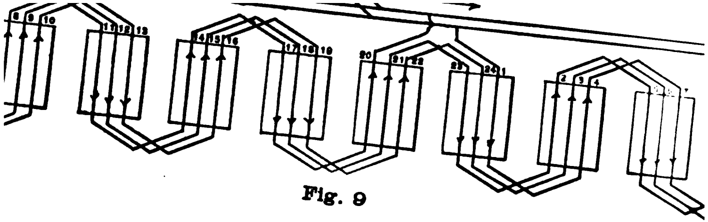
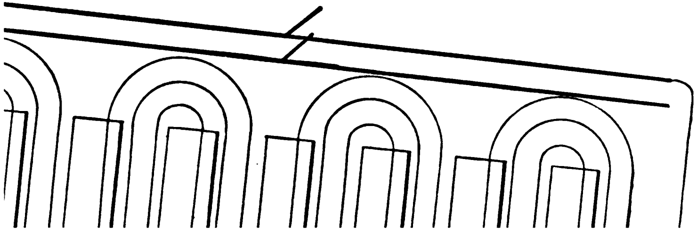


Fig. 9



ably called *undulatory winding* after Fritsche,* or better still, **Continuous Winding.** The same result may evidently be obtained by turning the wires back at the ends of the armature core so that they form isolated coils (Fig. 11), and then properly connecting the coils. When there are as many coils as poles, alternate coils

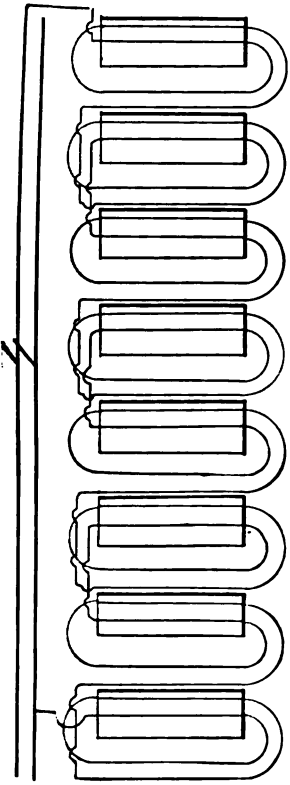


Fig. 11

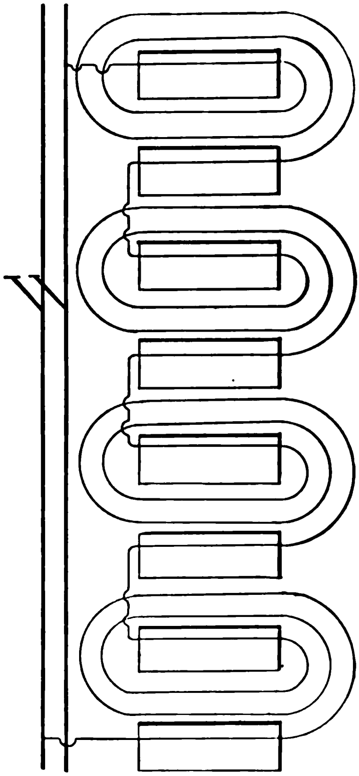


Fig. 11a

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* Fritsche's *Die Gleichstrom Dynamomachine*, p. 43.

are moving at any instant under poles of opposite signs, and the electric pressures developed in them are in opposite directions. They must therefore be so connected that they are alternately right- and left-handed (Figs. 11 and 12); and likewise when there are half as many coils as poles, the coils must all be connected in the same di and 12*a*). This arrangement may be called **Coil Winding,** or, after E. Arnold and S. P. Thompson,[†] *Loop Winding* or *Lap Winding.*

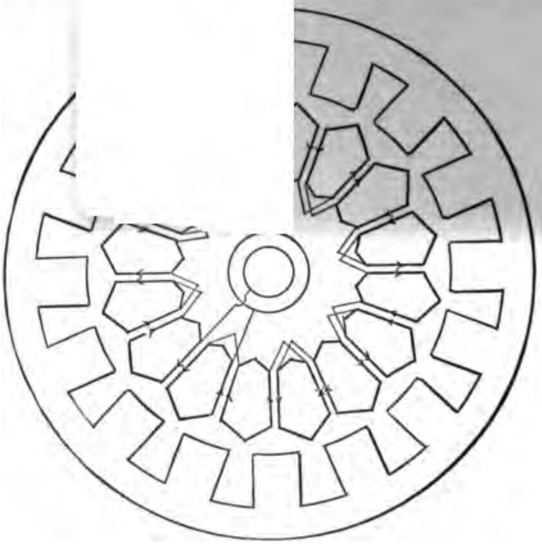

Fig. 12

As already stated (Sect. 4), alternator armatures a frequently connected with the two halves of the win

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[*] *Die Ankerwickelungen der Gleichstrom-Dynamomachinen,* p. 13.

[†] *Dynamo-Electric Machinery,* 4th ed., p. 314.

[‡] For numerous diagrams of alternator windings see Parshall a Hobart's *Armature Windings,* Chap. 12.

ings in parallel. In this case, the coils must be connected in each half so that they are alternately right- and left-handed, but where the halves join, both the coils must be either right- or left-handed (Fig. 13). When the coils are all connected in series, the first and last coils lie side by side; consequently in armatures built for high pressures a severe strain is thrown

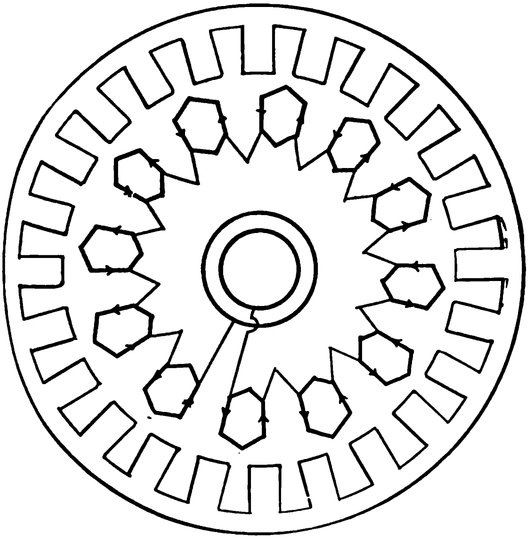


Fig. 12a

upon the insulation separating them. When the halves of the armature are connected in parallel, the first and last coils lie on opposite sides of the armature, and effective insulation is therefore less difficult of attainment. In this case, the pressure between adjacent coils cannot be greater than the total pressure divided by half the number of coils. When the coils are all in series the pressure between any adjacent coils, excepting between the two end coils, is equal to the total pressure divided by the total number of coils. When an alternator armature is connected with its halves in parallel, every precaution must be taken to make the pressures d [illegible] two halves equal at every instant; otl [illegible] nger of one portion of th armature ov [illegible] other, exactly as is some-

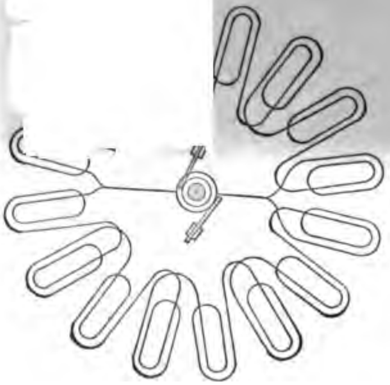


Fig. 13

times the case in multipolar continuous-current armatures (Vol. I., p. 275).

The armature conductors may lie upon the surface o the core or be imbedded below it. Where toothed core are used, the teeth are equal in number to the numbe of coils, and the armature approaches the fourth clas or pole type. Toothing increases armature reaction and self-inductance, thus interfering with regulation but it is in general quite advantageous and is comin

:eneral use by the better class of manufacturers ect. 11).

: coils for alternator drum armatures are ordinarily l on formers and then fastened upon the arma- after having been well insulated. Where surface ıgs are used, the coils are frequently arranged to down over the ends of the core, as in Fig. 14, : they are securely fastened by end plates, or blocks ιod or fibre. It is usual to fill the spaces in the

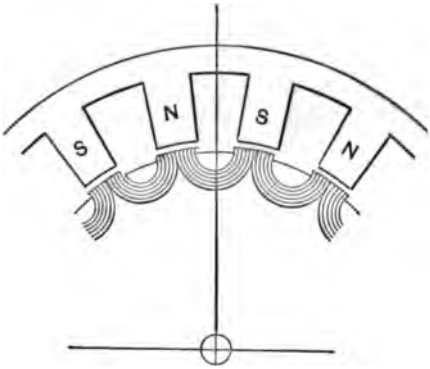


Fig. 14

's of the coils with wood blocks, which are screwed cores or are held by the binding wires. (Exam- Westinghouse and National alternators.) In some ıes the coils are flat, or pancake-like, and of the length as the armature core. In this case they d upon the cylindrical surface of the armature ıd securely bound with wire bands. (Examples: ιon-Houston and Elwell-Parker alternators.) The eriphery velocities of alternator armatures make bands essential to the preservation of surface windings, and all blocking must be securely fastened. The wood blocks which fill the centre of the coils make excellent driving teeth, and therefore serve a good purpose. When imbedded coils are used, they may be made upon formers (lathe-wound), and then applied to the core, or the conductors may be threaded through the grooves, which are well insulated. When the core teeth are T-shaped, the coils may be wound of sufficient width to slip over the head, and when in place they may be narrowed by squeezing. The coils for this purpose must be wound with ends of such shape that they will bend without injury, as is shown, for example, in Fig. 15. Lathe-wound coils are of decided advantage for armatures designed for high pressures, as their insulation may be made particularly safe. Such coils are usually served with layers of japanned canvas, fuller or press board, vulcanized fibre, and mica. The slots between the core teeth may be made of sufficient area to permit the use of any desirable thickness of insulation, and the teeth make a very complete mechanical protection for the coils. Toothed armatures with lathe-

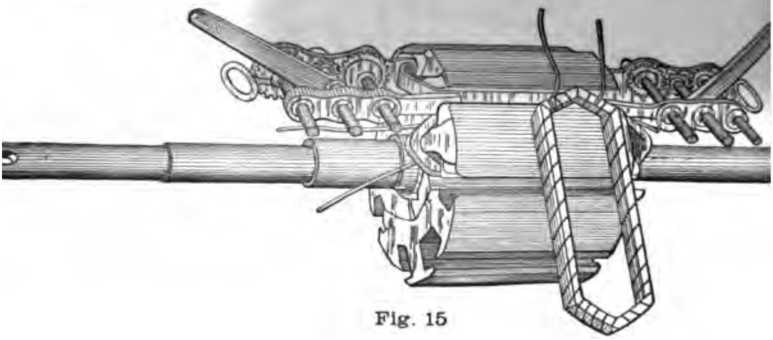


Fig. 15

wound coils should therefore be thoroughly reliable. If the magnetic surface of the armature is fairly complete, the wires are protected from magnetic drag (p. 153, Vol. I.), which decreases the chances of the conductors chafing and therefore injuring the insulation.

9. **Ring Armatures.** — Ring-wound alternator armatures were early used with commercial success, and some of the old **Magneto Machines** of the De Meritens type with permanent field magnets and ring armatures are still in operation. The invention of the ring armature for alternating-current machines is usually ascribed to either Gramme or Wilde, who independently patented the form in France and England in 1878. In America, ring armatures have not been viewed with as great favor as have drum armatures, probably on account of the smaller mechanical stability in the ring and the greater self-induction in its windings. As in drum armatures, the coils of ring-alternator armatures must be connected alternately right-handed and left-handed (Fig. 16). The ring may be arranged so that the field-magnets surround it, as in Fig. 16; so that it surrounds the fields, as in Fig. 17, in which case the latter usually revolve (examples: Gramme and Siemens & Halske alternators); or with the field poles as a crown upon both

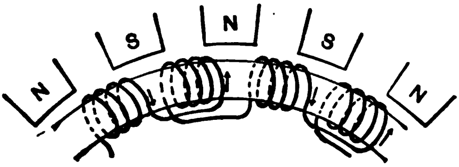


Fig. 16

ures must require more wire t
s for a given output, other things b
efore they must have greater self-ind
hanical arrangement of surface-woun

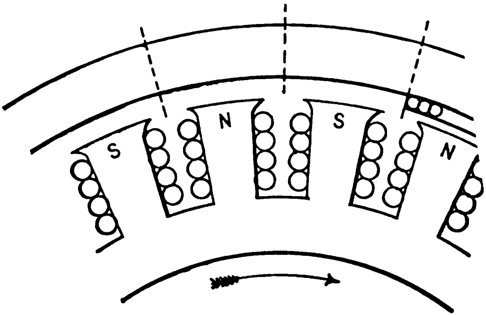
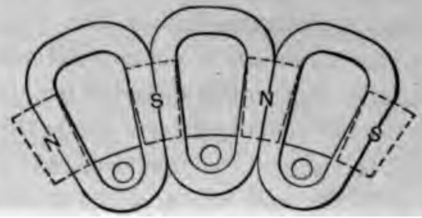


Fig. 17

**, more secure on the ring, but the ad
winding cannot be secured. The pos
aratively slow speed of revolution wh
l by the ring has commended it to
and English build**

dynamo. This was originally devised as early as 1849, but was not developed into commercial form until after 1860. In 1867 Wilde built an alternator with electromagnets and a disc armature. Since that time the disc armature has received much attention in Europe and England, and has been an element of many successful machines. (Examples: Siemens, Ferranti, and Mordey alternators.) The disc has received less attention in America than it deserves, and is here represented by but few machines that are in commercial service. This may be partially due to the essential peculiarity of the disc, which admits of no iron core, and is therefore difficult to build in a substantial and workmanlike manner. That this difficulty can be overcome is shown by the success of foreign alternators with disc armatures. Disc armatures may be wound either with coils or with continuous windings, as shown in Fig. 18. Either the armature or the field may revolve. (Examples: Ferranti alternator, Mordey alternator, Brush alternator.) The absence of iron in disc armatures reduces the losses due to hysteresis and foucault currents to a minimum, but it is likely to require an increase in the depth of the air gap. Hence a greater magnetizing current is likely to be required, or many turns must be placed upon the armature. That this difficulty also can be overcome is shown by the small amount of energy required to magnetize the Mordey alternators. In a 75-kilowatt machine of this type, the C^2R loss in the armature is 2.3 per cent and in the field 1.5 per cent, which compares favorably with continuous-current machines (see Vol. I., pp. 108 and 138). The curve of electric pressure in disc armatures is, in general, quite near to that o sine function. The first experimental determinatio the form of the curve was made in 1880 by Joub who experimented upon a Siemens machine havin disc armature. The curve proved to be practicall sinusoid. This is also true of the curve of press developed by a Mordey alternator. In iron-cored



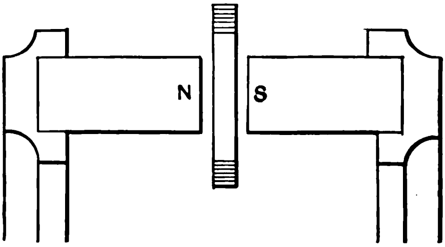


Fig. 18a

chines, armature reactions have a more apparent torting effect and therefore modify the form of curve (sometimes to a considerable degree). The ation is usually not great in surface-wound machi but when imbedded armature conductors are used, curves deviate widely from a sinusoid and someti become quite irregular.

11. Pole Armatures. — As already stated (Sect. there is, in general, no distinct division between armatures and drum armatures with imbedded conductors. In order to avoid foucault currents in the pole pieces, and sharp corners in the curves of armature pressure, precautions must be taken to make the magnetic surface of toothed armatures uniform, exactly as is done in the case of toothed continuous-current armature cores (Vol. I., p. 154). If this is not done, it is necessary to thoroughly laminate the field magnets, since the fluctuations of the induction in the pole pieces, and sometimes in the whole magnetic circuit, are likely to become very much greater and more rapid than ever occurs in continuous-current machines. (Example: Ganz alternator.) This ordinarily entails an excessive expense, but in the Ganz alternator the use of segmental punchings for both field and armature

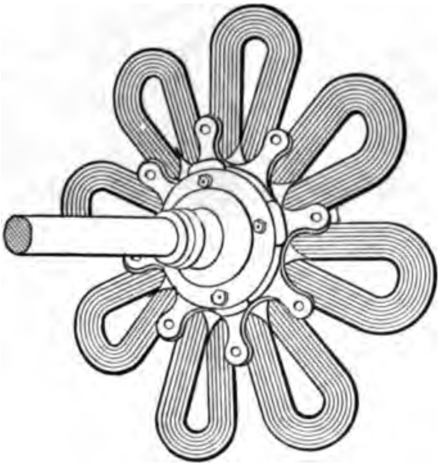


Fig. 18b

(Fig. 19) probably reduces the extra cost of the construction to some extent. The effect of a uniform reluctance in the magnetic circuit may be gained by placing narrow, deep armature coils in grooves as shown in Fig. 20. (Example: Gülcher alternator.) This is similar in many respects to the earliest alternator in which the pole type of armature was used, which was built by Lontin without provision being made to secure an unvarying reluctance. If the pole pieces of both field and armature are made wide and quite close together, the changes in the reluctance of the magnetic circuit probably may be reduced so as to be practically negligible (example: Hopkinson alternator), but armature reactions and magnetic leakage

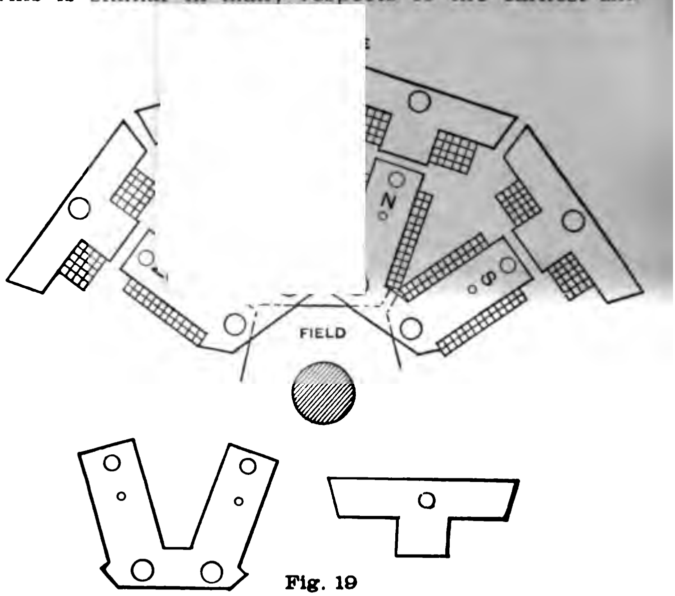


Fig. 19

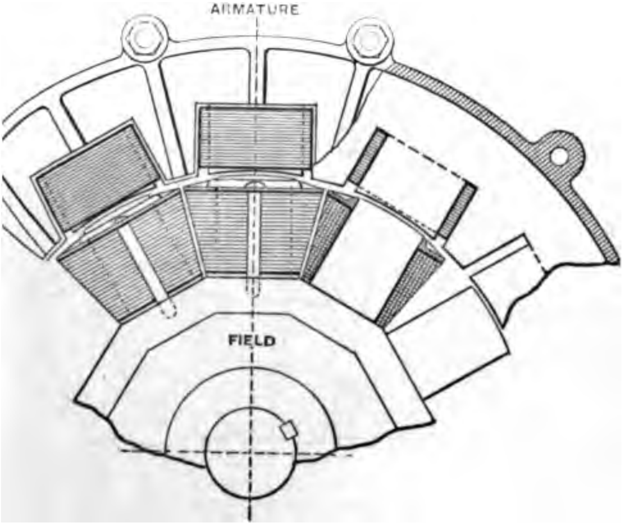


Fig. 20

hich carries either the field or the armature wind-
ust be arranged to revolve. In some cases where
ld magnets compose the revolving part it is pos-

This makes essential the use of some means for conveying the current to and from the revolving coils. In either case, no commutation is required, and therefore plain, insulated **Collecting Rings** serve the purpose when they are properly mounted on the shaft so that brushes may be arranged to bear against them. These rings, often called **Collec** made of copper or bronze.

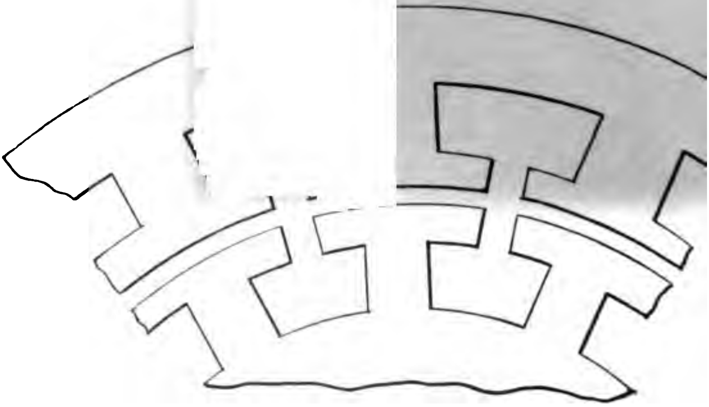


Fig. 21

Instead of brushes, the collection may be effected b means of flexible, weighted copper bands hung over th rings or by similar arrangements (Fig. 22). By the la ter construction a large collecting area may be gaine without unduly increasing the width of the rings. Th current collected, per square inch of collecting contac usually varies between 100 and 500 amperes. The sai limit is fixed by heating alone, and therefore may b quite large without danger. If mechanical consider tions required it, doubtless more than 500 amperes p

e inch of contact might be satisfactorily collected.
s such extreme cases do not often arise, it is not
ally advisable to exceed 200 amperes per square

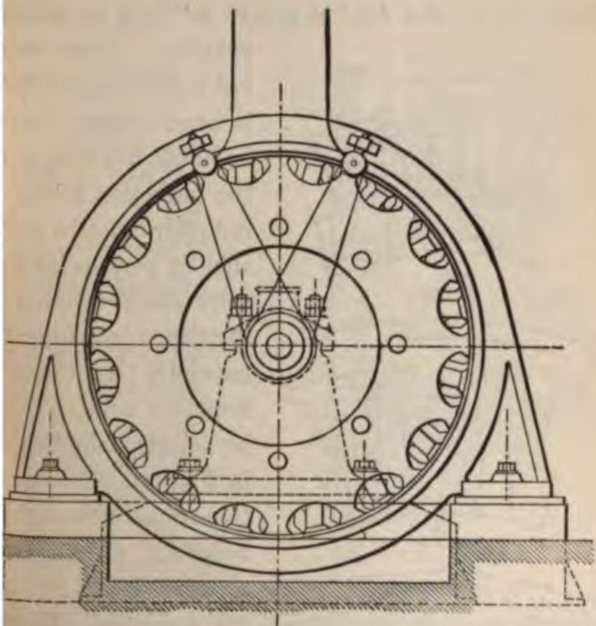
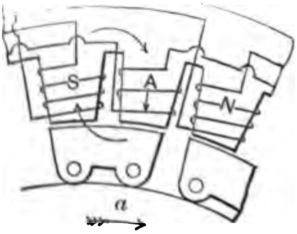


Fig. 22

Inductor Alternators. — The windings of inductor
ators may be made entirely stationary, thus avoid-
ollecting devices. (Examples: Stanley and War-
lternators.) These devices, however, are of so
expense in construction and material that there
marked advantage in suppressing them, and the
nical and magnetic difficulties encountered in the
and construction of inductor alternators has not

magnets be k



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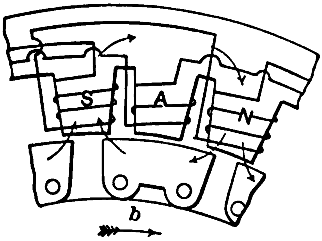

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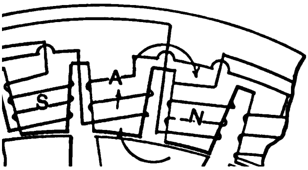

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it of constant reluctance, but in the latter the iron in e fields has been reduced to the minimum bulk. Each these forms gains some economy in construction by iting the coils. (Examples: Stanley, Royal, and War- n alternators.)

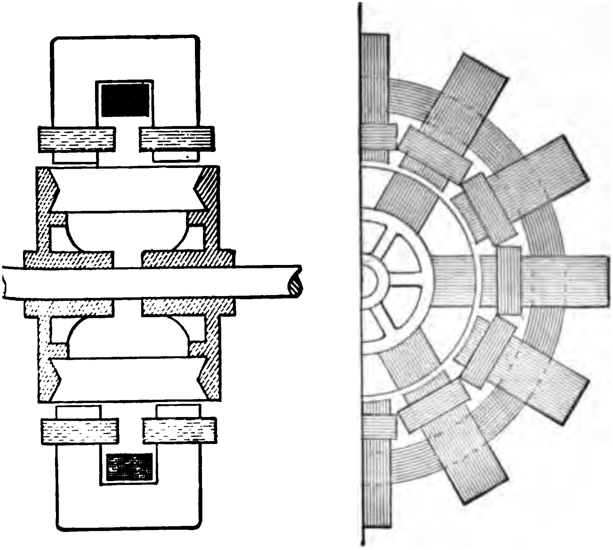

Fig. 24

In the Stanley and Warren alternators lines of force : caused by the motion of the inductor to sweep ross the armature coils, while the total magnetism in : inductor remains fairly constant. The field wind- ;s embrace the inductor core, and the machines are : true inductor alternators but would be properly ssified as machines having drum armatures and field gnets of a modified Mordey type.

4. **Armature Insulating and Core Materials.**—Before conductors are placed on an armature core it is

usual to insulate it, very much as in the case of a continuous-current armature (Vol. I., p. 104), but more thoroughly on account of the high pressures usually produced in alternator armatures. For this purpose mica, micanite, mica cloth, shellacked canvas, fuller board, oiled paper, sheets of vulcanite, vulcabeston, vulcanized ur insulating materials ar

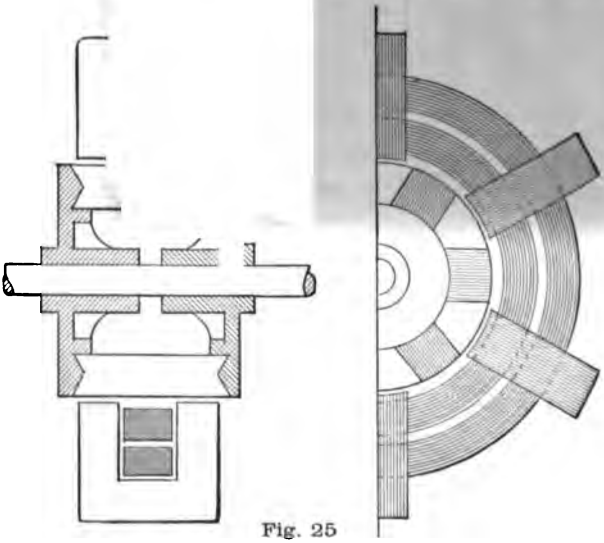

Fig. 25

used. Mica, micanite, vulcanite, vulcabeston, bonsilat vulcanized fibre, asbestos paper, and similar materia are also used to insulate collecting rings and brus holders, and for insulation between the armature coil The wire used for high-pressure alternator armatures often triple-cotton covered, and is thoroughly japanne during the process of winding. Vulcanized fibre is mac from paper fibre by a chemical process and is furnishe in sheets and tubes. Its convenient form, cheapness, and ease of working have brought it into extensive use. It unfortunately absorbs moisture when exposed to the air, which causes it to expand and contract to a remarkable degree. The moisture also reduces its insulating qualities to a large degree. It is therefore unsafe to place entire reliance upon it where continuously high insulation is required. Of the various available insulating materials, mica is the only thoroughly reliable one,

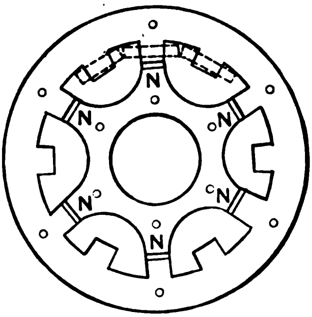

Fig. 26

but it is unduly expensive and of poor mechanical qualities. On the latter account it is generally used in combination with other materials. When made up in the form of micanite by treating with varnish and subjecting to high pressure and heat, its mechanical qualities are somewhat improved, and it may be formed into sheets and tubes or moulded as desired.* Materials such as vulcabeston and bonsilate are advantageous for insulating collector rings and similar details, since they can be moulded into any desired form. Vulcabeston, which is a

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* *Trans. Amer. Inst. E. E.*, Vol. 9, p. 798.

compour l containing rubber and asbestos, is also m
factured n sheets and has quite satisfactory mechai
and elec rical properties. Bonsilate is not sufficie
tough foı very general service. Boxwood and paraff
maple or mahogany are frequently used for insula
where co ısiderable bulk is required and their mecl
cal prope ugh, as they are liable t
more or ıe surrounding conditio
humidity ed with care.

The aı :mselves may be mad
punched ınchings of special sha
iron wire, American machines the
of discs i (Examples: General I
tric and Westingh
machines.) The K
machines have arma
cores made of iron ta
Special punchings
used in machines r
by Ganz & Co. (Fig.
Siemens & Halske, Gülcher, Elwell & Parker, and o
foreign manufacturers, although foreign manufactı
also use punched discs to some extent. In the la
sizes of American machines, which are built for
low speeds to connect directly to steam engines,
therefore have armatures of large diameters, the
are usually built up out of segmental punchings
together in such a way that the segments of alter
layers break joints (Fig. 27).

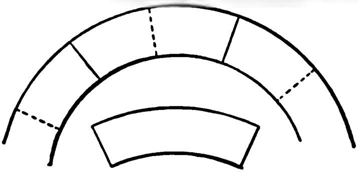
Fig. 27

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* Kapp's *Dynamos, Alternators, and Transformers*, p. 467.

CHAPTER III.

SELF-INDUCTION AND CAPACITY.

15. **Self-Induction.** — Before proceeding with the development of alternator design, it is essential to discuss the relation which exists between electric pressures and currents in a circuit which carries an alternating current. It is to be remembered that the current produced by cutting lines of force sets up in turn magnetic lines, which are in opposition to the original field. Again, when a current is introduced into a circuit, it produces a magnetic field the rise of which causes a counter electric pressure. These conditions are necessary under the law of conservation of energy and its corollary, Lenz's Law (see Vol. I., p. 77). This counter electric pressure is called the **Electric Pressure** or **Electromotive Force of Self-Induction,** and the phenomenon as a whole is called **Self-Induction.** Therefore self-induction may be defined as the inherent quality of electric currents which tends to impede the introduction, variation, or extinction of an electric current passing through an electric circuit. The electric pressure in a circuit which is due at any instant to self-induction is evidently proportional to the rate of change of the magnetization set up by the current of the circuit; therefore, magnetic reluctance remaining constant, the

electric pressure of self-induction is proportional to the rate of change of current in the circuit, which makes it again proportional to the rate of change of instantaneous pressure producing the current. The pressure which is effective in producing current may be called the **Active Pr**[illegible]uish it from the pressure which is **Im**[illegible]e circuit. The latter is called the **Im**[illegible]. It is evident that the active pressu[illegible]is equal to the difference between corre[illegible]taneous values of the impressed press[illegible] counter pressure. The corresponding instantaneous current in the circuit is given by Ohm's law, $c = \frac{e}{R}$, where c and e are instantaneous current and pressure in amperes and volts, and R is the resistance of the circuit in ohms. A little consideration will show that the **Phase** of the electric pressure of self-induction is not in unison with that of the current, although their periods are the same; because, the counter pressure is proportional to the rate of change of magnetism caused by the current, and, supposing the current to be an alternating one, its rate of

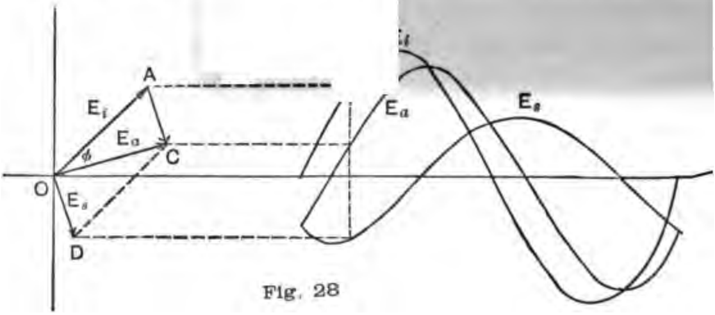

Fig. 28

is zero when it is a maximum, and the pressure -induction is therefore zero at the same time. the curve of current is a sinusoid, the rate of of its ordinates at any point is proportional to $) = \frac{\cos a\,da}{dt} = -\frac{\sin(a-90^\circ)\,da}{dt}$. Hence the curve ıter electric pressure is a sinusoid, the phase of lags 90° behind the phase of the current, and ıre of the active pressure. Suppose the line

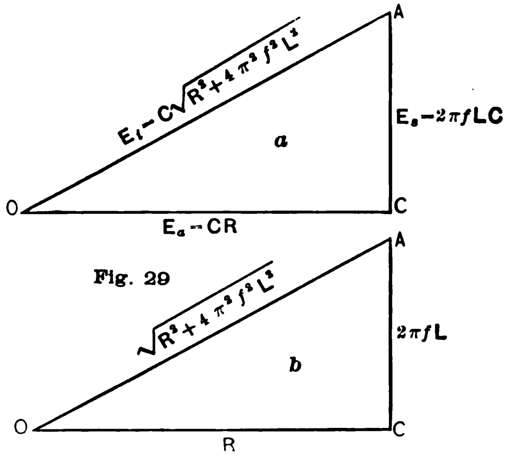

Fig. 29

ig. 28) represents the maximum value of the pressure. If the line be uniformly rotated the point *O*, its end *C* describes a circle and :ical projection a simple harmonic motion. At ıstant the vertical projection of the line pro- ally represents the magnitude of the instanta- active pressure corresponding to the angular of the line. The projection of the line *OD*, s 90° behind *OC*, likewise represents the instan-

[illegible] have been assu
ional to maximum values of pressur
e values of the pressures hold the sa
ach effective value is equal to the
ied by .707. Consequently, the activ
ng in a circuit with self-induction, is e
root of the difference between the
ressed and the inductive electric press
E_a, E_i, and E_s are respectively the a
l, and inductive effective pressures,

$$E_a = \sqrt{E_i^2 - E_s^2}.$$

angle ϕ between the lines OA and OC s
by which the phase of the active pres
that of the impressed pressure. Th
it evident that this lag is caused by
the self-induction line, and that its m
s upon the length of that line. The
angle is $\tan \phi = \frac{AC}{OC} = \frac{\text{Inductive Pressu}}{\text{Active P}}$

per unit current, the number of turns composing the circuit, and the current flowing therein. From the reference just given, $E=\frac{ndN}{10^8dt}$, where E is the pressure of self-induction for the given rate of change of magnetization, n is the number of turns in the coil, and, as before, N is the magnetic flux. But in a long solenoid, $N=\frac{4\pi n'CA}{10}$, where n' is the number of turns per centimeter, and A is the area of the solenoid. Then

$$dN=\frac{4\pi n'AdC}{10} \text{ and } E=\frac{4\pi nn'A}{10^9}\times\frac{dC}{dt}.$$

$4\pi nn'A$ is called the absolute **Self-Inductance** or the **Coefficient of Self-Induction** of the coil, and is usually represented by the capital letter L. $4\pi n'A$ is equal to the number of lines of force passing through the solenoid when the current is one C.G.S. unit. Hence *the C.G.S. value of the self-inductance of any circuit which is in a magnetic medium of unit permeability, may be defined as the product of the number of lines of force enclosed by the circuit when carrying a unit current, by the number of turns in the circuit,* or $L_s=nN_1$. Since the number of lines of force developed by a coil is proportional to the number of turns composing it, its self-inductance is proportional to the square of its turns. In order that the formula $E=\frac{4\pi nn'A}{10^9}\times\frac{dC}{dt}$ may be written $E=\frac{LdC}{dt}$, the values of C and E being given in amperes and volts, the practical value of L is made 10^9 times as large as that of the absolute unit. The Chicago Electrical Congress has formally declared the name of this practical value to be the **Henry.** The formula plainly shows that the absolute dimension of the C.G.S. unit of self-inductance is one centimeter, and the practical unit is therefore 10^9 centimeters or one theoretical earth quadrant. The nam [illegible] s therefore at one time assigned to th [illegible] the absolute dimensions of the ohm ar [illegible] city equal to one earth quadrant per s [illegible] nsions of the henry are equal to the ol [illegible] one second, and therefore the term [illegible] hm), a name suggested by Ayrton and [illegible] nit, came into some use. The use of tl [illegible] in honor of Professor Joseph Henry of Princeton College, was suggested by the American Institute of Electrical Engineers, and was officially adopted by the International Electrical Congress at Chicago.[†] The name *henry* is therefore now the proper and only name for the unit.

The definition of the henry is developed for a long solenoid in a medium of unit permeability. If the circuit does not comprise a long solenoid, the general definition still holds, as already said, but the summation of the number of lines of force passing through each turn individually must be taken, since the number of lines passing through the turns is a variable which depends upon their position in the coil. Thus, suppose Fig. 30 to represent a short solenoid of eight turns in which are developed ten lines of force when one ampere flows through the coil. Assuming the distribution of the

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[*] Thompson's *Elect. and Mag.*, *revised edition*, Art. 357.

[†] *Proceedings International Electrical Congress of Chicago*, p. 18.

lines shown in the figure, the inductance is calculated as follows:

$$\overline{10 \times 2} + \overline{8 \times 2} + \overline{6 \times 2} + \overline{4 \times 2}$$

$$= 56 \text{ C.G.S. units, or } \frac{56}{10^9} \text{ henrys.}$$

If an iron core be now placed in the coil, the number of lines of force will be increased directly as the reluctance of the magnetic circuit is decreased. Hence, assuming the distribution of the lines to remain unchanged, the inductance becomes $\frac{56P'}{10^9 P}$ henrys, where $\frac{P'}{P}$ is the ratio of the reluctance before and after the iron core is inserted. In the case of a long solenoid $L = \frac{4\pi n n' A}{10^9}$, when the permeability is unity, but when the permeability of

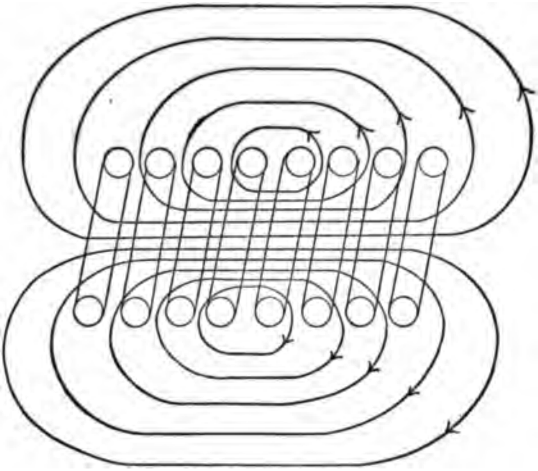

Fig. 30

the magnetic circuit taken as a whole is μ, the nu
ber of lines of force due to unit current is $\frac{4\pi n'}{10}$
and therefore $L = \frac{4\pi n n' A\mu}{10^9}$. In general, where t
magnetic circuit is composed wholly of non-magne
material, the self-inductance is $L = \frac{10nN}{10^9 C} = \frac{nN}{10^8 C}$, a
is a const of C. When iron or oth
magnetic ded in the magnetic ci
cuit, the nductance varies with tl
value of $= \frac{10nN}{10^9 C} = \frac{nN}{10^8 C}$, but th
may have for each value of C, sin
$\frac{N}{C} = \frac{4\pi n'}{10}$ with C. In this case t
inductance for any value of C is μ times as great
when no magnetic material is included in the magne
circuit, the value of μ taken being that correspondi
to the particular value of C. The self-inductance
a long solenoid which contains an iron core, when car
ing a certain current, may therefore be defined as t
number of turns in the solenoid multiplied by t
number of lines of force set up by the current divid
by the current. As an example of the calculation
the value of L, consider a uniform ring of wroug
iron 100 centimeters in mean circumference and
square centimeters in cross-section. Suppose a c
of 2500 turns be uniformly wound on the ring, and
current of two amperes be passed through the ma
netizing coil. Taking μ as equal to 250, which is a f
value, $\frac{10N}{C} = 4\pi \times 25 \times 20 \times 250 = 1{,}571{,}000$. Hen
$L = \frac{2500 \times 1{,}571{,}000}{10^9} = 3.93$ henrys. If the current

the magnetizing coil be taken as $1\frac{1}{2}$, the value of μ becomes roughly 350, and $L = \frac{2500 \times 2,199,400}{10^9} = 5.50$ henrys. If the ring be of brass or other non-magnetic material, the values figure out as follows:

$$\frac{10N}{C} = 4\pi \times 25 \times 20 = 6284,$$

and
$$L = \frac{2500 \times 6284}{10^9} = .0157 \text{ henrys.}$$

In the usual practical problems that are met, the conformation and numerical constants of the magnetic circuit and its windings are unknown, or are so irregular that the self-inductance cannot be determined by calculation, and experimental determination must be resorted to.

At the meeting of the Chicago Electrical Congress a general definition was given for the henry as follows: "As a unit of induction, the henry, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt, while the inducing current varies at the rate of one ampere per second." This is in agreement with the definition already presented.

17. Examples of Self-Inductances.—Ordinary practical experience in electrical measurements and in handling wires soon gives a capacity for estimating the value of resistances; in the same way facility is soon gained in roughly estimating electrostatic capacities, or the current which may be safely carried by a wire, or even he ampere-turns required to produce a given magnetizaion in a magnetic circuit. Ordinary practice, however, gives little clue to estimating the self-inductance in a

a way as to ai
inductances. The following valu
are therefore presented here to g
judgment.*

The range of self-inductances met
wide. The smallest which are practi
doubly wound resistance coils use
bridges and similar devices. Since t
doubled back upon itself, the magneti
rent is almost neutral and the induc
than a microhenry. The inductance o
call-bell of 2.5 ohms resistance has be
microhenrys; a telephone call-bell of
ance, 1.4 henrys; the armature of a
generator of 550 ohms resistance, from
the plane of the coil lay in the plane o
to 7.3 henrys when the plane of the
dicular to the plane of the pole pieces;
receiver measuring 75 ohms, with diap
millihenrys, without diaphragm

ance, 3.6 henrys; astatic mirror galvanometers of 5000 ohms resistance average about 2 henrys. The single coil of a Thomson galvanometer of 2700 ohms resistance measured 2.56 henrys; the coil of another Thomson galvanometer having 100,000 ohms resistance measured 70 henrys; the coil of an Ayrton and Perry spring voltmeter, without iron core, measured 1.462 henrys. This coil had a length of 2.88 inches, an external diameter of 3 inches, was wound on a brass tube .58 inch in external diameter, and had a resistance of 333.5 ohms. Each of the above measurements was made with a current of a few milliamperes. The following are measurements of telegraphic apparatus:

POLARIZED RELAYS OF VARIOUS TYPES.

Type.	Resistance in Ohms.	Self-inductance in Henrys.	Testing Current in Milliamperes.
1	419	1.99	6.3
2	423	1.89	6.3
3	413	1.69	6.3
4	413	1.31	6.3

All armatures were 4 mils from poles.

A common Morse relay of 148 ohms resistance measured 10.47 henrys with the armature against the poles, and 3.71 henrys with the armature 20 mils from the poles, the measuring current being 6.3 milliamperes. In ordinary working adjustment, the inductance of a Morse relay is about 5 henrys. Telegraph sounders with bobbins, respectively, $1\frac{1}{4}$ by 1 and $1\frac{1}{2}$ by $1\frac{1}{4}$ inches, each

wound to 20 ohms resistance, measured 191 and 150 millihenrys, the armatures being 4 mils from the poles and the measuring current being 125 milliamperes. A single coil of a Morse sounder with a resistance of 32 ohms, and having an iron core .31 inch in diameter and 3 in
obbin being .94 inch in
diameter, was
a self-inductance of 94
millihenrys.
der with a core like that
of the precedi
h a bobbin of 50 ohms
resistance hav
of 1.25 inches, was found
to have a self-i
4 millihenrys. The self-
inductance of a
ler of 14 ohms resistance
measured 265

Bare No. 12 B. and S. gauge copper wire erected on a pole line about 23 feet from the ground, is calculated by Kennelly to measure about 8.5 ohms and 3.15 millihenrys per mile; number 6 copper wire under similar conditions is calculated to measure about 2.1 ohms and 2.95 millihenrys. A quadruplex telegraph line, with all instruments in circuit, measures approximately 10 henrys.

The largest self-inductances met in practice are usually in the windings of induction coils or of electrical machinery. The secondary of an induction coil capable of giving a two-inch spark and having a resistance of 5700 ohms, measured 51.2 henrys. The primary of an induction coil which is 19 inches long and 8 inches in diameter, measured .145 ohm and 13 millihenrys, while its secondary measured 30,600 ohms and 2000 henrys. The inductance of dynamo fields is likely to vary from 1 to 1000 henrys; continuous-current dynamo armatures measure between the brushes from .02 to 50 henrys; the fields of a shunt-wound Mather and Platt continuous-current dynamo built for an output of 100 volts and 35 amperes, measured 44 ohms and 13.6 henrys at a small excitation; the armature of the same machine measured .215 ohm and .005 henry; a Mordey alternator armature of the disc type, with a capacity for 18 amperes at a pressure of 2000 volts, measured 2 ohms and .035 henry; a Kapp alternator armature of the ring type, with a capacity of 60 kilowatts at 2000 volts pressure, measured 1.94 ohms and .069 henry; another Kapp machine, 30 kilowatts 2000 volts, measured 7 ohms and .0977 henry; the fields of a Ferranti alternator measured 3 ohms and .61 henry, while the armature of the same machine built for an output of 200 volts and 40 amperes measured .0011 to .0013 henry, with no current in fields; the primary and secondary windings of transformers measure roughly from .001 of a henry up to 50 henrys, depending upon their output and the pressure for which they are designed.

The effect of the field magnetism upon the self-inductance of a disc-alternator armature is shown by some measurements taken by Dr. Duncan* on a small Siemens eight-pole alternator, the results of which are given in the table on the following page.

Professor Ayrton found that the self-inductance of an unexcited Mordey alternator armature varied between .033 and .038 henry, and that this decreased about 10 per cent when the fields were excited.†

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* *Electrical World*, Vol. 11, p. 212.

† *Jour. Institution of E. E.*, Vol. 18, p. 662, also ibid, p. 654.

[illegible] amperes	.120	.112
2.5 "	.125	.115
4.5 "	.128	.115

Self-induction of armature removed f
nry; resistance of armature, .7 ohm
les 45°.

18. The Energy of the Self-Induced Mag

has been shown (Vol. I., p. 69) that tl
ainst the electric current when the numb
rce passing through a circuit is changed,

$$dW = \frac{-nCdN}{10^8}.$$

L has a fixed value, then $CdN = NdC$, ar

$$dW = \frac{-nNdC}{10^8} = -LCdC.$$

ence the work stored in the magnetic fie
rrent changes from a zero value to value

the work stored in the field still remains $\frac{LC^2}{2}$ if L is given its average value between the limiting values of the current. If there is hysteresis, the average value of L, when going up the curve, is greater than when going down the curve, and the work stored in the field by the increasing current is not all recovered when the current falls again to zero. If a coil be wound on a closed ring of soft iron, which exhibits great retentiveness and coercive force in this form, the value of L is very great if the ring be magnetized by an alternating current. If the ring be magnetized by a rectified periodic current, that is, one which varies uniformly between zero and a maximum, the value of L is practically the same as though the iron core were not present. This behavior is due to the ring continuously retaining the magnetization caused by the maximum current, and since the induction in the core therefore remains constant it does not set up a counter electric pressure. By making a cut in the ring, its coercive force may be reduced so much that the average value of L is practically the same for rectified and alternating currents.

19. Curves of Rising and Falling Currents in a Self-Inductive Circuit. — The work done by the effect of self-inductance is manifested, as already explained, by a counter electric pressure which tends to retard a rising current and to accelerate or continue a falling current. That is, it produces an effect in many respects analogous to the inertia of tangible matter. In the case of the latter, $\frac{MV^2}{2}$, MV, and $\frac{MdV}{dt}$ are respectively the energy, momentum, and rate of change of momentum of the mass M, when moving at a velocity V; while the case of the electric circuit, $\frac{LC^2}{2}$, LC, and $\frac{LdC}{dt}$, n be called the energy, momentum, and rate of chang momentum (counter electric pressure) of its magn field. If a circuit having self-inductance be sudde connected to a source of constant electric pressu the curre nstantly to the value $C=$ but it is r s rise is always along a lo rithmic c hen the current has reacl its full value, a smaller quantity of electricity has pas through the circuit during the interval than would h passed if the retardation, or momentum effect, had been present. This decreased amount of electricit proportional to the area OYQ between the curve current and the horizontal line YQ (Fig. 31). counter electric pressure at any instant during the of the current is $\frac{-ndN}{10^8 dt}$, and the instantaneous curr which would flow through the circuit under its ir ence is $c = \frac{-ndN}{10^8 dtR}$. The total quantity of electri

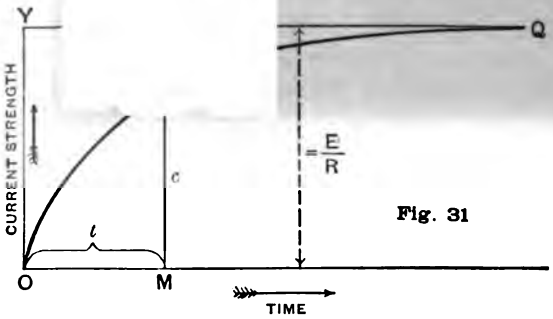

Fig. 31

which would be transferred through the circuit due to a change of the induction from 0 to N is therefore $\int_0^t cdt = n\int_0^N \frac{-dN}{10^8 R} = \frac{-nN}{10^8 R} = -\frac{LC}{R}$. This is equal to the deficit of electricity which flows through the circuit in the period during which the current is rising to its permanent value $C = \frac{E}{R}$. If the pressure be suddenly reduced to zero, the current does not stop immediately, but falls off along a logarithmic curve, and the quantity of electricity passing through the circuit is increased on this account. The increased quantity is proportional to the area between the curve and the X axis. That this quantity is equal to the quantity of electricity lost in starting the current is shown thus: The counter electric pressure is, as before, $\frac{-ndN}{10^8 dt}$, and $c = \frac{-ndN}{10^8 dtR}$. Whence $\int_t^0 cdt = n\int_N^0 \frac{-dN}{10^8 R} = \frac{nN}{10^8 R} = \frac{LC}{R}$. This is equal and opposite in sign to the quantity lost upon starting the current. Hence, if the induction passing through the circuit returns to its initial value, exactly the same total number of coulombs passes through a circuit having self-inductance as would pass were there no self-inductance. In the same way if a mass of moving matter be raised from a velocity V to a velocity V', a certain amount of work is done in accelerating the body; but if after a certain distance has been traversed the velocity be allowed to fall to V again, the work of acceleration is returned and the total work during the cycle is exactly the same as if inertia did not exist in the mass. If the electric circuit have an iron core, the value of the induction may not fall to its initial value upon breaking the circuit, and the energy given up is then not equal to that absorbed in building up the magnetization. The difference in the energy remains stored in the magnetic field in the form of residual magnetism. As an analogue, suppose that when the moving mass assumed above [illegible] it comes to a velocity V'', which is g [illegible] initial velocity V. Then some of the e [illegible] l in acceleration is retained and the [illegible] the work done during the cycle is i [illegible] count of the inertia of the body.

The conditio [illegible] he curves of rising and falling current are loga [illegible] and of exactly the same dimensions when pressure is applied and withdrawn, instantaneously, from a circuit requires that the resistance of the circuit remain constant. If the circuit is broken, by opening a switch or otherwise, it is an experimental fact that the counter pressure rises much higher than the original impressed pressure, frequently rising to many times its value. The extreme severity of the shock which may be received upon breaking a circuit of large inductance attests the fact. This is due to the exceedingly large increase of resistance in the circuit, introduced by the break. The increase in resistance causes the current to fall off more quickly, and hence a greater rate of change of magnetism. However, as before, the work given out by the field must be

$$W=\int_0^C LCdC=\frac{LC^2}{2},$$

and is equal to the energy stored in the circuit when the current was introduced; and $q=\int_0^t c\,dt=\frac{LC}{R}$, which is the same as before. The total number of coulombs transferred being the same and the induced pressure being greater upon breaking a circuit than upon making it, the period of action, *t*, must be shorter upon the break.

20. The Effect of Self-Inductance in Divided Circuits. Application to a Shunted Ballistic Galvanometer. — The fact that the total quantity of electricity which passes through a wire when subjected to a transient electric pressure is independent of the self-inductance of the circuit, as is shown above, has a bearing upon the distribution of current in divided circuits. With no external disturbing factors, it is apparent that where a transient electric pressure is impressed upon parallel circuits of different inductances, the number of coulombs which flow through each circuit would also flow were the circuits without self-inductance, but the phase of the flow in each circuit is retarded so as to lag behind that of the pressure by an amount which is proportional to the self-inductance of the circuit. This reasoning would make it appear that shunting a ballistic galvanometer must change the constant in the ratio of the resistances of galvanometer and shunt without regard to their self-inductances, as is true when continuous currents are in question. This, however, is not correct, because the movement of the needle which occurs before the end of the discharge generates a counter electric pressure in the galvanometer coils. This reduces the proportion of the discharge which passes through the galvanometer. Assuming

that the number of lines of force due to the nee which cut the coils are proportional to the sine of t deflection ; calling r_g and L the resistance and self-i ductance of the galvanometer coils ; r_s the resistance the shunt (the inductance of the latter being assum negligible on account of its being wound with doubl
wire) ; an he respective instantaneo
currents ; taneous impressed elect
pressure i stantaneous active pressu
causing cu ough the galvanometer co
is $c_g r_g$, and impressed pressure less t
counter el aused by self-induction a
the swing herefore

$$c_g r_g = c_s r_s - \left(\frac{L dc_g}{dt} + \frac{kd(\sin a)}{dt}\right),$$

where k is a constant which depends on the stren of the needle. Whence $r_s c_s dt - r_g c_g dt = L dc_g + kd(\sin$

and $r_s \int_0^t c_s dt - r_g \int_0^t c_g dt = L \int_0^0 dc_g + k \int_0^a d(\sin a)$.

This is $q_s r_s - q_g r_g = k \sin a$. If the deflection be sn then $\sin a$ is sensibly equal to $2 \sin \frac{a}{2}$, but $K \sin \frac{a}{2} =$ where K is the ordinary constant of the ballistic vanometer (Vol. I., p. 46). Hence $q_s r_s - q_g r_g = \frac{2}{}$ Calling Q the total discharge, which is equal to $q_s +$ this becomes

$$r_s(Q - q_g) - r_g q_g = \frac{2 k q_g}{K}, \text{ and } q_g = \frac{Q r_s}{r_g + r_s + \frac{2k}{K}}.$$

This discussion shows that the coulombs of a discharge which pass through a shunted ballastic galvanometer are less than the ratio of the resistances or $q_g < \frac{Qr_s}{r_g + r_s}$. The deficit is caused by the lines of force from the needle cutting the galvanometer coils while the discharge is passing, and its value is

$$\frac{Qr_s}{(r_g+r_s)\left(1+\frac{K}{2k}(r_g+r_s)\right)}.$$

The shunted ballistic galvanometer therefore gives readings which are too small, unless the duration of the discharge is very small compared with the time of vibration of the needle.*

In the case of coils connected in parallel each having self-inductances, it is difficult to assign a true fixed value to the self-inductance of the circuit. In fact, only under special conditions can a single coil with self-inductance be substituted for the coils in parallel so as to produce the same effect as the latter upon transient currents of every duration. These conditions are fulfilled when the ratio of $\frac{L}{R}$ is constant for all the coils, and an equivalent coil may then be substituted for the parallel circuits. In this case the resistance of the equivalent coil must be

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{etc.},$$

and its self-inductance must be

$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \text{etc.}$$

---

* See Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I., p. 213.

These values make $\frac{L_c}{R_c}$ equal to the constant value of the ratio for the individual coils.*

21. Rate of Work in a Self-Inductive Circuit when the Current is rising or falling. — The effect of self-inductance has been the effect of inertia in a moving solid. ect of water flowing in a pipe, as sugg ly, also represents many analogies. Th ng an electric pressure upon a circuit, not rise to its full value instantly, but i garithmic function, the constant of whi n the self-inductance of the circuit. In , if pressure be exerted upon water filling a pip ater cannot begin its full flow instantly on account of inertia. If a gate be suddenly closed in the pipe after the flow is fully under way, the momentum of the liquid tends to continue the flow, and the gate suffers a severe blow. In the same way, upon opening an electric circuit a bright spark passes on account of the so-called extra current caused by the tendency of self-inductance to uphold the flow. It must always be remembered that the analogies between the flow of electric current and moving solids or liquids are by no means exact (Vol. I., p. 11), but they are quite useful in fixing the meaning of the phenomena. There is a marked difference between the effect of bends on the inertia effect in the pipe containing water and in the electric circuit. Thus, in the electric circuit, a solenoid has much more self-inductance than has the same wire straightened out. On the other hand, bends in a water pipe cause the inertia effect to be absorbed by friction. Notwithstanding the differences, the analogies are worthy of further consideration.

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* Hospitalier's *Traité de l'Énergie Électrique*, Vol. 1, p. 496; Ledeboer & Maneuvrier, *Académie des Sciences*, 1887.

When water in a pipe is set in motion part of the force exerted upon it at any moment is utilized in accelerating its mass $\left(\frac{Mdv}{dt}\right)$, and the remainder in overcoming frictional resistances (Av). That is,

$$F = Av + \frac{Mdv}{dt},$$

where F is the pressure exerted, v the instantaneous velocity of the water, M is its mass, and A is a constant. It is here assumed that the frictional resistance is proportional to the velocity, which is true only when v is small. When the velocity of the water has become so great that $Av_1 = F$, where v_1 is the final velocity, the acceleration ceases, and the water continues to flow at a uniform velocity v_1 as long as the force is applied. In the case of the electric circuit, the impressed electric pressure is expended in overcoming the counter electric pressure due to self-inductance $\left(\frac{Ldc}{dt}\right)$ and in causing the current to flow through the resistance (R) of the circuit, or $E = cR + \frac{Ldc}{dt}$. This is similar to the expression for the flow of a liquid as given above. cR represents the electric pressure exerted in overcoming the electric resistance or electric friction of the conductor (the active pressure), and $\frac{Ldc}{dt}\left(=\frac{d(LC)}{dt}\right)$, the pressure exerted in storing energy in the magnetic field, or in

changing the momentum of the magnetic field (com
Sect. 19) Likewise $\frac{Mdv}{dt}\left(=\frac{d(MV)}{dt}\right)$ in the for
relating to the flow of water represents, of course
pressure or force exerted in storing energy in the w
by increasing its momentum. The power expende
the elect instant is $Ec = c^2R +$
in which r expended in heating
conductor power expended in sto
energy in d. In this discussion,
assumed t esistance of a conductor
constant or a variable current as
a constant one. rrect within practical li
provided the rate of variation of the current is not great and the conductor is not too thick.

22. The Time Constant of a Self-Inductive Circui From the equation

$$E = cR + \frac{Ldc}{dt} \quad \text{is given} \quad c = \frac{E - \frac{Ldc}{dt}}{R},$$

which represents the instantaneous value of the cur flowing at any moment while the pressure E is applie a circuit of inductance L. To find the value of the stantaneous current at any particular time t, we l from the same equation, by transposition, $\frac{dc}{E-cR}=$ whence

$$\int_0^c \frac{dc}{E-cR} = \int_0^t \frac{dt}{L},$$

which gives $-\frac{1}{R}\log(E-cR)\Big]_0^c = \frac{t}{L}\Big]_0^t,$

$$\log\left(\frac{E-cR}{E}\right)=-\frac{Rt}{L},$$

ily $c=\frac{E}{R}(1-\epsilon^{-\frac{Rt}{L}})$, where ϵ is the base of the ı logarithms.* This shows, as already stated, theoretical curve representing the rise or fall current is logarithmic. The formula shows en L is very small the current almost im- ly takes its full value $C=\frac{E}{R}$, since $\epsilon^{-\frac{Rt}{L}}$ quickly negligible in comparison with unity. Theoreti- ıen inductance is present, the current can only its full value after an infinite time, yet $\epsilon^{-\frac{Rt}{L}}$ practically negligible after a comparatively terval. Since resistance has the absolute dimen- a velocity (a length divided by a time) and ıce has the dimensions of a length, the ratio $\frac{L}{R}$ dimensions of a time; this ratio, in the case circuit is, therefore, generally called the **Time** t of the circuit, and may be represented by the etter τ. In the preceding equation, $\frac{E}{R}$ repre- e value which the current would instantly reach ınder the constant impressed pressure, were ɔ inductance in the circuit. This is the same ultimate value when there is inductance. The ı may therefore be written

$$c=C(1-\epsilon^{-\frac{t}{\tau}})\quad\text{or}\quad C-c=C\epsilon^{-\frac{t}{\tau}}.$$

:e Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I., p. 207.

When $t = \tau$, this becomes

$$C - c = \frac{C}{2.718} = .368\,C.$$

This is the deficit of the current after a time in seconds equal to τ, and the current at that instant is therefore .632 of its ultimate or full value. The value of the time constant is therefore a *measure of the growth* of the current in a circuit, and it is obvious that in a circuit of great inductance and also great resistance, the current practically reaches its full value as quickly as in a circuit of small inductance and proportionally small resistance.

23. Examples of Time Constants. — The following are the time constants of some of the circuits for which inductances have previously been given (Sect. 17). Wheatstone bridge resistances, when properly wound, generally have a time constant of a millionth of a second or less; electric bell, 4.8 millionths of a second; telephone call-bell, nearly .02 of a second; armature of a small magneto generator, from .005 to .013 of a second; telephone receiver, with diaphragm, about .001 of a second; mirror galvanometer for marine signalling, .0016 of a second; mirror galvanometer of 5000 ohms resistance, .0004 of a second; 2700-ohm coil of a mirror galvanometer, .001 of a second; 100,000-ohm coil of a mirror galvanometer, .0007 of a second; coil of Ayrton and Perry spring voltmeter, .0044 of a second; polarized relays, types 1, 2, 3, and 4, respectively, .0048, .0045, .0041, and .0052 of a second; Morse relays, from about .070 to .026 of a second, with about .034 of a second as an average for instruments in working adjustment; two telegraph sounders, .0095 and .0065 of a second; bare No. 12 B. and S. gauge copper wire on a pole line, .00037 of a second; No. 6 wire in a similar position, .0014 of a second; primary of large induction coil, .09 of a second; secondary of same, .065 of a second; dynamo fields, from about .01 to 10 seconds; continuous-current dynamo armatures, from .005 to 5 seconds; Mordey 36-kilowatt 2000-volt alternator armature, .017 of a second; Kapp 60-kilowatt 2000-volt alternator armature, .035 of a second; primary and secondary windings of transformers, from several thousandths of a second to a number of seconds. Finally, suppose 6 ohms is the resistance of the magnetizing coil figuring in the problem of Section 16. Then assuming the value of L to be constant, which is not exact when the core is iron, the time constant becomes in the three cases, respectively, .65, .92, and .0026 of a second.

24. Equation for Current in a Self-Inductive Circuit when an Alternating Sinusoidal Pressure is applied. — The total quantity of electricity which is transferred through a circuit when a periodic electric pressure is impressed upon it has been shown to be independent of the inductance of the circuit, provided the period gives sufficient time for the current to follow its natural curve of rise and fall; the only change, in this case, in the flow caused by inductance being a retardation of the phase of the current relative to the pressure (Sect. 19). If, however, the pressure be an alternating one the quarter period of which is not materially greater than period of the impressed

In the case of an alternating curre cuit by an impressed alternating pr reduces the current uniformly in c effect is therefore one which make crease in the resistance of the circui

Returning now to the formula $c = \frac{E}{}$ sidering c and E instantaneous value pressure which vary according to a sin ing for E its value $e_m \sin a$, where e_m value of the sinusoidal pressure; then

$$c = \frac{e_m \sin a - L\left(\frac{dc}{dt}\right)}{R}, \text{ or } \quad dc + \frac{R}{L} c dt$$

It is desired to find from this equatio in terms of $\frac{R}{L}\left(=\frac{1}{T}\right)$, t, e, and $\sin a$. this, the equation

ι and v are entirely arbitrary and only their prod-
fixed by the assumed conditions, we are at lib-
make further assumptions regarding the value of
them. Therefore, for further convenience in
ating, we will assume such a value for v that
$dv = 0$, and the value of v is derived from this
egration as follows: $\log v = -\frac{t}{\tau} + \log A'$, where
constant of integration.
ce $v = A'\epsilon^{-\frac{t}{\tau}}$. Since $\frac{vdt}{\tau} + dv$ is taken equal to
he principal equation reduces to $vdu = \frac{e_m}{L} \sin adt$,
$= \frac{1}{A'}\epsilon^{\frac{t}{\tau}}\frac{e_m}{L} \sin adt$ and $u = A'' + \frac{1}{A'}\int \epsilon^{\frac{t}{\tau}}\frac{e_m}{L} \sin adt.$
e, placing $A'A''$ equal to A,

$$uv = c = \epsilon^{-\frac{t}{\tau}}\left[A + \int \epsilon^{\frac{t}{\tau}}\frac{e_m}{L} \sin adt\right]$$

$$c = A\epsilon^{-\frac{t}{\tau}} + \frac{e_m}{L}\epsilon^{-\frac{t}{\tau}}\int \epsilon^{\frac{t}{\tau}} \sin adt.$$

dt may be most readily integrated by parts as
:

$$\int \epsilon^{\frac{t}{\tau}} \sin adt = \int ydz = yz - \int zdy.*$$

g $\sin a = y$ and $\epsilon^{\frac{t}{\tau}}dt = dz$, makes by integration
, and by differentiation $dy = \cos ada$, but $a = \omega t$
, p. 80), and therefore $da = \omega dt$, whence

$$dy = \omega \cos \omega t dt.$$

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[*] See Price's *Calculus*, Vol. II., p. 358.

last term may again be integrated b
s $a = y$ and $\epsilon^{\frac{t}{\tau}}dt = dz$, and the origina
mes

$$\epsilon^{\frac{t}{\tau}} \sin a dt = \tau\epsilon^{\frac{t}{\tau}} \sin \omega t - \tau^2\omega\epsilon^{\frac{t}{\tau}} \cos \omega t - \tau^2\omega$$

ansposing, and substituting a for ωt, giv

$$(1 + \tau^2\omega^2)\int \epsilon^{\frac{t}{\tau}} \sin a dt = \tau^2\epsilon^{\frac{t}{\tau}}\left(\frac{1}{\tau} \sin a -\right.$$

$$\int \epsilon^{\frac{t}{\tau}} \sin a dt = \left[\frac{\epsilon^{\frac{t}{\tau}}}{\frac{1}{\tau^2} + \omega^2}\right]\left(\frac{1}{\tau} \sin a - \omega \text{ co}\right.$$

Substituting the value of this integral in
n for the current, found on the preceding

$$c = A\epsilon^{-\frac{t}{\tau}} + \frac{e_m}{L\left(\frac{1}{\tau^2} + \omega^2\right)}\left(\frac{1}{\tau} \sin a - \omega \text{ c}\right.$$

Now $$\left[\frac{\frac{1}{\tau}}{\sqrt{\frac{1}{\tau^2}+\omega^2}}\right]^2+\left[\frac{\omega}{\sqrt{\frac{1}{\tau^2}+\omega^2}}\right]^2=1,$$

and this may be written

$$\left[\frac{\frac{1}{\tau}}{\sqrt{\frac{1}{\tau^2}+\omega^2}}\right]^2+\left[\frac{\omega}{\sqrt{\frac{1}{\tau^2}+\omega^2}}\right]^2=\cos^2\phi+\sin^2\phi,$$

where ϕ is an angle whose cosine and sine equal respectively the first and second terms in the left-hand side of the equation.* Substituting $\sin\phi$ and $\cos\phi$ for their equivalents in the last term in the equation for current as developed, there results

$$\frac{e_m}{L\sqrt{\frac{1}{\tau^2}+\omega^2}}(\sin a\cos\phi-\sin\phi\cos a)$$

$$=\frac{e_m}{L\sqrt{\frac{1}{\tau^2}+\omega^2}}\sin(a-\phi).$$

Consequently, $c=A\epsilon^{-\frac{Rt}{L}}+\frac{e_m}{\sqrt{R^2+4\pi^2f^2L^2}}\sin(a-\phi),$ since $\frac{L}{\tau}=R$, and $\omega=\frac{2\pi}{T}=2\pi f$, where T is the period and $f=\frac{1}{T}$ is the frequency of the alternating current under consideration. The angle ϕ is determined by the condition that

$$\tan\phi=\frac{\sin\phi}{\cos\phi}=\frac{\omega}{\frac{1}{\tau}},$$

---

* Chauvenet's *Trigonometry*, p. 90.

and from this $\tan \phi = \omega \tau = \frac{2 \pi f L}{R}$

25. Exponential term is practically exponential member of the equation shows the natural rise of current pressure is first introduced in the c may generally be entirely neglected ure is an alternating one, since its negligible within a small interval aft is introduced. This is shown by tak equal to zero, as it is at the instant impressed upon the circuit, and then value of A. This is readily shown to

$$A = - \frac{e_m}{\sqrt{R^2 + 4 \pi^2 f^2 L^2}} \epsilon^{\frac{Rt_1}{L}} \sin (a$$

where a_1 is the phase of the alternating introduced in the circuit, and t_1 is the t duction. Substituting the value of A formula gives

$$c = \frac{e_m}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} \sin(a - \phi),$$

where e_m is the maximum value of the pressure. Therefore,

$$c_m = \frac{e_m}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}, \text{ and } C = \frac{E}{\sqrt{R^2 + 4\pi^2 f^2 L^2}},$$

where C and E are the effective values of current and pressure. Since $\frac{2\pi f L}{R} = \tan\phi$,

$$\sqrt{R^2 + 4\pi^2 f^2 L^2} = R\sqrt{1 + \tan^2\phi} = \frac{R}{\cos\phi}.$$

Therefore, $c_m = \frac{e_m \cos\phi}{R}$ and $C = \frac{E\cos\phi}{R}$.

It is thus shown that when a sinusoidal electric pressure is impressed in an electric circuit having a constant inductance L, the current is also sinusoidal and lags behind the pressure by an angle ϕ, the tangent of which equals $\frac{2\pi f L}{R}$, and which is therefore directly dependent upon the inductance of the circuit and the frequency of the impressed pressure; the maximum and effective currents are less than the maximum and effective pressures divided by R, by an amount dependent upon the frequency and the inductance.

26. Definition of Impedance and Reactance. — The quantity $\sqrt{R^2 + 4\pi^2 f^2 L^2}$ is generally called the **Impedance** of the circuit and sometimes its **Apparent Resistance**, while $2\pi f L$ is sometimes called the **Reactance** or **Inductive Resistance**. The square of the impedance of a circuit is therefore equal to the sum of the squares of its resistance and reactance. Impedance and reactance are both of the dimensions of resistance and are therefore expressed in ohms. **Impedance** may be defined for self-inductive circuits in general, as the total opposition in a circuit to the flow of an alternating electric current, and **reactance**, as the component of the impedance caused by the self-inductance of the circuit.

27. Circuits **onstants in Parallel and**
Series. — The of circuits combined in
parallel may be ı the impedances of the
individual circ *ıe angle of lag is the*
same for all aı ave no magnetic effect
on each other. tive electric pressure at
the common ter cuits is

$$E = C_1\sqrt{R_1^2 + 4\pi^2f^2L_1^2} = C_2\sqrt{R_2^2 + 4\pi^2f^2L_2^2}, \text{ etc.}$$

Also $E = C\sqrt{R^2 + 4\pi^2f^2L^2}$, and $C = C_1 + C_2 + \text{etc.}$ In these expressions R and L are the joint resistance and apparent joint self-inductance, R_1, L_1, etc., are the resistances and self-inductances of the individual circuits, C is the effective value of the total current, and C_1, C_2, etc., are the effective currents in the different circuits.

The above formulas may be transformed as follows:

$$\frac{C_1}{E} = \frac{1}{\sqrt{R_1^2 + 4\pi^2f^2L_1^2}}, \quad \frac{C_2}{E} = \frac{1}{\sqrt{R_2^2 + 4\pi^2f^2L_2^2}}, \quad \text{etc.}$$

Adding these together gives

$$\frac{C_1 + C_2 + \text{etc.}}{E} = \frac{C}{E} = \frac{1}{\sqrt{R_1^2 + 4\pi^2f^2L_1^2}} + \frac{1}{\sqrt{R_2^2 + 4\pi^2f^2L_2^2}} + \text{etc.}$$

But
$$\frac{C}{E}=\frac{1}{\sqrt{R^2+4\pi^2f^2L^2}},$$

whence
$$\frac{1}{\sqrt{R^2+4\pi^2f^2L^2}}=\frac{1}{\sqrt{R_1^2+4\pi^2f^2L_1^2}}$$
$$+\frac{1}{\sqrt{R_2^2+4\pi^2f^2L_2^2}}+\text{etc.}$$

This expression is similar to that giving the joint resistance of divided circuits, $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\text{etc.}$ The apparent joint self-inductance of this formula will evidently be dependent upon the frequency except when the time constants of the circuits are the same (compare Sect. 20); and when the time constants and therefore the angles of lag of the individual circuits are not equal, the *geometrical sum* instead of the *arithmetical sum* of the reciprocals of the individual impedances must be taken to get the reciprocal of the joint impedance. This is fully developed later (Chap. IV.).

The impedance of circuits in series is always calculated from the summed resistances and self-inductances, provided the circuits have no magnetic effect on each other and contain no capacity.

Thus,
$$\sqrt{R^2+4\pi^2f^2L^2}$$
$$=\sqrt{(R_1+R_2+\text{etc.})^2+4\pi^2f^2(L_1+L_2+\text{etc.})^2}.$$

This is correct whether the time constants of the individual circuits are equal or unequal.

28. Triangles of Resistance and Pressure. — In Section 15 it is shown that the impressed pressure in a circuit is equal to the square root of the sum of the

squares of the active pressure and the pressure of s
inductance. Dividing the effective values of the th
pressures by the current gives, in each case, the equi
lent of resistance, so that the three sides of the trian
of electromotive forces are also proportional to
impedance, reactance, and resistance of the circuit (

Fig. 29). $\frac{L}{}=\frac{E_s}{E_a}$. The electric pre
ure of sel dently equal to the rate
which self orce cut the turns of a c
multiplied f turns, or $\omega LC = 2\pi fL$
Reactance luctive pressure divided
current, o Since the line represe
ing active nind that representing i

pressed pressure by the angle ϕ, the length of
former is equal to that of the latter multiplied by cos
or $E_a = E_t \cos\phi$. Therefore,

$$C = \frac{E_a}{R} = \frac{E_t \cos\phi}{R}, \text{ and } c_m = \frac{e_m \cos\phi}{R},$$

exactly as shown by analysis (compare Sect. 25). T
triangle of electric pressures (Fig. 29*a*) and the equi
lent triangle of resistances (Fig. 29*b*), therefore, foret
the more important of the results that can be glean
from the rather laborious integrations which have j
been performed.

29. Application. — The application to circuits in g
eral, and to alternator armatures in particular, of
deductions which are thus made is evident.

Thus, suppose it is desired to design an alterna
which is to generate 25 amperes at an effective pre
ure of 1000 volts at its terminals, the frequency bei

100. Take first, for example, a disc armature without iron in its core, with a resistance of 1 ohm and an average self-inductance of .01 henry. The effective value of the total pressure to be developed in this armature at full load is then

$$\sqrt{(1000 + 25 \times 1)^2 + (2\pi \times 100 \times .01 \times 25)^2},$$

which is equal to 1037 volts. Consequently, the effect of self-inductance is to demand an increase of the total pressure equal to 12 volts. Suppose, however, the armature is of a type having an iron core and has an average working inductance of .05 henry, the total pressure then becomes

$$\sqrt{(1000 + 25 \times 1)^2 + (2\pi \times 100 \times .05 \times 25)^2},$$

which is equal to 1291. Hence, the total pressure must be increased by 266 volts on account of self-inductance. If the two machines were worked at full load upon resistances of absolutely no inductance or capacity, the lag of the currents with respect to the impressed pressure in the circuit in the two cases would be respectively 8° 43′, and 37° 27′ (Fig. 32).

30. Resolution of an Irregular Curve into Component Sinusoids or an Equivalent Sinusoid. — As already said (Sect. 5), it is not safe to assume a sinusoidal form for the curve of pressure developed by an alternator. In general, it is safe to say that the curve produced by nearly all machines having smooth core or disc armatures, is sufficiently close to a sinusoid to make the deductions applicable with some degree of accuracy.

When the curve does not follow a sinusoid, it is possible to resolve it into a number of component sinusoids according to Fourier's theorem, the effect of which can, to some extent, be separately estimated. The general analytical expression for the instantaneous current becomes $c = a \sin \alpha + b \sin 2\alpha + c \sin 3\alpha + \text{etc.} + a' \cos \alpha + b' \cos 2\alpha + c' \cos 3\alpha + \text{etc.}$, which is too complex for general use. The separation is often more readily effected by plotting sinusoids by trial and approximation (Fig. 33). The first or fundamental component sinusoid has the same period as the primary curve, and the other components are regular harmonics of the first. When even harmonics are present the

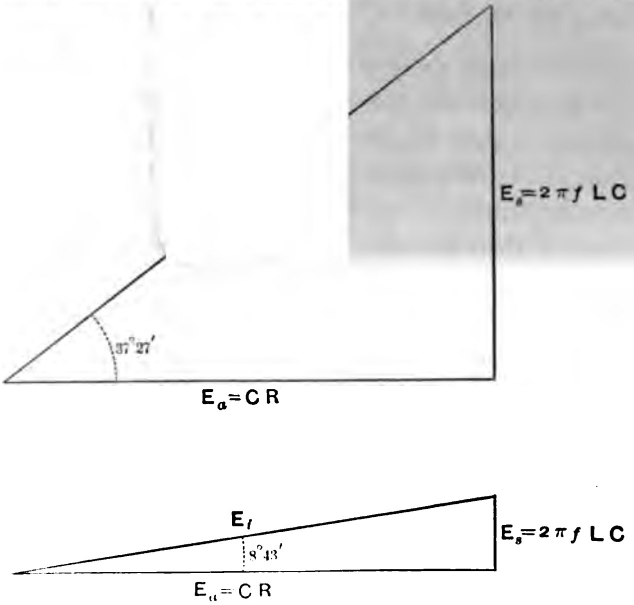

Fig. 32

successive loops of the primary curve are dissimilar, but they are similar when only odd harmonics are present. Since the successive loops of alternating current curves are always similar, it is evident that only the odd harmonics need be looked for in distorted curves. In fact, such curves may nearly always be considered as composed of the fundamental sinusoid combined with sinusoids of three times and five times the frequency, higher harmonics being represented not at all or only by a small residual. The sines and cosines of the Fourier formula when taken in combination, cause the lack of symmetry of alternating current curves. Even if the curve of pressure is an exact sinusoid, if L is not absolutely constant, the current curve varies from the sinusoidal form. In circuits containing iron cores, L varies with the value of the current on account of the variations of μ, and the current curve therefore takes an irregular form. The amount of irregularity depends upon the amount of iron present and upon the extent of its saturation. The expression for curves of pressure

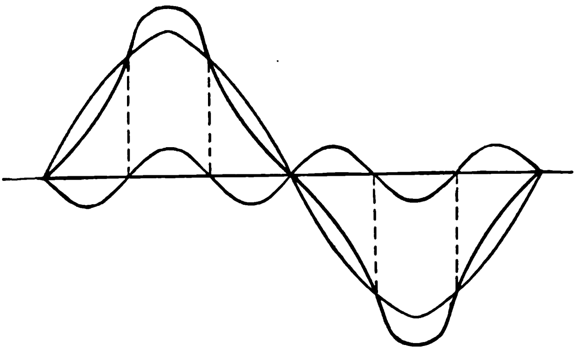

Fig. 33

have the sa
ffective values as the curves which t
eir relative phase positions must be
epresent an equal amount of power.
inusoids of curves that do not vary
inusoidal form are practically the sam
ental harmonic, but when the prima
idely from the sine form the equival
kely to differ in both magnitude and po
ndamental harmonic.*

31. The Effect of Capacity in a Circu
ted conductors have the property of bein
lectricity in its static form. When such
onnected to a source of a different poten
ill flow into or from it, until its potentia
s that of the source. The measure of t
lectricity which is held by the conductor
otential is its **Capacity,** and the C.G.S. u
ay be defined as the capacity of a co
ntains a unit charge of electricity wh
ntial. The

after Faraday, and is $\frac{1}{10^9}$ times as large as the C.G.S. unit of capacity. The farad is too large a unit of capacity to be convenient in practice, and the microfarad, or millionth of a farad, is commonly used as the unit of measurement. The capacity of a conductor depends upon its conformation and surroundings. The term **Condenser** is applied to any insulated conductor having an appreciable capacity, although it is more strictly used to designate a combination of thin sheets of conducting material, insulated, and laid together with the alternate layers connected in parallel. In the following discussion the term condenser will be used in its broader sense.

From the foregoing is at once derived the fundamental relation $Q = sE$, where Q is the quantity of electricity in coulombs, s the capacity in farads, and E the potential in volts.

When a condenser is connected to a source of alternating electric pressure, as indicated in Fig. 34, a current will flow into and out of the condenser, the value of which at any instant is proportional to the rate of change of the active pressure (E_a); because the charge in the condenser at any instant is proportional to the electrical pressure between the terminals of the con-

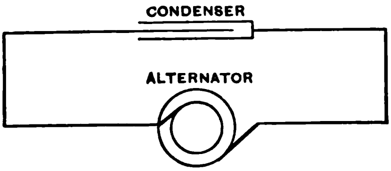

Fig. 34

ıe current flowing into or out of the cc
. any instant the condenser current (c_s)

$$c_s = s\frac{de}{dt};$$

ıd since, when the alternating pressur
$\frac{e}{t} = 2\pi fe_m$ cos a, where e_m is the maxi
:ting on the condenser, there results

$$c_s = 2\pi fse_m \cos a,$$

ıd
$$\frac{c_s}{2\pi fs} = e_m \cos a = e_s.$$

This pressure, which is in phase with t
ırrent, may be called the **Capacity Pre
›nser Pressure.** It is 90° in advance of th
:e, as cos a = sin (a+90°). That it must
ay be readily seen from the reactions tha
rcuit. When a sinusoidal pressure appl
inals of a condenser is rising, *a current*
ndenser. This

ance in the circuit the current takes a short time to build up; however, the principle remains the same (Sect. 33). From the formula for instantaneous current in the condenser the maximum current is seen to be,

$$c_{sm} = 2\pi f s e_m,$$

and the effective current

$$C_s = 2\pi f s E_t.$$

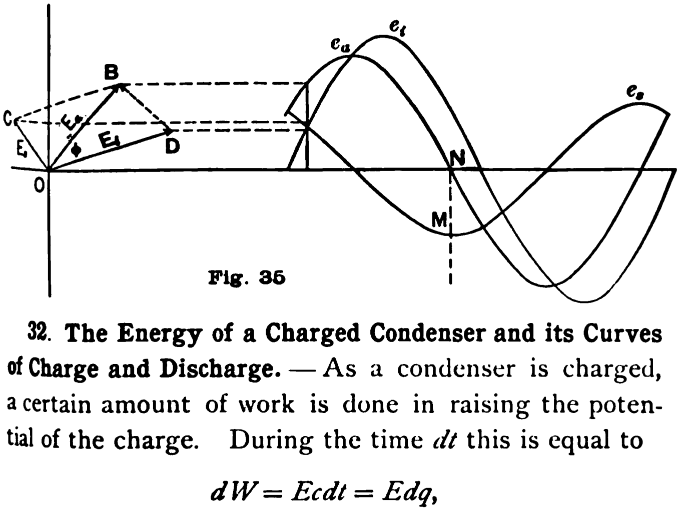

Fig. 35

32. The Energy of a Charged Condenser and its Curves of Charge and Discharge. — As a condenser is charged, a certain amount of work is done in raising the potential of the charge. During the time *dt* this is equal to

$$dW = Ecdt = Edq,$$

or

$$= \frac{1}{s}qdq,$$

(in which E is a constant pressure impressed on the condenser terminals, c is the current flowing into the condenser at the instant t, and q is the final charge in the condenser), from which, by integration, $W = \frac{1}{2}\left(\frac{1}{s}\right)q^2$. This represents a certain amount of work which is stored in the condenser when its charge is increased

electro-magnetic field, but
harge is truly potential and analogous
compressed spring. The total work
uit, containing resistance and capacity,
impressed during a time of charge *dt*,

$$ecdt = Rc^2dt + \frac{1}{s}qdq,$$

hen e, c, and q are the instantaneous
ent, and charge, and where the last te
tored in the charge dq. If this equation
$t = dq$, there results an equation of pre

$$e = Rc + \frac{q}{s},$$

r

$$e = R\frac{dq}{dt} + \frac{q}{s}.$$

rom these equations the charge at any i
etermined when the applied pressure
uring charge; and

$$E = R\frac{dq}{} + \frac{q}{}$$

solving for A' when $t = 0$, and therefore $q = 0$, there results
$$A' = -sE = -Q;$$
hence,
$$q = Q\left(1 - \epsilon^{-\frac{t}{Rs}}\right).$$

During discharge the condenser pressure is zero, and therefore
$$0 = \frac{dq}{dt} + \frac{q}{Rs};$$
from which, by integration,
$$\log q = -\frac{t}{Rs} + \log A'',$$
or
$$q = A''\epsilon^{-\frac{t}{Rs}}.$$

Solving for A'' when $t = 0$, and therefore $q = Q$ (the total charge), there results $A'' = Q$; hence,
$$q = Q\epsilon^{-\frac{t}{Rs}}.$$

From these equations, which are exactly similar to those for self-induction (Sect. 21), it is seen that the curves of charge and discharge are logarithmic when an unvarying pressure is applied to the system (see Fig. 36 *a* and *b*). In many cases of practice Rs is so small that the charge and discharge of a condenser are practically instantaneous.

33. Time Constant of a Circuit containing Capacity.—In these equations Rs has the same relation as $\frac{L}{R}$ in the similar equations for self-inductance (Sect. 22), and therefore Rs may be termed the time constant of the

$$Q - q = .368\,Q\,;$$

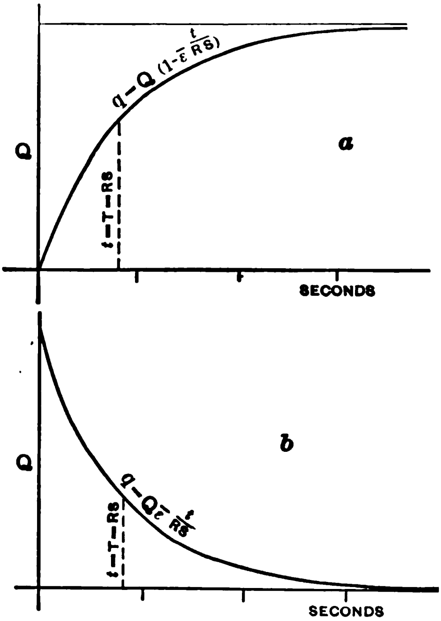

Fig. 36

34. Equation for the Current in a Circuit containing Capacity when an Alternating Sinusoidal Pressure is applied. — In the case when a sinusoidal pressure is impressed upon the circuit the equation of pressure

$$e = Rc + \frac{q}{s}$$

may be differentiated, giving

$$de = Rdc + \frac{dq}{s} = Rdc + \frac{cdt}{s},$$

and as $e = e_m \sin a$, there results

$$\frac{cdt}{Rs} + dc = \frac{e_m}{R} \cos a da.$$

This is a differential equation similar to that for self-induction and may be integrated in the same manner. The formula reduces to the practical form

$$c = \frac{e_m}{\sqrt{R^2 + \frac{1}{4\pi^2 f^2 s^2}}} \sin (a + \phi') + A\epsilon^{-\frac{t}{Rs}},$$

where ϕ' is the angle by which the current is in advance of the impressed pressure. The tangent of ϕ' is found during the development to be equal to

$$\frac{1}{2\pi f s} \div R$$

(see treatment on self-induction, Sect. 24).

The exponential term $A\epsilon^{-\frac{t}{Rs}}$ in the general equation represents the irregularity due to the fact that the current and impressed pressure must start at the same instant. That the term must usually disappear in an indefinitely short time, in practice, may be shown as in Sect. 25 in a similar instance.

; the impedance or apparent resistan
ıe reactance due to capacity. The fi
e written

$$E = C\sqrt{R^2 + \frac{1}{4\pi^2 f^2 s^2}} \text{ and } \frac{E}{C} = \sqrt{R^2$$

rom which triangles of pressures, simi
nd of resistances may be constructed
nd *b*).

Since $$\frac{1}{2\pi f s} \div R = \tan\phi',$$

$$C = \frac{E}{R\sqrt{1 + \tan^2\phi'}} = \frac{E\cos\phi}{R}$$

It is thus shown that when a sinusoidal
re is impressed in an electric circuit, hav
, the current is also sinusoidal, and *lead*
y an angle ϕ', the tangent of which i
hich is therefore *inversely* dependent upo
the circuit and the frequency of the im
re: the maxim

In Fig. 35 suppose the line *OB* represents E_a, the active pressure in a circuit containing a condenser,

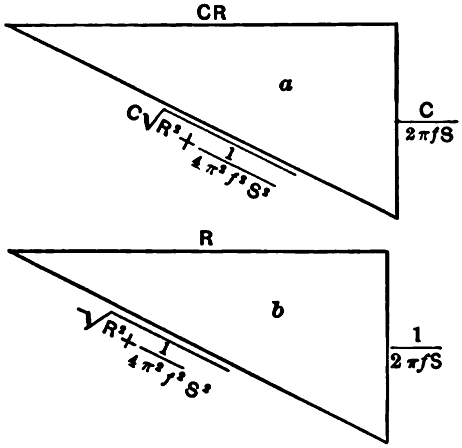
CR
a
$C\sqrt{R^2+\frac{1}{4\pi^2f^2S^2}}$
$\frac{C}{2\pi fS}$
R
b
$\sqrt{R^2+\frac{1}{4\pi^2f^2S^2}}$
$\frac{1}{2\pi fS}$

Fig. 37

then *OC* laid off 90° in advance of *OB* and equal to $E_c = \frac{c_c}{2\pi fs}$ will represent the capacity pressure.

The resultant, or *OD*, will be the effective impressed pressure (E_t).

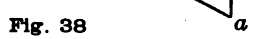

Fig. 38

It will be seen from this figure that E_a leads E_t and is shorter. From the relations shown in the construction, the expression $E_t = \sqrt{E_a + E_c}$ may be formed, and a triangle of pressure laid off as in Fig. 38, where the angle *cba* shows the current lead.

35. Effect of Capacity and Self-Inductance combine in a Circuit, and the Equation for the Current flowin when an Alternating Sinusoidal Pressure is applied. — In the preceding discussions it has been shown tha the instantaneous pressure of self-induction when sinusoidal pressure, E, is applied to the circuit is

$- 90^\circ$),

while the in: enser pressure is

$+ 90^\circ$).

It is therefr and e_t are directly opposed, and their d ress the effect when both are in a circ

$$C = \frac{E}{\sqrt{R^2+\left(2\pi fL - \frac{1}{2\pi fs}\right)^2}},$$

and the instantaneous current is

$$c = \frac{e_m}{\sqrt{R^2+\left(2\pi fL - \frac{1}{2\pi fs}\right)^2}} \sin(a - \phi'') + A\epsilon^{-\frac{t}{r}},$$

in which ϕ'' is an angle whose tangent is

$$\left(2\pi fL - \frac{1}{2\pi fs}\right) \div R.$$

In this case $\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fs}\right)^2}$ is the impedanc and $\left(2\pi fL - \frac{1}{2\pi fs}\right)$ is the combined reactance o self-induction and capacity.

The term $A\epsilon^{-\frac{t}{r}}$ may be shown to disappear in a ver short time, as has been done in the similar case unde self-induction. τ'' is the positive difference between τ and τ'.

Since $$\left(2\pi fL - \frac{1}{2\pi fs}\right) \div R = \tan\phi'',$$

the active pressure will be

$$E_a = E\cos\phi''.$$

As the equations are similar to those of self-induction and capacity, triangles of pressure and resistance may be drawn (see Sect. 28).

When $2\pi fL$ is greater than $\frac{1}{2\pi fs}$, the angle ϕ'' is positive, and the current lags behind the pressure, but when $\frac{1}{2\pi fs}$ is greater than $2\pi fL$, the angle ϕ'' is negative, and the current leads the pressure. Finally, when $2\pi fL = \frac{1}{2\pi fs}$, the angle ϕ'' is zero, and the circuit acts towards an alternating current as though it contained neither self-induction nor capacity, but only resistance; that is, the self-induction and capacity exactly neutralize each other. In this case, the relation between L and s is $L = \frac{1}{4\pi^2 f^2 s}$, or $4\pi^2 f^2 L = \frac{1}{s}$.

35*a*. **Effect, on the Transient State in a Circuit, of Self-Inductance and Capacity combined.** — When an inductive coil of inductance L, is included in a circuit of resistance R, and a condenser of capacity s, is shunted across a portion of the circuit of resistance r, the following conditions are set up:

The condenser is charged with a quantity of electricity $Q = sCr$, where C is the steady value of the current. Now if the impressed pressure be suddenly removed, the

At the same time the self-inductance w tity of electricity to be transferred thr in the opposite direction, which is equal

$$q_1 = \frac{LC}{R}.$$

Hence the total quantity of electri through the circuit is

$$q_1 - q_2 = \frac{C}{R}(L - sr^2),$$

and the effect of the condenser is to ap the self-inductance by an amount equal of the condenser multiplied by the squar ance around which it is shunted.

36. Methods of measuring Self-Induct considering self-inductance and capacity, to discuss the various available and pra of measuring the magnitude of the ind cuits. These methods are based upon a the unknown inductance, either with a kn or resistances; with a known capacity; or gible inductance. This may be gained by using a straight strip of German silver, or thin strips bent back on themselves, separated by thin silk or oiled paper for insulation. The pressures at the terminals of the standard and inductive resistances are measured by an electrometer or by a high-resistance voltmeter of negligible inductance. Then, if the impressed pressure in the circuit is approximately sinusoidal, the following relation holds:

$$\frac{E_1}{E} = \frac{C(R_1^2 + 4\pi^2 f^2 L_1^2)^{\frac{1}{2}}}{CR},$$

where E_1, R_1, and E, R are the respective pressures at the terminals and the resistances of the inductive and standard resistances; C is the current flowing through them; f is the frequency of the circuit; and L_1 is the inductance to be determined. Hence,

$$\frac{E_1^2}{E^2} = \frac{R_1^2 + 4\pi^2 f^2 L_1^2}{R^2},$$

and $L_1^2 = \frac{R^2 E_1^2 - R_1^2 E^2}{4\pi^2 f^2 E^2}$, or $L_1 = \frac{R_1}{2\pi f}\left(\frac{R^2 E_1^2}{R_1^2 E^2} - 1\right)^{\frac{1}{2}}$.

R_1 is measured by means of a Wheatstone bridge, or by some other usual method, and f is determined from the speed and number of poles of the alternator producing the pressure. The measurement of pressure at the terminals of the non-inductive resistance is equivalent to measuring the current which flows through the non-inductive resistance; for $C = \frac{E}{R}$. Substituting C for $\frac{E}{R}$ in the expression for L gives

$$L = \frac{R_1}{2\pi f}\left(\frac{E_1^2}{C^2 R_1^2} - 1\right)^{\frac{1}{2}}.$$

The current may be measured by an electrodynamometer, instead of taking the pressure at the terminals of a standard known resistance.

A modification of this method may be used to determine the working inductance of alternator armatures. Thus, first measure the pressure at the terminals of the alternator whe it and normally excited. This measurer de by a high-resistance voltmeter of n nce, such as a Weston voltmeter for nts, a Cardew voltmeter with a consider e resistance in series, or some type of e neter. The latter follow in general the Thomson (Kelvin) quadrant electrometer, nstructed as to be portable and direct reading. If the armature current does not have too great a demagnetizing effect on the field, the open circuit pressure may be taken as the total pressure which acts when the armature is connected to a circuit ; that is, it is the impressed pressure. Hence, connect the armature to a load composed of a known resistance with negligible or known constant inductance, and measure the current which flows. Then

$$C = \frac{E}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$$

if the curves of pressure and current are approximately sinusoidal. From this the inductance in the circuit is found to be

$$L = \left(\frac{E^2 - C^2R^2}{4\pi^2 f^2 C^2}\right)^{\frac{1}{2}}.$$

If the load be an inductive resistance, the value of the armature inductance is found by subtracting the known load inductance from the circuit inductance as determined above. For, the inductive pressure of the load is $2\pi fL'C$ and that of the armature is $2\pi fL''C$, while the total inductive pressure is $2\pi fLC$, which is equal to the sum of the other two. Hence,

$$2\pi fLC = 2\pi f(L' + L'')C,$$

and

$$L'' = L - L'.$$

If the armature reactions of the machine thus tested be considerable, the value of the inductance given is too great, but in their effect upon regulation, armature reactions and inductance are inextricably mixed, and therefore cannot be entirely separated.

This method of measuring inductance is a convenient one, as the instruments used are an electrodynamometer, or other amperemeter reading effective currents, and a voltmeter reading effective pressures, which are portable and may be used where convenience dictates. The result given by this method is the actual working inductance, which is an important feature when the circuit contains iron and the inductance therefore depends upon the volume of the testing current. On the other hand, the accuracy of the method is not great. Under the most favorable circumstances an accuracy of two or three per cent is attainable. This is sufficiently close for many purposes where the method may be advantageously used.*

2. *Comparison with Resistance by Bridge* (Method of Maxwell and Rayleigh). The coil of unknown self-inductance L and resistance R is placed in one arm of

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* Compare *London Electrician*, Vol. 33, p. 6.

a bridge (Fig. 39). The other resistance arms of the bridge are non-inductive and of values A, B, and R'. The bridge is first balanced in the usual way to determine the resistance R. With the balance for constant currents retained, the galvanometer key is depressed before the battery key. This causes a throw of the galvanometer r e pressure of self-inductance or reactar the coil. When C is the current in the ity of electricity passed

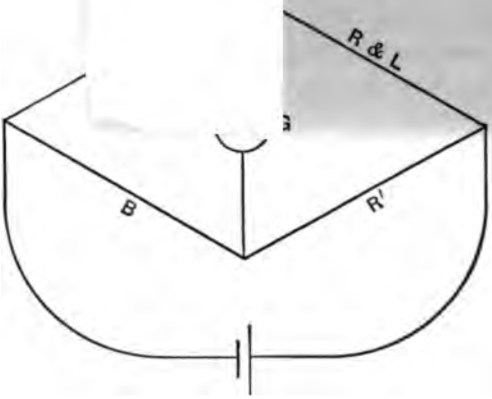

Fig. 39

through the bridge coils due to this pressure, is LC divided by the resistance of the bridge network (see Sect. 19). The flow of this electricity is through R and R', in series with the divided circuit made up of A plus B in parallel with G, where G is the resistance of the galvanometer. The resistance of the network is therefore $R + R' + \frac{G(A+B)}{G+A+B}$, and the quantity of electricity in coulombs which passes through the circuit is

$$Q = \frac{LC}{R + R' + \dfrac{G(A+B)}{G+A+B}}.$$

The proportion of this which passes through the galvanometer is

$$q : Q = \frac{G(A+B)}{G+A+B} : G, \text{ or } q = \frac{(A+B)Q}{G+A+B},$$

provided the galvanometer needle does not move appreciably until the impulse is past (Sect. 20). Hence,

$$q = \frac{LC}{R + R' + \dfrac{G(A+B)}{G+A+B}} \times \frac{A+B}{G+A+B} = K \sin \tfrac{1}{2}\theta,$$

where K is the ballistic constant of the galvanometer. Now the balance for steady currents is disturbed a small amount by the introduction of a small resistance r in the bridge arm with R. Suppose C' is the current which now flows through R; the effect of the disturbance of the balance is the same as though a steady electric pressure $C'r$, opposed to the battery pressure, had been introduced into the arm R'. The current flowing through the galvanometer on this account is

$$C_{,} = \frac{C'r}{R + R' + \dfrac{G(A+B)}{G+A+B}} \times \frac{A+B}{G+A+B} = K'\delta,$$

where K' is the galvanometer constant for steady currents and δ the deflection. From these two equations of flow, we get

$$\frac{C'r}{K'\delta} = \frac{CL}{K \sin \frac{1}{2}\theta},$$

whence $$L = r\frac{C'}{C}\frac{K \sin \frac{1}{2}\theta}{K'\delta}.$$

When the galvanometer is sensitive and r is made very small, the difference between C and C' becomes small and their ratio becomes sensibly equal to unity, whence $L = \frac{rK\theta}{K'\delta}$, provided the ballistic throw is sufficiently small. Since [illegible] uals $\frac{T}{2\pi}$ (Vol. I., pp. 17 and 39), this [illegible] $L = \frac{rT\theta}{2\pi\delta}$.

When $\frac{C'}{C}$ ca [illegible] ed as unity its value may evidently be ta [illegible] $\frac{A+R}{A+R+r}$. It is evident that a dead bea [illegible] r cannot be used in this work.*

3. *Comparison of Two Self-Inductances by Bridge* (Maxwell's Method). The two inductive resistances having self-inductances L and L' are connected in two arms of a bridge, together with variable resistances which are non-inductive (Fig. 40). We will call the resistances of these arms R and R'. The other arms of the bridge are non-inductive and of values A and B. First balancing the bridge in the usual way for steady currents, the proportion $R : R' = A : B$ is given. Now the galvanometer key is depressed before the battery key, and if the ratio of the impedances of the inductive arms is not equal to the ratio of A and B, the galvanometer needle will throw. R and R' must then be adjusted until a balance is obtained for transient currents. This being done, the balance for steady currents

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* See Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I., p. 322, and Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., p. 477.

must again be gained by adjusting A and B. This will again disturb the balance for transient currents, which must be adjusted by changing R and R'. This process of trial and approximation is repeated until the balance exists for both steady and transient currents, when

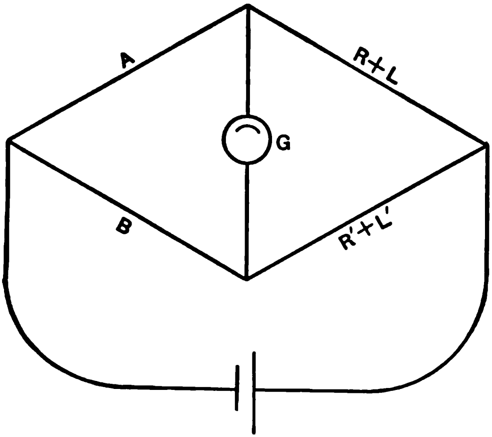

Fig. 40

$$\frac{\sqrt{R^2+\frac{4\pi^2L^2}{T^2}}}{\sqrt{R'^2+\frac{4\pi^2L'^2}{T^2}}}=\frac{A}{B}, \text{ or } \frac{R^2+\frac{4\pi^2L^2}{T^2}}{R'^2+\frac{4\pi^2L'^2}{T^2}}=\frac{A^2}{B^2}.$$

But
$$\frac{R^2}{R'^2}=\frac{A^2}{B^2}.$$

Hence,
$$\frac{L^2}{L'^2}=\frac{A^2}{B^2}, \text{ or } \frac{L}{L'}=\frac{A}{B}.$$

Various modifications of the bridge arrangement have been made in order to facilitate the balancing, but under the best circumstances the process is a laborious one.*

3*a*. (Ayrton and Perry's Standard Inductance.) The annoyances incident to making the adjustments required in the preceding method may be eliminated by making the sta djustable self-inductance. In this case, tl nced for steady currents by adjusting A alance for transient currents is then ltering any of the resistances by adjus f L'. This being done, we have as bef is method has been quite fully developed Ayrton and Perry,† who, following the m essor Hughes and Lord Rayleigh, designed a very satisfactory inductance standard. This consists of three coils, two fixed side by side and the third mounted so as to rotate within the others (Fig. 41). Calling the rotating coil A and the others B and C, evidently four arrangements can be made; thus, A may be connected with B alone, C alone, B and C in unison, or B and C in opposition. Each of these arrangements gives a maximum inductance when coil A lies within the plane of coils B and C and the field due to its current reinforces that due to the fixed coils. A smoothly graded variation of the inductance may then be gained by revolving coil A until a minimum value for the arrangement is reached with A at 180° from its preceding position. By means of a properly graduated circle the value of the inductance may be directly indicated for any position of A in either arrangement. If the ratio of the resistances of the unknown coil and the inductance standard does not give a value of $\frac{A}{B}$ which brings the required value of L' within the range of the standard, some non-inductive resistance may be included in one of the arms R or R'. Thus, suppose the unknown inductance to be nearly ten times as great as the highest value of the standard, then adjusting the resistances of R and R' so that $\frac{R}{R'}$ is greater than 10 makes $\frac{A}{B} = \frac{L}{L'}$ greater than 10, and the range of the standard inductance is sufficient. A somewhat sim-

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* Maxwell's *Electricity and Magnetism*, 2d ed., Vol. II., p. 367; Gray *Absolute Measurements*, Vol. II., p. 455.

† See *Jour. Inst. E. E.*, Vol. 18, p. 290.

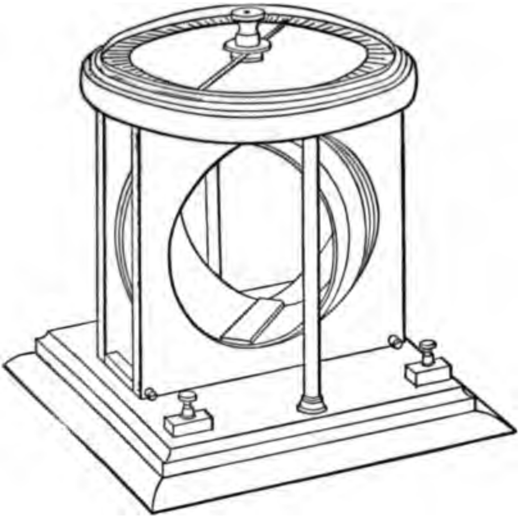

Fig. 41

950

ilar standard may be readily made by using two solenoids that telescope each other.

4. *Comparison with a Known Capacity.* In the case of condensers, the capacity may be said, in general, to be independent of the charge, that is, the capacity is constant. The ch nser (that is, the quantity
of electricity h directly proportional to
the difference een its plates; and if the
condenser has its terminals a transient
electric pressur arging is proportional to
the rate at wh changes. That is, the
current flowing r is proportional to the
rate of chang ire impressed upon it.
Therefore, as the pressure rises the current is flowing in a positive direction, and when the pressure reaches its maximum the current ceases. The phase of the charging current is consequently 90° in *advance* of the phase of the pressure impressed at the condenser terminals. If the condenser is shunted around a non-inductive resistance, the charging current is 90° in advance of the active pressure which causes current to flow through the resistance around which the condenser is shunted. In this respect a capacity is exactly the opposite of an inductance. If a conductor be shunted by a capacity, the quantity of electricity transferred in charging the condenser during a transient current is evidently $Q = sE = sCR$, where s is the capacity, R the resistance of the conductor, and C the current flowing through the latter. This quantity causes an apparent increase in the current passing through the conductor, and therefore an apparent decrease in the resistance of the con-

ductor (see Sect. 31). Here again the property of capacity is opposed to that of inductance, which when placed in a circuit increases its apparent resistance to a transient current. It is therefore possible to practically neutralize the effect of inductance by placing a proper condenser in circuit with it. If the inductance varies with the current, the capacity required for neutralization will also depend upon the current. Therefore, when alternating currents are used, the neutralization may be complete for the integral of the current taken over a full period, while the neutralization is by no means complete at any instant. The latter can be effected only by making a condenser with a capacity which varies with the charge in the same way as the inductance varies with the current.

These relations between the effects of a capacity and of an inductance lead to several methods of measuring the value of one in terms of the other. The original method suggested by Maxwell[*] is as follows: The unknown inductance is placed in the arm R of a bridge; the known capacity is shunted around a variable non-inductive resistance in arm B; and the arms R' and A are variable non-inductive resistances (Fig. 42). By a process of trial and approximation similar to that of the third method, a common balance is obtained for both steady and transient currents, when $L = R'As = RBs$.

This is proved as follows: When balance exists for steady currents, $RB = R'A$, while balance for transient currents also requires that at every instant

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[*] *Electricity and Magnetism*, 2d ed., Vol. II., p. 387.

$$Ac_A = Bc_B \text{ and } \frac{Ldc_R}{dt} + Rc_R = R'c_{R'}.$$

The charging current of the condenser when balance exists, is

$$c_{R'} - c_B = \frac{sBdc_B}{dt} = \frac{sAdc_A}{dt}$$

Whence

$$\frac{sAdc_A}{dt};$$

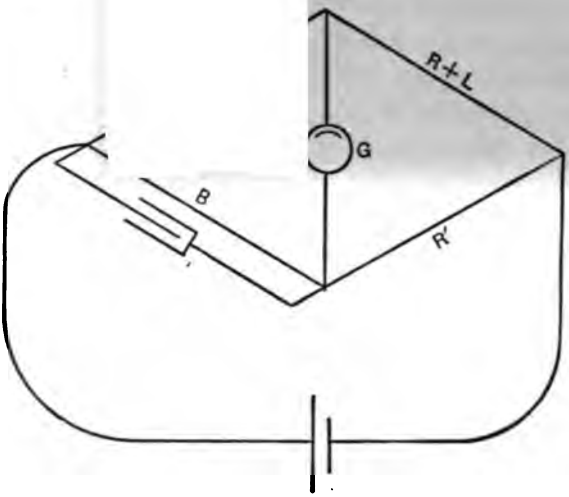

Fig. 42

but $c_B = \frac{Ac_A}{B}$ and $c_A = c_R$, so that

$$R'c_{R'} = \frac{R'Ac_R}{B} + \frac{R'sAdc_R}{dt},$$

or
$$\frac{BLdc_R}{dt} + RBc_R = R'Ac_R + \frac{R'sABdc_R}{dt}$$

nce $R'A - RB = 0$, this becomes $L = R'As = RBs$.

The correctness of this formula may also be seen from the fact that balance for transient currents only holds when the quantity of electricity transferred through the non-inductive half of the bridge is increased by the condenser by an amount equal to the deficit which is caused by the inductance in the other half of the bridge times $\frac{R}{R'}$, or $Q = se_B = \frac{R}{R'} \times \frac{L}{R} \times \frac{e_B}{R}$; hence, $L = RBs$.[*]

The formula $L = RBs$ may be written $\frac{L}{R} = Bs$, which shows that the time constants of the branches of the bridge which contain the inductance and capacity must be equal when the transient balance is obtained.

4*a*. (Pirani's Modification.) To avoid the annoyances incident to the adjustment of a simultaneous balance for steady and transient currents, the following modification of the fourth method is advantageous.

The three branches A, B, R' of the bridge contain non-inductive resistances only. The fourth branch contains the inductive resistance in series with a non-inductive resistance r (Fig. 43). The condenser is shunted around the latter. The balance for steady currents being obtained, the balance for transient currents is gained by changing the connections of the condenser so as to alter that portion of r which is shunted by the condenser. Then, if r' is the value in ohms of that portion (Fig. 43), $L = sr'^2$. For, to give a balance for transient currents the charging current of the condenser must be equal and opposite to the effect of the inductance in the circuit. Hence, if x represent the eter terminals between each closing and opening. The closing and opening of the battery circuit (or what is equivalent, the reversal of the battery) may be effected in synchronism with the reversals of the galvanometer by means of two commutators mounted upon a rotating shaft This is in effect the device designed by Professors ry, and called by them a **Secohmmeter** n diagrammatically in Fig. 44. The a bridge with standard variable induct mmeter are shown in Fig. 45. Whe er is used in comparing an inductance, her inductance or with a capacity, the ve the commutators rotate does not affect the result, except to vary the sensibility of the test, provided that time is given between the reversals for the current to rise to its full value. This is evident from the fact that the total quantity of electricity moved under the influence of self-inductance depends only upon the integral taken over the current-curve from zero to C, and from C to zero, and time does not enter as a factor of this total quantity. When, however, the secohmmeter is used in the second method, where an inductance is compared with a resistance, the number of reversals enters directly as a factor of the ult. The expression for the inductance is then

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[*] Hospitalier's *Traité de l'Énergie Électrique*, Vol. I., p. 469.

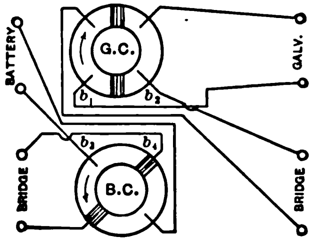

Fig. 44

Jour. Inst. E. E., Vol. 18, p. 284; *Electrical World*, Vol. 13, p. 232.

$$L = k\frac{r\beta}{V\delta}\frac{A}{B},$$

where β is the deflection when the secohmmeter is rotated at V revolutions per second, and the bridge is balanced for steady currents, while δ is the galvanometer deflection for steady currents when the balance is disturbed by altering B to $B + r$. k is a constant depending upon the relative angular positions of the two commutators, and can be determined by calibration.

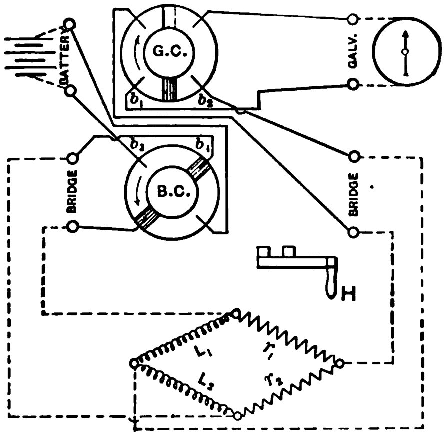

Fig. 45

When the secohmmeter is used, the galvanometer may always be dead beat, which gives an additional advantage to its use in the methods where it is required to read the galvanometer deflections for transient currents.

38. The Effect of a Varying Permeability in an Alternating-Current Circuit. — In the theoretical discussion of this chapter, the counter electric pressure in an electric circuit due to self-induction has been taken equal to $\frac{LdC}{dt}$, L being taken proportional to the permeability of the magnetic circuit. Thus, if e_1 and L_1 are the counter electric pressure and self-inductance of an electric circuit without an iron core, the formula gives

$$C$$

Now, if e_2 and unter pressure and self-inductance for n iron core is within the circuit, the forn

$$\frac{L_1dC}{dt},$$

where μ is the permeability of the magnetic circuit (compare Sect. 16). The formula $e_2 = \frac{L_2dC}{dt}$ is really incorrect, since μ varies with C, so that it should be

$$e_2 = \frac{d(L_2C)}{dt} = \frac{L_1d(\mu C)}{dt} = \left(\mu + C\frac{d\mu}{dC}\right)L_1\frac{dC}{dt}.$$

In using the formula $e_2 = \frac{L_2dC}{dt} = \mu L_1\frac{dC}{dt}$ we have omitted the effect on the counter electric pressure of self-induction, which is caused by the rate of change of permeability with the current. The magnitude of this is represented by the term

$$\frac{Cd\mu}{dC}\frac{L_1dC}{dt}, \text{ or } \left(\frac{Cd\mu}{\mu dC}\right)L_2\left(\frac{dC}{dt}\right).$$

In general, it may be said that $\frac{Cd\mu}{dC}$ is small compared with μ, for the values of the induction in iron which are ordinarily used in practice, and $\frac{Cd\mu}{\mu dC}$ is quite small compared with unity. Under some practical conditions it is possible that $\frac{Cd\mu}{dC}$ may be as great as $\frac{1}{5}$ or $\frac{1}{6}$ of the value of μ, but this is not common. The formulas as they have been worked out, therefore, may be accepted as indicative of the action in such circuits containing iron cores as are likely to be met in actual alternating-current machinery. The definition of self-inductance adopted by the Chicago Electrical Congress takes into account the variability of μ.

39. The Power expended in a Circuit on which a Sinusoidal Alternating Pressure is impressed. — If the circuit be without inductance or capacity the current wave agrees in phase with the pressure which sets it up. The rate of expenditure of energy in the circuit at any moment is equal to the product of the corresponding instantaneous current and pressure. The average rate of expenditure of energy, or the average value of the power expended, in the circuit during a complete period is equal to the average of all the instantaneous products. Or,

$$W = \frac{2}{T}\Sigma_0^{\frac{1}{2}T} ce, \text{ but } e = e_m \sin a \text{ and } c = \frac{e}{R},$$

where T is the time of a complete period and e_m is the maximum instantaneous pressure ordinate; hence,

$$W = \frac{2}{T}\Sigma_0^{\frac{1}{2}T} ce = \frac{e_m^2}{\pi R}\int_0^{\pi} \sin^2 a da = \frac{e_m^2}{2R},$$

but $$E = \frac{e_m}{\sqrt{2}} \text{ and } C = \frac{E}{R} = \frac{e_m}{\sqrt{2}R}.$$

Hence, $$W = \frac{e_m^2}{2R} = CE,$$

C and E being the effective values of the current and pressure.

If the circuit under consideration is reactive, the current is caused to lag behind or lead the pressure by the angle ϕ. The [illegible] ure of energy in the circuit at any inst [illegible] still equal to the product of the corresp [illegible] eous values of the current and press [illegible] ession for the average power expende [illegible] s therefore, as before,

$$[illegible]\ c.$$

In this case, however, $e = e_m \sin a$, and $c = \frac{e_m}{I} \sin (a \mp \phi)$ (Sect. 24), where I is the impedance of the circuit.

Hence, $$W = \frac{e_m^2}{\pi I} \int_0^\pi \sin a \sin (a \mp \phi)\, da$$

$$= \frac{e_m^2 \cos\phi}{\pi I} \int_0^\pi \sin^2 a da$$

$$\mp \frac{e_m^2 \sin\phi}{\pi I} \int_0^\pi \sin a \cos a da = \frac{e_m^2 \cos\phi}{2I},$$

and since $e_m = E\sqrt{2}$, and $\frac{e_m}{I} = c_m = C\sqrt{2}$, there follows

$$W = CE \cos \phi.^*$$

Assuming the current and pressure curves to have equal positive and negative loops (Sect. 80), the expression thus derived for the power expended in a circuit during one-half period applies to every half period, and therefore to continuous operation. In the ordinary measurement of current and pressure the effective values of the quantities are determined. Consequently, *the product of amperes and volts*, thus determined, *does not represent the power expended in a reactive circuit*, *but the product must be multiplied by the cosine of the angle of lag*. On the other hand, a **Wattmeter**, that is, an electrodynamometer with one coil of low resistance connected in series with the circuit and another coil of high resistance connected in shunt with the circuit, averages the instantaneous products, and therefore gives readings that are directly proportional to the power absorbed.

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* Compare Picou's *Machines Dynamo Électriques*, p. 261; Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I., p. 224; Kapp's *Alternating Currents of Electricity*, p. 46; etc.

40. Method for Measuring the Angle of Lag. — We have here a ready method for determining the angle of lag of the current flowing in a circuit. Measure the current flowing in a circuit by an electrodynamometer; measure the pressure at its terminals by an electrostatic voltmeter or some type of non-inductive voltmeter of very high resistance. Finally, measure the power absorbed in the circuit by means of a wattmeter the pressure coil of which is non-inductive and of very high resistance. The power in watts determined by the wattmeter when divided by the product of volts and amperes gives the cosine of the angle of lag. If the curves of current and pressure are of irregular form this measurement will give the angle of lag between the equivalent sine curves (Sect. 30). We will later take up the effect of inductance in the pressure coil of the wattmeter (Sect. 45).

41. Blakesley's Graphical Proof.—Blakesley has given a neat proof of the formula $W = CE \cos \phi$.* Returning to the graphical representation of alternating pressures or currents by means of rotating lines, let AB and AC (Fig. 46) repr y the maximum value of

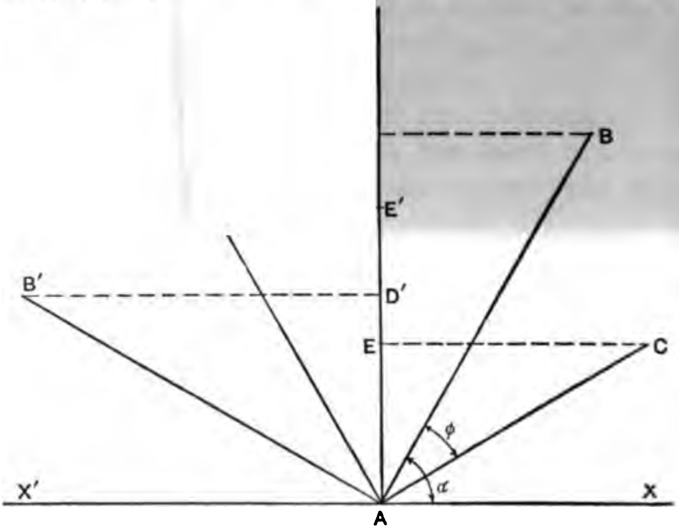

Fig. 46

the impressed electric pressure in a circuit and the maximum value of the resulting current. The angle BAC is the angle of lag. If the lines rotate about the point A, counter-clockwise, the instantaneous projections of the lines AB and AC upon the axis of Y represent the instantaneous values of the pressure and current, when a is measured from the X axis. It is therefore desired to determine the average value of the products of these projections. Draw AB' and AC' respectively perpendicular and equal to AB and AC. These lines represent the positions of AB and AC after revolving through 90°. In the figure the angle BAX represents a, and CAX represents $a - \phi$. Also the angle $B'AD' = BAX$, and $C'AE' = CAX$. It is then seen from the figure that

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* Blakesley's *Alternating Currents of Electricity*, 2d ed., p. 6.

$$AE \times AD = AC \sin CAX \times AB \sin BAX,$$

or $$ce = c_m \sin (a - \phi) \times e_m \sin a;$$

and in the same way

$$AE' \times AD' = AC' \cos C'AE' \times AB' \cos B'AD',$$

or $$c'e' = c_m \cos (a - \phi) \times e_m \cos a.$$

The mean of these expressions is

$$\frac{ce + c'e'}{2} = \frac{c_m e_m}{2} [\sin a \sin (a - \phi) + \cos a \cos (a - \phi)]$$

$$= \frac{c_m e_m}{2} \cos [a - (a - \phi)] = \frac{c_m e_m}{2} \cos \phi = CE \cos \phi.$$

This is the expression for the mean of the products of e and c for two values of a which are 90° apart. This mean value is independent of the positions of the lines in the figure, and is therefore the mean for all positions.*

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* The maximum power that can be expended in an inductive circuit when a given pressure is applied, may be shown thus: $W = CE \cos \phi$, and since $\cos \phi = \frac{R}{I}$, where I is impedance, and $C = \frac{E}{I}$, there results $W = \frac{E^2 R}{R^2 + 4\pi^2 f^2 L^2}$. This is a maximum when $R = 2\pi f L$. Hence, $\phi = 45°$,

Power loops or curves may be plotted as in Figs. to 50, the ordinates of which represent the produc of the corresponding ordinates of the current and pres ure curves. Figure 47 shows the power loops for a no inductive circuit in which the pressure and current r verse their directions at the same time, and the pow

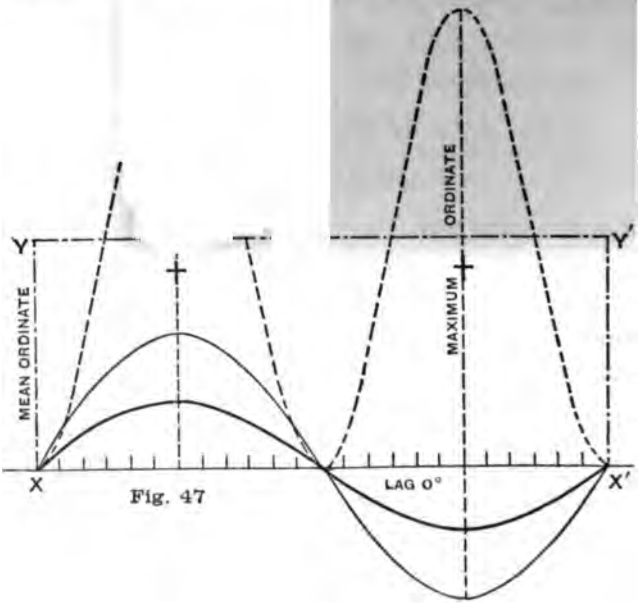

Fig. 47

ordinates are therefore always positive but their nume ical value varies in each half period from 0 to $c_m e_m$ ar back to 0, so that the power absorbed by the circu

---

and the power factor is 70.7 per cent. This expression for maximu power is of no practical importance, as in the operation of electrical c cuits and machinery the highest possible operating efficiency or *pla efficiency* is usually desired. A high plant efficiency is incompatible wi a low power factor.

varies continually during each half period. In this case $\phi = 0$, $\cos\phi = 1$, and the average power is $W = CE$. Figure 48 shows the power loops for a reactive circuit in which the angle of lag is 45°. This may be taken to equally represent the condition when the current leads or lags. It will be seen in this case, that during a portion of each half period the current and pressure are in opposite directions, and some of the ordinates of the power loops are therefore negative. This must always be the case when the current and pressure do not coincide in phase. During the portion of the half period in which the ordinates of the power loop are positive the circuit absorbs power, but during the portion in which the ordinates are negative the circuit gives out power which was stored as magnetic field or condenser charge, and returns it to the source. The total energy given to the circuit during the half period is equal to the differ-

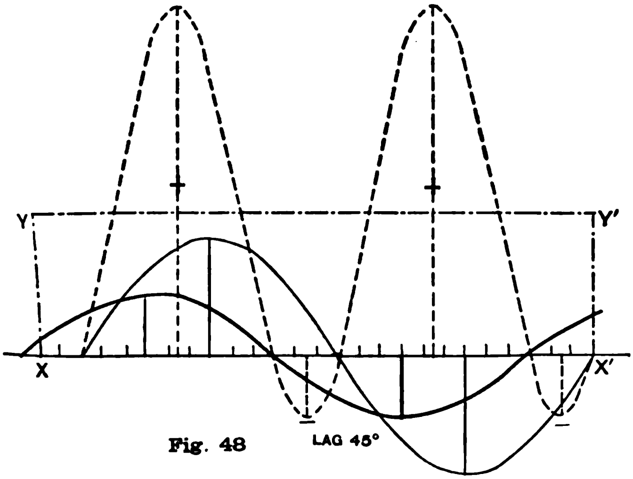

Fig. 48

ence of that represented by the positive and negative loops, and the average power absorbed by the circuit is equal to this difference divided by the length of the half period. When $\phi = 45°$, $W = CE \cos 45° = .707\, CE$. Figure 49 shows the power loops for a circuit in which the current and pressure differ in phase by 90°. In this case the negative [illegible] to the positive ones, or the circuit an[illegible]tely give and take equal

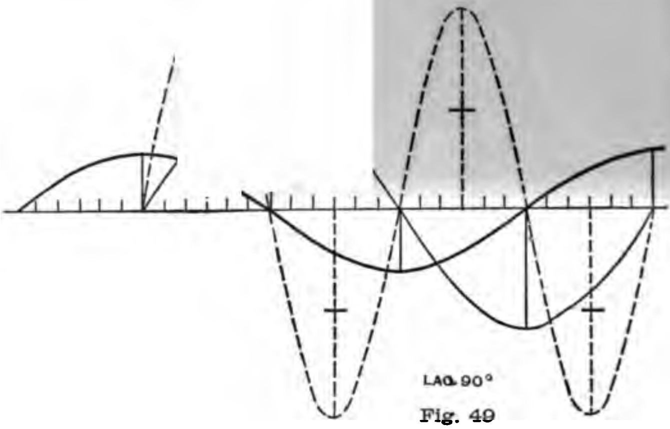

Fig. 49

amounts of energy, so that taking each half period as a whole, no power is absorbed by the circuit. In this case $\cos \phi = 0$, and therefore $W = 0$.

42. Definition of Power Factor. — The product of the effective values of the current and pressure, CE, in a reactive circuit is called the **Apparent Energy** or **Apparent Watts** in the circuit. The reading of a wattmeter applied to the circuit, which gives the value of $CE \cos \phi$, gives the **True Energy** or **True Watts** in the circuit. The ratio of the true watts to the apparent watts in a circuit is generally called the **Power Factor,** as originally suggested by Fleming. The power loops for a circuit are exactly symmetrical, provided the original current and pressure curves are sinusoids (Figs. 47 to 49). When the pressure and current are in unison of phase the average ordinate of the power loops is equal to one-half of the maximum ordinate, since the maximum ordinate is equal to $c_m e_m$, and the average ordinate is equal to $CE = \frac{c_m e_m}{2}$. When the original curves are not in unison of phase, the average power ordinate is equal to one-half of the difference between the maximum positive ordinate and the maximum negative ordinate.* When the current and pressure curves are not sinusoids, the power loops are not symmetrical and the average power ordinate does not necessarily depend at all upon the maximum ordinate (Fig. 50). It is evident from the power loops that the torque on an alternator armature which is delivering current to a circuit varies from zero to a maximum value which is much greater than the average. The torque on a continuous-current machine is uniform, and the armatures of alternators are therefore subjected to severer strains than are continuous-current armatures.

43. Wattless Current.—The preceding expressions show that the energy expended in an inductive circuit is equal to the effective value of the impressed pressure and a component of the current which is in phase with the pressure, and has a value of $C \cos \phi$. This may be called the **Active** or **Working Current.** The remaining component of the current does no work, and therefo must be in quadrature with the pressure. This gives a value of $C\cos(\phi+90^\circ)=C\sin\phi$. This componen which does no work during a full period, is often calle the **Wattless or Idle Current.** For illustration, suppos in Fig. 51 that OS is a pressure applied to a circuit, O the current gle. Resolving OA into it

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* Fleming's *Alternate Current Transformer*, Vol. I., p. 124.

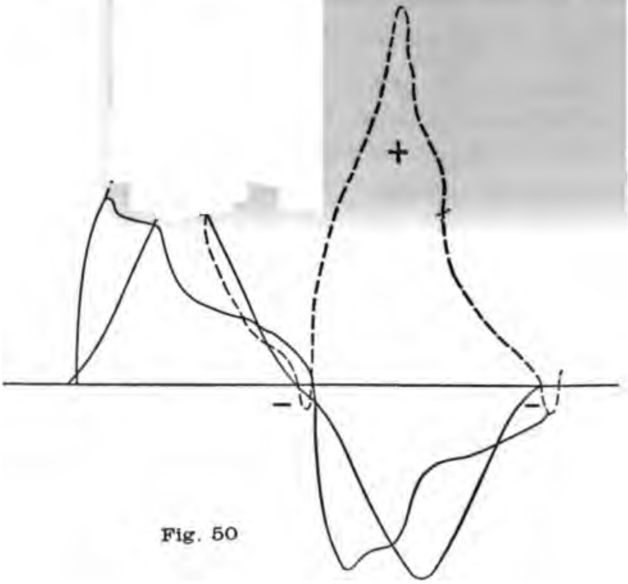

Fig. 50

components, OW and OI, in phase with and at rig angles to OS, the component OW multiplied by t pressure will give the power absorbed by the circuit, a OI will be wattless.* If ϕ were 90°, the total curre would be in quadrature with the pressure and therefc wattless. While a wattless current may do a considerable amount of work in one quarter period, during the next quarter period the circuit returns an equal amount, and the total work for the period is zero. (Compare power loops.) A lag of 90° would only be possible in a circuit having no electrical resistance, since otherwise some energy would necessarily be expended in heating the conductors. It is possible, however, to make the ratio of inductive resistance, $2\pi fL$, so great in comparison with the true resistance R, that the lag is very nearly 90°. It is also possible to make the capacity of a circuit so great in comparison with its resistance that ϕ is a lead of nearly 90°. The latter condition is one not met in practice, but the former may quite easily be brought about in circuits including underloaded transformers of poor design.

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* Thompson's *Dynamo-Electric Machinery*, 4th ed., p. 636.

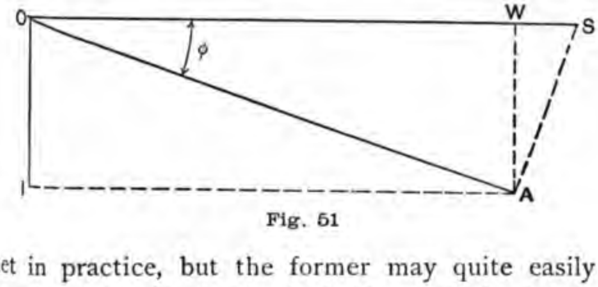

Fig. 51

The value of the power factor of a circuit is evidently equal in numerical value to cos ϕ, for, power factor equals

$$\frac{\text{True Watts}}{\text{Apparent Watts}} = \frac{CE\cos\phi}{CE} = \cos\phi.$$

The total current in a circuit multiplied by the power factor is, therefore, equal to the active component of the current. A factor, which in the same way is proporof the potential at the points when varying synchronously as a sine function, the deflection becomes

$$d = \frac{k}{T}\int_0^T (v_1 - v_2)\left(v_3 - \frac{v_1 + v_2}{2}\right)dt.$$

If it is desired to measure the energy absorbed by an inductive circui [illegible] ter may be used in the following mann [illegible]

The inductiv [illegible] is connected in series with the non-in [illegible] e AB (Fig. 52). Let the potential of the [illegible] nd C at any instant be represented res [illegible] v_2, and v_3, when B is the junction betwee [illegible] and non-inductive resistance. Then if [illegible] ctrometer be connected with its quadrants to A and B, and its needle and case to C (Fig. 52 *a*), the deflection is

$$d = \frac{k}{T}\int_0^T (v_1 - v_2)\left(v_3 - \frac{v_1 + v_2}{2}\right)dt.$$

If the connection of the needle be interchanged so that it is connected to B while the connections of the quadrants remain unchanged (Fig. 52 *b*), this becomes

$$d' = \frac{k}{T}\int_0^T (v_1 - v_2)\left(v_2 - \frac{v_1 + v_2}{2}\right)dt.$$

By subtraction, this results in

$$d' - d = \frac{k}{T}\int_0^T (v_1 - v_2)(v_2 - v_3)\,dt.$$

Dividing this by kR, where R is the resistance of AB, gives

$$\frac{d' - d}{kR} = \frac{1}{T}\int_0^T \frac{v_1 - v_2}{R}(v_2 - v_3)\,dt.$$

Now $\frac{v_1 - v_2}{R}$ is equal to the instantaneous value of the current passing through the circuit, and $v_2 - v_3$ is the

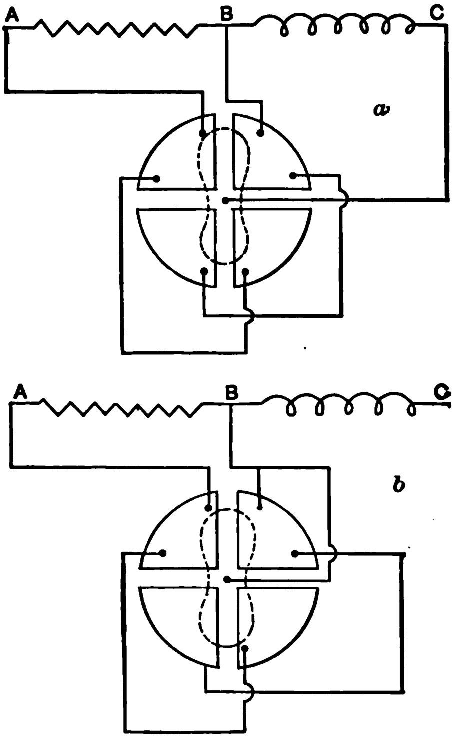

Fig. 52

instantaneous value of the difference of pressure between the terminals of the inductive resistance *BC*.

Consequently, $\frac{d - d}{kR} = \frac{1}{T}\int_0^T ccdt = W$, where W is the power absorbed by the inductive part of the circuit.*

On account of structural defects, the deflections of electrometer needles do not always follow the theoretical law. Consequently, it is necessary to determine how great the deviation is before the instrument may be relied upon.† nt may be calibrated by the use of cont passing through known resistances, whi ed that v_1, v_2, and v_3 are nearly the effec e tests.

1 *a*. *Electrost* A modification of the quadrant electro ade which reads directly as a wattmeter.‡ he needle box is divided diametrically into two parts instead of into quadrants. The needle consists of a disc divided diametrically into two parts (Fig. 53). The parts of the circuit A and B are connected to the two halves of the needle, and B and C to the two halves of the needle box.

Then the force which causes the deflection of the needle is theoretically proportional to the product

$$(v_1 - v_2)(v_2 - v_3), \text{ or } W = \frac{d}{kR} = \frac{1}{T}\int_0^T \frac{(v_1 - v_2)}{R}(v_2 - v_3)\,dt.$$

This instrument may also be calibrated, as explained above, by passing a known continuous current through a known resistance. A wattmeter of this type has been designed by Swinburne which may be made direct reading or may be read by means of a torsion head so that the needles will always remain in a fixed position in relation to the needle boxes.*

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* Ayrton, *Jour. Inst. E. E.*, Vol. 17, p. 163; Gray's *Absolute Measurements*, Vol. II., p. 698; Swinburne, Note on Electrostatic Wattmeters, *London Electrician*, Vol. 26, p. 571.

† Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., pp. 662 and 699.

‡ Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I., p. 611; Hospitalier's *Traité de l'Énergie Électrique*, Vol. I., pp. 205 and 567.

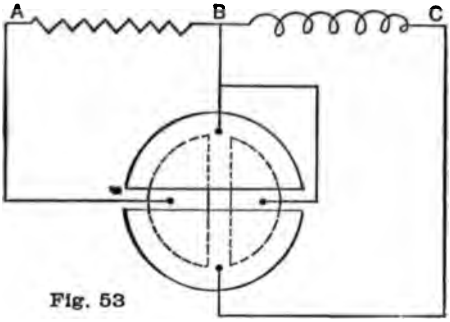

Fig. 53

2. *Three-Voltmeter Method.*† As in the previous method, a non-inductive resistance must be connected in series with the inductive circuit to be tested. (Fig. 54.) Voltmeters are then respectively connected between the points A and B, B and C, and A and C. Letting e_1, e_2, and e represent instantaneous pressures at the three voltmeters, then $e = e_1 + e_2$, whence

$$e^2 - e^2_1 - e^2_2 = 2\,e_1 e_2.$$

But the instantaneous value of the power in the inductive circuit is $w = ce_2 = \frac{e_1}{R}\,e_2$. Substituting the value of e_1e_2 already found gives $w = \frac{1}{2R}(e^2 - e_1^2 - e_2^2)$, and the mean power absorbed during a period is

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* Swinburne, Note on Electrostatic Wattmeters, *London Electrician*, Vol. 26, p. 571; *Electrical World*, Vol. 17, p. 257, and Vol. 19, p. 44.

† Suggested by Ayrton and Sumpner, *London Electrician*, Vol. 26, p. 736; *Electrical World*, Vol. 17, p. 329.

$$W = \frac{1}{T}\int_0^T w\,dt = \frac{1}{2RT}\int_0^T (e^2 - e_1^2 - e_2^2)\,dt$$

$= \frac{1}{2}$ $^2),$

where E, E_1, respective readings of
the voltmeters. nown and the value of
the current C i nula may be written

$E_1^2 - E_2^2).$

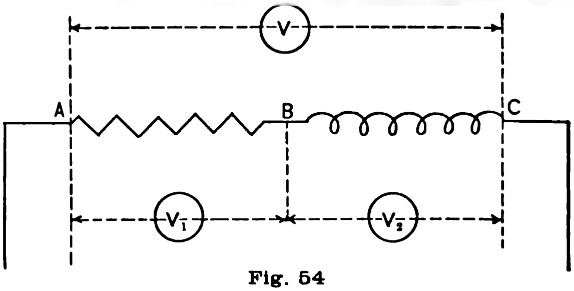

Fig. 54

In order that the results of measurements may be the most accurate possible, E_1 should equal E_2, which makes the method inconvenient for use in ordinary testing. Neither is the method sufficiently accurate to compensate for its disadvantages. The accuracy of any particular measurement made by this method may be checked by inserting a known non-inductive resistance in place of the inductive circuit.

3. *Three-Ammeter Method.** Instead of putting a non-inductive resistance in series with the inductive circuit, it may be placed in parallel with it. In this case amperemeters must replace the voltmeters of the preceding method (Fig. 55). One amperemeter measures the whole current C, another measures the current C_1 in the non-inductive resistance R, and another measures the current C_2 in the inductive circuit. It is evidently essential that the amperemeter which is in series with the non-inductive resistance shall be of negligible inductance. Supposing c, c_1, c_2 be the instantaneous values of the currents at any moment, we have $c = c_1 + c_2$ and $c^2 - c_1^2 - c_2^2 = 2\,c_1c_2$, while

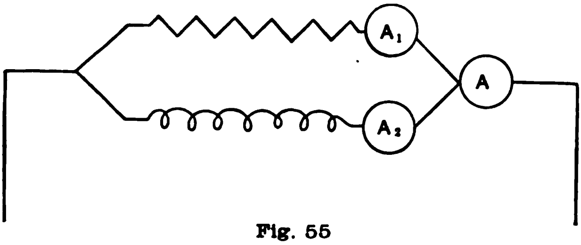

Fig. 55

$$w = ec_2 = Rc_1c_2 = \frac{R}{2}\,(c^2 - c_1^2 - c_2^2).$$

Whence $$W = \frac{1}{T}\int_0^T w\,dt = \frac{R}{2}\,(C^2 - C_1^2 - C_2^2).$$

This may also be written,

$$W = \frac{E}{2\,C_1}\,(C^2 - C_1^2 - C_2^2).$$

In this case the greatest accuracy is given when C_1 and

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* Suggested by Fleming, *London Electrician*, Vol. 27, p. 9; *Electrical World*, Vol. 17, p. 423.

ncations of the last two methods ha
by Ayrton, Sumpner, Blakesley, and
the obvious arrangements is to omit

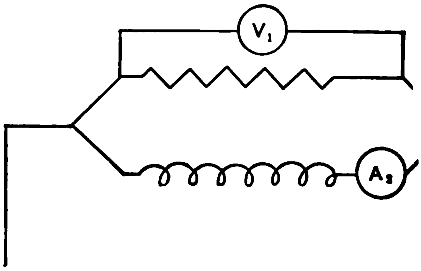

Fig. 56

in series with the non-inductive resista
method, and connect a voltmeter acros
in Fig. 56. In this case, the power b

$$W = \frac{R}{2}\left(C^2 - C_2^2 - \frac{E^2}{R^2}\right).$$

5. *Split Dynamometer Methods.* If
nating currents of the

$$c_1 = \sqrt{2}\, C_1 \sin a \quad \text{and} \quad c_2 = \sqrt{2}\, C_2 \sin (a - \phi),$$

where ϕ is the angle between the two current waves. Substituting, gives

$$\frac{1}{T}\int_0^T c_1 c_2\, dt = \frac{2}{T}\int_0^T C_1\, C_2 \sin a \sin (a - \phi)\, dt,$$

which is equal to

$$\frac{2\, C_1\, C_2}{\pi}\int_0^\pi \sin a \sin (a - \phi)\, da = C_1\, C_2 \cos \phi.$$

An electrodynamometer used in this manner is called a **Split Dynamometer.** Now suppose we determine the values of C_1 and C_2 by means of the dynamometer used as an amperemeter or by other instruments, then the value of $\cos \phi$ is at once found. If the measurements are all made by the same electrodynamometer, its constant does not need to be known. Suppose the readings in the two circuits are $C_1^2 = k\delta_1$ and $C_2^2 = k\delta_2$, and the reading as a split dynamometer is $C_1 C_2 \cos \phi = k\delta_3$, then $\cos \phi = \frac{\delta_3}{\sqrt{\delta_1 \delta_2}}$. This plan was first suggested by Blakesley.*

5*a*. Blakesley planned various methods for using a split dynamometer in measuring the power absorbed by an inductive circuit.† In one of the methods a non-inductive resistance is connected in parallel with the inductive circuit to be tested (Fig. 57), and a split dynamometer is connected so that one coil carries the total current, and the other carries the current of the inductive branch. An amperemeter is also placed in the induc-of $\frac{E}{R_1}$, where E is the pressure in the circuit, ϕ_1 the angle of lag in the pressure coil which is dependent on the relation $\tan\phi_1 = \frac{2\pi f L_1}{R_1}$, and R_1 and L_1 are respectively the resistance and self-inductance of the pressure coil. The currents in the series and pressure coils now have a difference of p ch is equal to $\phi - \phi_1$ instead of ϕ, wh ϕ ngle of lag in the main circuit. The inductive wattmeter is therefore propo s $\phi_1 \cos(\phi - \phi_1)$, while a correct reading is propo to $CE\cos\phi$. The readings of an inductive wat must therefore be multiplied by a factor equal to

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* *Alternating Currents of Electricity*, 2d ed., p. 97.

† *Phil. Mag.*, Vol. 31, p. 346.

$$\frac{\cos\phi}{\cos\phi_1\cos(\phi-\phi_1)} = \frac{\cos\phi}{\cos\phi_1(\cos\phi\cos\phi_1 + \sin\phi\sin\phi_1)},$$

in order that they may give the true power. This multiplier may be called the "correcting factor" of an inductive wattmeter. But

$$\cos\phi = \frac{R}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}, \quad \sin\phi = \frac{2\pi f L}{\sqrt{R^2 + 4\pi^2 f^2 L^2}},$$

$$\cos\phi_1 = \frac{R_1}{\sqrt{R_1^2 + 4\pi^2 f^2 L_1^2}}, \quad \sin\phi_1 = \frac{2\pi f L_1}{\sqrt{R_1^2 + 4\pi^2 f^2 L_1^2}},$$

and the correcting factor therefore reduces to *

$$\frac{RR_1^2 + 4\pi^2 f^2 L_1^2 R}{RR_1^2 + 4\pi^2 f^2 L L_1 R_1} = \frac{1 + (2\pi f)^2 \tau_1^2}{1 + (2\pi f)^2 \tau\tau_1}.$$

---

* Fleming's *Alternate Current Transformer*, Vol. I., p. 139; Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., p. 680; Loppé et Bouquet's *Courants Alternatifs Industriels*, Vol. I., p. 55.

The formulas show that *when* τ_1 *is negligibly small* (in which case ϕ_1 is practically equal to zero), *or* τ_1 *is equal to* τ (in which case $\phi_1 = \phi$), the correcting factor reduces to unity, and *the readings of the wattmeter are directly proportional to power. When* τ_1 *is less than* τ, the correcting factor is less than unity, and *the wattmeter reads too high,* and *when* τ_1 *is greater than* τ, the correcting factor is greater than unity, and *the wattmeter reads too low.* The indications of an inductive wattmeter may, therefore, be either correct, too high, or too low, depending upon the algebraical value of the time constant of the circuit upon which measurements are being made. As a general rule, the time constant, τ, of the circuit is likely to be positive and greater than that of the wattmeter, so that the readings of an inductive wattmeter are generally found in practice to be too high; but in ordinary measurements it is impossible to determine the value of τ, so that the correcting factor of the wattmeter is unknown. The only safety in wattmeter measurements of power in alternating-current circuits, therefore, lies in the use of a wattmeter with such a very small time constant in the pressure coil that it may be considered absolutely negligible.

Another correction due to the power used by the wattmeter itself is also necessary. Thus, if the pressure coil be connected to the circuit between the current coil and the test circuit (Fig. 58), it is evident that the power measured includes that absorbed by the pressure coil. If the current coil be included between the point of connection of the pressure coil and the test circuit (Fig. 59), the power measured includes

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break in order to avoid burning u
There are four methods of reducing

a. The break may be made gradu
resistance into the circuit before th
This resistance should vary gradual
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This device is used quite largely t
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contact is of much higher resistance
per contact, and the extra current sp
flowing through it. Therefore wher
tact is broken but little spark passe
pass causes comparatively little burni
of the switch which may be readily r

b. A coil of high resistance and wc

break, and thus the spark is reduced or entirely suppressed.

c. The switch may be shunted by a fine wire which acts as a fuse. When the switch is opened, breaking the circuit, the extra current spends itself by flowing through the fine wire shunt, which it burns off at the same time. This arrangement makes it necessary to replace the fuse before the time of each break. The arrangement is used to some extent upon the fuse blocks (Fig. 62) intended for use in high-pressure electric-light mains. The main fuse of comparatively low resistance is shunted by a fine high-resistance fuse.

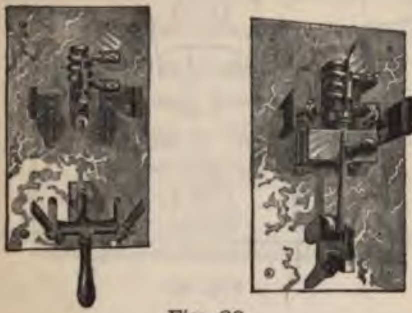

Fig. 60

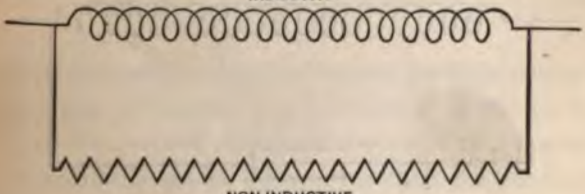

Fig. 61

that absorbed by the current coil. In either case thi power should be small and usually may be neglected but when this is not the case it is easily determine from the resistance of the coil included, if the pressure or current is known. In some wattmeters special correcting coil wound over the series coil is introduced in [illegible] ressure coil which correct

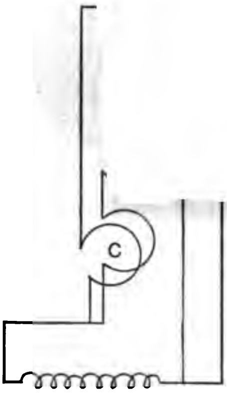

Fig. 58

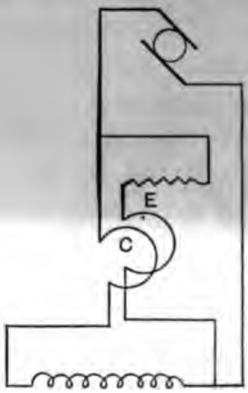

Fig. 59

for the current in the pressure coil, the instrument being connected as in Fig. 58. (Example: Weston wattmeter.)

As in the case of the electrodynamometer, or othe instruments operated by electrodynamic action, it i necessary that a wattmeter of the type here discusse shall have no metal in its frame in which foucault currents may be developed (Sect. 73). If this precautio is not carefully looked after, the constant of the instrument will vary with the frequency, and a calibration i necessary for every frequency. For a properly buil wattmeter, which is used at a point near which there are no masses of metal, a single calibration with continuous currents is sufficient.

46. The Spark caused by breaking a Self-Inductive Circuit.—It is to be expected (sêe Sect. 19) that a severe spark will pass upon breaking a circuit when it is carrying a continuous current, if it has a great self-inductance, since the self-generated electric pressure tends to uphold the falling current. This is indeed a well-known effect observed upon breaking circuits containing self-inductance. It is seen in exaggerated form in circuits containing such enormous self-inductances as those found in dynamo field windings. Again, breaking the external circuit of a continuous-current series dynamo causes a much more severe spark than breaking the external circuit of a shunt machine. In the latter case the *extra current*, or transfer of electricity due to the self-induction, flows from the field coils through the armature instead of attempting to jump across the break. It may therefore be dangerous to break the circuit of a series dynamo even while the normal working pressure is entirely harmless, while no special danger is likely to come from breaking the external circuit of a shunt dynamo. On the other hand, it is possible to get an exceedingly severe shock by breaking the field circuit of a shunt dynamo in which the working pressure may be quite low. The high pressure due to self-induction which is generated in the shunt field coils when the circuit is broken is a frequent source of injury to the insulation. The extra current, having no outlet, makes one by jumping from

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eak in order to avoid burning up the
here are four methods of reducing the

a. The break may be made gradually
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This device is used quite largely to redu
used by opening switches or automatic
s in high-pressure electric-light or electri
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r contact, and the extra current spends i
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ct is broken but little spark passes, whil
ss causes comparatively little burning up
the switch which may be readily renewe

b. A coil of high

break, and thus the spark is reduced or entirely suppressed.

c. The switch may be shunted by a fine wire which acts as a fuse. When the switch is opened, breaking the circuit, the extra current spends itself by flowing through the fine wire shunt, which it burns off at the same time. This arrangement makes it necessary to replace the fuse before the time of each break. The arrangement is used to some extent upon the fuse blocks (Fig. 62) intended for use in high-pressure electric-light mains. The main fuse of comparatively low resistance is shunted by a fine high-resistance fuse.

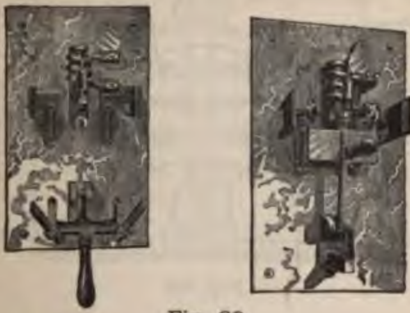


Fig. 60

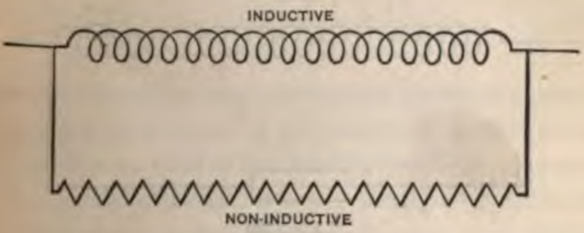
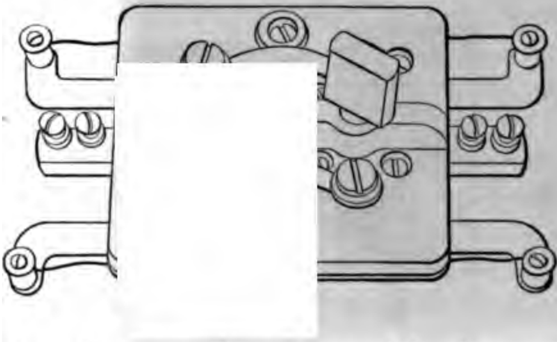


Fig. 61

When the main fuse blows out, the extra current, instead of causing a vicious spark, spends itself by flowing through the shunt fuse, at the same time blowing it out.



d. A condenser may be so arranged that it neutralizes the effect of the self-inductance at the time of the break. This may be done in two ways: 1. The condenser may be connected in parallel with the inductive circuit (Fig. 63). Then upon the break the capacity of the condenser tends to neutralize the effect of the inductance since the charging current of the condenser due to the rise of inductive pressure is opposite in direction to the extra current due to self-inductance, and the spark i therefore reduced or suppressed. 2. The condense

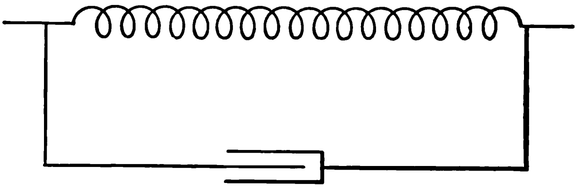


Fig. 63

may be connected in parallel with the switch (Fig. 64). In this case the extra current flows directly into the condenser, and the spark is reduced or suppressed. The effect of a condenser of fixed capacity in suppressing the spark at break due to a fixed self-inductance is evidently the same in the two positions. Upon closing the circuit, however, the condenser assists the rise of current in the circuit when in the first position, but has no effect whatever when in the second position, since it is short-circuited when the switch is closed. A condenser is ordinarily connected across the terminals of the primary circuit-breaker of a Ruhmkorff induction coil.

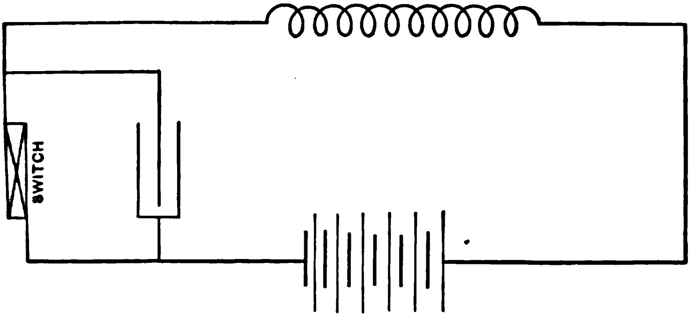


Fig. 64

There is a marked difference between the amount of spark ordinarily produced upon breaking a continuous current and an equal alternating one. For instance, breaking a continuous current of 25 amperes at 1000 volts pressure upon an ordinary hand switch without an especially quick break is likely to cause a lively arc, while breaking an equal alternating current ordinarily causes little more than an observable spark. Sometimes, however, a destructive arc is caused in breaking an alternating current. This is particularly true when fuses blow in high-pressure alternating-current mains where the metallic vapor from the fuse serves as a path for the arc. This difference in behavior on the part of alternating-current circuits is due to the fact that the circuit may be [illegible] rent instants when the current, and tl[illegible] t up by it, have widely different value[illegible]

47. The Sel[illegible] rallel Wires.— The self-inductance of [illegible] s, hanging upon a pole line or otherwi[illegible] roduces serious difficulties into the o[illegible] -distance telephones or telegraphs. In the ordinary alternating systems for lighting and the transmission of power, the effects are not so serious, though when the transmission is over a long distance the self-inductance of the line cannot be neglected. An expression for the self-inductance of two parallel wires may be developed thus: suppose that two parallel conductors A and A' form a circuit of indefinitely great length. Let C be the current flowing through the conductors, r their radius, and ∂ the distance between their axes. Also let μ and μ' be respectively the permeability of the medium surrounding the wires and of the wires themselves. The strength of the magnetic field (H_a) at a point outside of the conductor A at a distance a from its centre, and due to the current in A is

$$H_a = \frac{2\,C}{a}.*$$

---

* The lines of force due to the current in the conductor are circles with their planes perpendicular to the conductor, and if the force at any point

The magnetic induction (B_a) at the point is therefore

$$B_a = \frac{2\,\mu C}{a}.$$

Now consider a space cut out by two planes perpendicular to the axes of the conductors and one centimeter

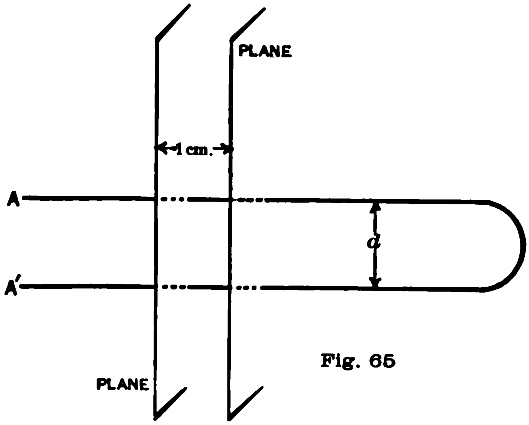


Fig. 65

apart (see Fig. 65). Within this space at a distance a from A a number of lines of force

$$B_a da = dN_a = \frac{2\,\mu C da}{a}$$

will pass through a radial width da. The total number of lines of force that will pass between the planes in the distance between the surface of A and the centre of A' will be,

$$N_a = \int_r^{\partial} \frac{2\,\mu C da}{a} = 2\,\mu C \log_e \frac{\partial}{r}.$$

---

at a distance a from the conductor is F, the work done against it in moving a unit magnet pole around the conductor is $W = 2\,\pi a F$. Also by Vol. I., p. 12, $W = 4\,\pi n C$, but in this case, $n = 1$, and therefore $2\,\pi a F = 4\,\pi C$, or $F = \frac{2\,C}{a} = H$.

At any point p within A and at a distance b from the centre of A, the magnetic effect will be as though the current within a circle of radius b were condensed at the centre, since the magnetic effect at the point, p, of the uniform layer of current in the conductor beyond b will be zero. The strength of field at p may therefore be wri

$$= \frac{2Cb}{r^2},$$

and the magneti ,

Proceeding as before,

$$B_c db = dN_c = \frac{2\mu' Cbdb}{r^2},$$

and

$$N_c = \int_0^r \frac{2\mu' Cbdb}{r^2} = \mu' C.$$

But this induction does not link with the whole current, but only with that within a circle of radius b. The product of the current with the number of lines of force enclosed by it is

$$\Sigma dN_c dC_c = \int_0^r \frac{2\mu' Cbdb}{r^2} \times \frac{\pi b^2}{\pi r^2} = \frac{1}{2}\mu' C.$$

The self-inductance of A per unit length is equal to 10^{-9} times the number of lines of force linking the current when it has a value of unity (Sect. 16), and therefore,

$$L_A = \left(2\mu \log_e \frac{d}{r} + \frac{\mu'}{2}\right) \div 10^9.$$

The effect of the return conductor A' is to exactly double the magnetism which is linked or enclosed by the current, and therefore the self-inductance per centimeter length of circuit or per two centimeters length of wire is

$$L_{AA'} = 2\left(2\,\mu \log_e \frac{\partial}{r} + \frac{\mu'}{2}\right) \div 10^9.$$

When the conductors are of copper suspended in the air, $\mu = \mu' = 1$, and

$$L_{AA'} = 2\left(2 \log_e \frac{\partial}{r} + \frac{1}{2}\right) \div 10^9.\text{*}$$

As a rule the value of $2 \log_e \frac{\partial}{r}$, derived from the distances apart and diameters of wires in ordinary electric circuits, is quite large compared with $\frac{1}{2}$, and the impedance of such circuits consisting of two parallel wires, each l centimeters in length, may therefore be approximately written,

$$I = \sqrt{R^2 + 4\,\pi^2 f^2 L^2} = \left\{R^2 + \frac{4\,\pi^2 f^2}{10^9}\left(4 \log_e \frac{\partial}{r}\right)^2 l^2\right\}^{\frac{1}{2}}.$$

The ratio of the impedance of a circuit to the resistance of the conductors $\left(\frac{I}{R}\right)$ has been called by Kennelly the **Impedance Factor,** and its value has been calculated by him for circuits having a wide range in the values of ∂ and r, and for various frequencies from 40 to 140.† Kennelly has also measured the resistance and impedance of a certain circuit, and found that the actual measurements with an approximately sinusoidal current fully agree with the computation.‡

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[*] Gerard's *Leçons sur l'Électricité*, Vol. I., p. 232; Kennelly, *Trans. Amer. Inst. E. E.*, Vol. 10, p. 213.

[†] Kennelly, Impedance, *Trans. Amer. Inst. E. E.*, Vol. 10, p. 175.

[‡] Kennelly, *Trans. Amer. Inst. E. E.*, Vol. 10, p. 215.

sumption of a sinusoidal current, but current is usually not sinusoidal, and t ance and impedance are therefore likely from 5 to 25 per cent, depending upo of the circuit and the distortions of th For average conditions, Emmet advises cent to the figures of the table on this a

48. The Distribution of Current in a tribution of the current over the normal a conductor along which it flows, is un the current is steady, as this is the dis gives the least loss of energy in the c proof of this theorem is as follows: T lost in the conductor is, according to J where R is equal to $\frac{l\rho}{A}$, l, ρ, and A bei specific resistance, and area of the cond ering that the conductor is divided into ments of equal area and resistance, r, a flowing in one of these is c, then the po

Gauge No. B. & S. Wire.	Resistance in Ohms per Mile of Wire.	Reactance and Impedance in Ohms per Mile of Wire at a Frequency of 60.						Reactance and Impedance in Ohms per Mile of Wire at a Frequency of 125.					
		12 Inches Between Centres.		18 Inches Between Centres.		24 Inches Between Centres.		12 Inches Between Centres.		18 Inches Between Centres.		24 Inches Between Centres.	
		React.	Imp.	React.	Imp.	React.	Imp.	React.	Imp.	React.	Imp.	React.	Imp.
0000	.259	.508	.570	.557	.615	.591	.646	1.06	1.092	1.17	1.190	1.23	1.260
000	.324	.523	.616	.573	.658	.607	.686	1.09	1.138	1.20	1.237	1.26	1.305
00	.412	.534	.682	.588	.725	.618	.749	1.12	1.194	1.23	1.297	1.29	1.357
0	.519	.550	.756	.603	.796	.633	.818	1.15	1.258	1.26	1.360	1.32	1.415
1	.655	.565	.865	.614	.896	.648	.920	1.18	1.349	1.28	1.436	1.35	1.50
2	.826	.580	1.008	.629	1.038	.663	1.06	1.21	1.466	1.31	1.55	1.38	1.61
3	1.041	.591	1.196	.644	1.223	.674	1.24	1.24	1.61	1.34	1.70	1.41	1.75
4	1.313	.606	1.448	.656	1.467	.690	1.48	1.26	1.82	1.37	1.89	1.44	1.94
5	1.656	.620	1.76	.670	1.78	.704	1.80	1.30	2.10	1.40	2.17	1.47	2.22
6	2.088	.633	2.18	.685	2.20	.720	2.21	1.32	2.46	1.43	2.51	1.49	2.56
7	2.633	.647	2.71	.700	2.72	.730	2.73	1.35	2.93	1.46	3.00	1.52	3.04
8	3.320	.662	3.38	.712	3.39	.742	3.40	1.38	3.59	1.48	3.63	1.55	3.66
9	4.186	.677	4.21	.727	4.22	.761	4.23	1.41	4.39	1.51	4.43	1.58	4.45
10	5.280	.688	5.32	.742	5.33	.776	5.34	1.44	5.47	1.54	5.50	1.62	5.53

which exhibits the number of lines
current C, shows that a greater nun
rounds the central filament of the
nearer the surface. In fact, the fila
the outside of the wire are surroundec
of force than the central filament. W
current flows through the wire, a cour
ure is set up in each filament whicl
where N_f is the number of lines of fc
current and which surround the filam
eration. Since N_f increases from th
wire towards the central filament, the
pressure is greatest at the centre and
face of the conductors. Consequent
dency for the current to forsake t
conductor and to take a place nearer t
tendency is directly proportional to th
the current is sinusoidal. It is oppc

by the current flowing through the conductor. The ratio of the resistance of a conductor to an alternating current, R_a, to its true resistance, or the resistance which it opposes to a continuous current, R_c, may be calculated by the following formula:[*]

$$\frac{R_a}{R_c} = 1 + \frac{1}{12}\frac{\mu^2 l^2 f^2}{R_c^2} - \frac{1}{180}\frac{\mu^4 l^4 f^4}{R_c^4} + \text{etc.}$$

When the wire is copper, μ is equal to unity, and the formula becomes

$$\frac{R_a}{R_c} = 1 + \frac{1}{12}\frac{l^2 f^2}{R_c^2} - \frac{1}{180}\frac{l^4 f^4}{R_c^4} + \text{etc.}$$

A table showing the increase in the resistance of wires when carrying alternating currents was first cal-

Diameter.		Area.		Increase over Ordinary Resistance.	Frequency.
MM.	Inches.	Sq. MM.	Sq. in.		
10	.3937	78.54	.122	less than $\frac{1}{100}$ %	80
15	.5905	176.7	.274	$2\frac{1}{2}$ %	
20	.7874	314.16	.487	8 %	
25	.9842	490.8	.760	$17\frac{1}{2}$ %	
40	1.575	1256.	1.95	68 %	
100	3.937	7854.	12.17	3.8 times	
1000	39.37	785400.	1217	35 times	
9	.3543	63.62	.098	less than $\frac{1}{100}$ %	100
13.4	.5280	141.3	.218	$2\frac{1}{2}$ %	
18	.7086	254.4	.394	8 %	
22.4	.8826	394.0	.611	$17\frac{1}{2}$ %	
7.75	.3013	47.2	.071	less than $\frac{1}{100}$ %	133
11.61	.4570	106	.164	$2\frac{1}{2}$ %	
15.5	.6102	189	.292	8 %	
19.36	.7622	294	.456	$17\frac{1}{2}$ %	

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[*] Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., p. 329; Gerard's *Leçons sur l'Électricité*, Vol. I., p. 236.

culated by Mordey[*] on data presented by Lord Kelvin[†] (see table on preceding page). From this table it is seen that R_a is practically the same as R_c for the sizes of wire and frequencies which are ordinarily used, but Emmet[‡] has calculated a table which may be conveniently used in any case where R_a differs from R_c. This table is given b

Product of Circular Mils and Frequency.		duct of Circular and Frequency.	$\frac{R_a}{R_c}$.
10,000,000	[illegible]	70,000,000	1.13
20,000,000	[illegible]	80,000,000	1.17
30,000,000	[illegible]	90,000,000	1.20
40,000,000	1.05	100,000,000	1.25
50,000,000	1.08	125,000,000	1.34
60,000,000	1.10	150,000,000	1.43

This table shows that the frequency or the diameter of the wire may be so great that no current at all will flow at the centre of the conductor, while if the frequency is very great, the current will all remain at the exact outer surface or skin of the wire. Thomson[§] shows that the value of the current at the distance x from the surface of a conductor is equal to

$$\epsilon^{-x\left(\frac{4\pi^2 f\mu}{\rho}\right)^{\frac{1}{2}}},$$

when ρ is the specific resistance of the material. Gray[||] shows that however great the diameter of a wire may be, its resistance to an alternating current will never be less than the true resistance of a wire of the diameter in centimeters given in the following table.

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[*] *Jour. Inst. E. E.*, Vol. 18, p. 603.

[†] *Jour. Inst. E. E.*, Vol. 18, pp. 14 and 35.

[‡] *Alternating Current Wiring and Distribution.*

[§] J. J. Thomson's *Elements of Electricity and Magnetism*, p. 418.

[||] *Absolute Measurements in Electricity and Magnetism*, Vol. II., p. 338.

Frequency.	Copper.	Lead.	Iron, $\mu = 300$.
80	1.43	4.98	.195
120	1.17	4.08	.159
160	1.02	3.52	.138
200	.91	3.16	.123

The specific resistance of lead is not far from that of ordinary German silver; it is about twice that of iron, and about twelve times that of copper. The remarkably large *skin effect* in the case of the iron is thus shown to be due to its large magnetic permeability. The permeability of 300 may be somewhat large to assume for an iron wire when under the ordinary circuit conditions. Kennelly states that iron telegraph or telephone wires show by measurement with small currents that μ is about 150. With increasing currents it would, of course, increase to about 1000, and then again diminish.

The value of the self-inductance of a wire was determined in the preceding section on the assumption of a uniform distribution of the current in the conductor. Any disturbance of this distribution on account of *skin effect* will reduce the value of the self-inductance by a small amount.* The correctness of these deductions in regard to self-inductance and *skin effect* in electrical conductors is proved by extensive experimental researches by Professor Hughes[*] and Lord Rayleigh.

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* Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., p. 329.

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[*] The Self-induction of an Electric Current in Relation to the Nature and Form of the Conductor, *Jour. Inst. E. E.*, Vol. 15, p. 6.

CHAPTER IV.

GRAPHICAL AND ANALYTICAL METHODS OF SOLVING PROBLEMS IN ALTERNATING CURRENT CIRCUITS.

49. Graphical Methods. — Graphical methods lend themselves very satisfactorily to the solution of problems relating to circuits upon which a sinusoidal electrical pressure is impressed. This application of the methods was first brought to general attention by T. H. Blakesley.* The methods are those of vector algebra, and are entirely analogous to those which are so largely used in the graphical solutions relating to the composition and resolution of forces in graphical statics and relating to the composition and resolution of velocities, etc., in graphical dynamics. To make the treatment as simple as possible, the use of the methods herein will be made to conform as closely as possible to their use in the treatises on graphical statics.†

If the line OA in Fig. 66 is conceived as swinging at a uniform angular velocity ω around the point O, the angle a which it makes with the horizontal axis OX at any instant is $a = \omega t$, where t is the interval of time during which the line describes the angle a. The instanta-

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* Blakesley's *Alternating Currents of Electricity*.

† See Dubois' *Graphical Statics*, Hoskins' *Elements of Graphic Statics*, etc.

neous projection Oa upon the vertical axis OY, of the line OA, has a value $Oa = OA \sin \alpha$. If OA is proportional to the maximum value of a sinusoidal function, its instantaneous values are proportionally represented by the instantaneous projections of OA; and if OA is proportional to the effective value of a sinusoidal function, the
instantaneous val tion are proportionally
represented by th 2 into the correspond-
ing instantaneous , t is therefore possible
to represent all th a sinusoidal function:

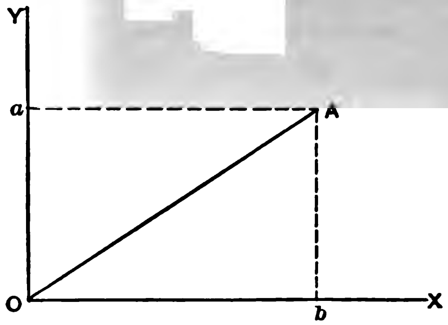


Fig. 66

(1) by a straight line which revolves at a uniform rate around one end; and (2) by the instantaneous projections of the line. It is evident that the motion which the projection of the end A of the revolving line makes along the axis OY is a simple harmonic motion, and that all the theorems relating to simple harmonic motion may be applied to these solutions. As is ordinarily done, the rotation of the line will always be considered to be left-handed in the following discussions; and angles measured from right to left will be considered

positive, while those measured from left to right will be considered negative.

If two sinusoidal electric pressures of the same frequency but having a phase difference ϕ, act in a circuit, the corresponding instantaneous values are,

$$e = \sqrt{2}\, E \sin a,$$
$$e' = \sqrt{2}\, E' (\sin a + \phi).$$

The total instantaneous electric pressure acting in the circuit is $e + e'$. In Fig. 67 the pressure E is represented by the line OA, and the pressure E' by the line OA'. Oa and Oa' are the instantaneous values of the pressure for the angular positions shown. The total instantaneous pressure in the circuit which corresponds to the angular position shown, is equal to $Oa + Oa'$, or Oa''. It is readily shown that Oa'' is the projection of the diagonal of the parallelogram constructed upon OA and OA'. This is true for all angular positions, since

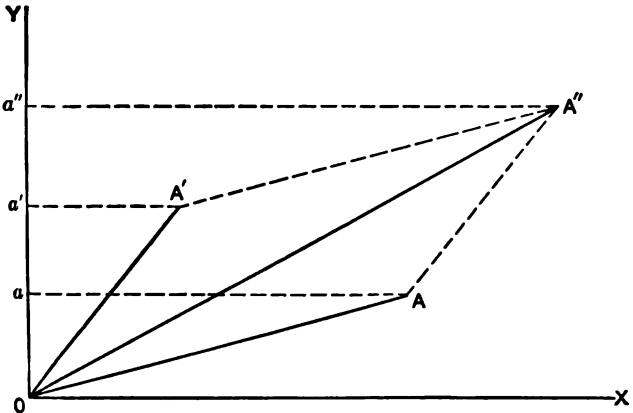
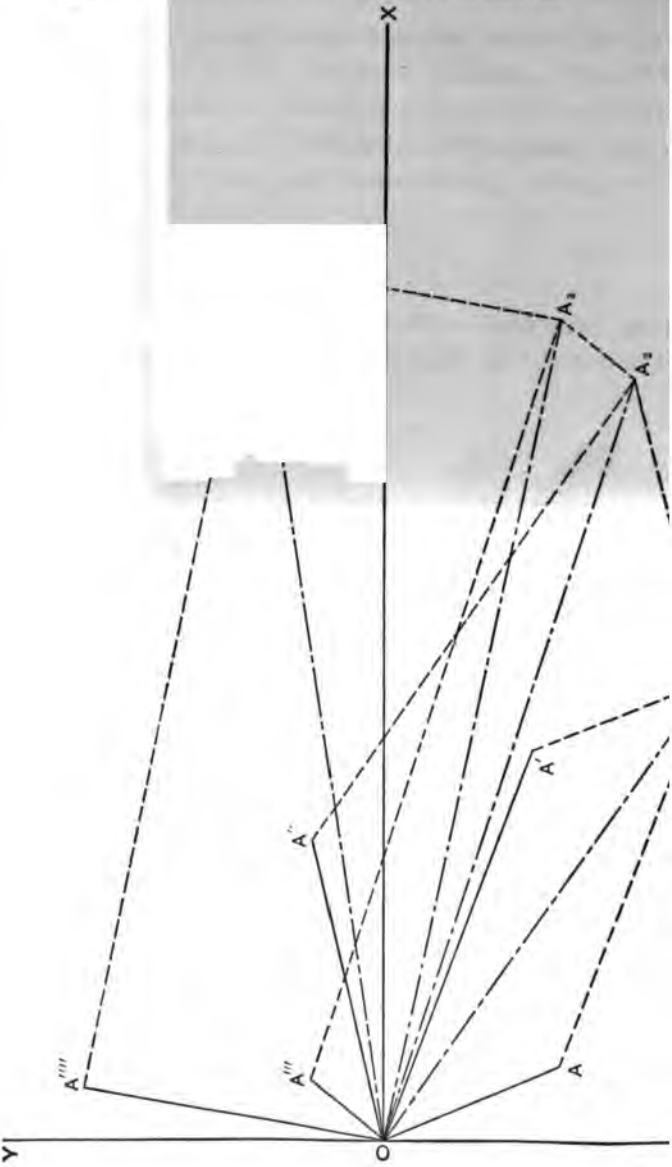


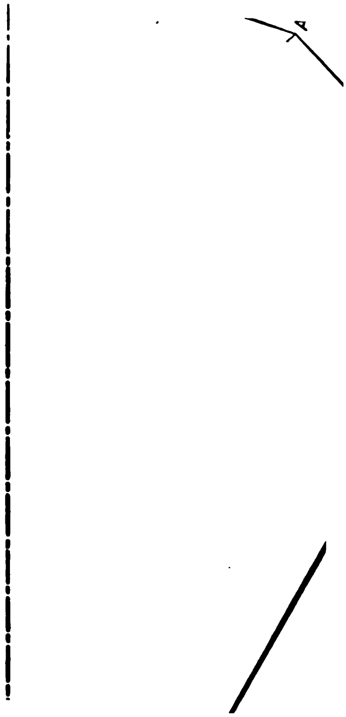
Fig. 67


X
A₂
A₃
A′
A″
A″
A‴
A
Y
O

the sum of the projections of the lines OA and OA' must be equal to the sum of the projections of the lines OA and AA'', which in turn is equal by construction to the projection of the diagonal OA''.

The length of the line OA'', therefore, proportionally represents the magnitude of the effective or maximum total electrical pressure in the circuit, and its position relative to that of OA and OA', represents the relative phase position of the total pressure. If instead of two pressures acting in a circuit, there are three or more, as OA, OA', OA'', OA''', and OA'''', in Fig. 68, the same construction is used. Thus, completing the parallelogram for OA and OA', their resultant OA_1 is found. Completing the parallelogram for OA_1 and OA'', their resultant OA_2 is found, and again with this and OA''' the resultant OA_3 is obtained; finally, OA_4, the final resultant, is obtained by combining OA_3 with OA''''. The figure shows that it is unnecessary to complete all the parallelograms. It is only necessary to draw the lines AA_1, A_1A_2, A_2A_3, A_3A_4 respectively parallel and equal in length to the lines OA', OA'', OA''', and OA'''', and the line drawn from O to the end of the last line laid off gives the phase-position and magnitude of the total pressure in the circuit, regardless of the number of the components from which it is derived (Fig. 69). The composition of electrical pressures is therefore exactly analogous to the composition of velocities or of forces. *As in the case of velocities or forces, the resultant of any number of electrical pressures may be determined by this method.*

The resultant of two sinusoidal alternating electric



rents differ from each other in phase by an angle ϕ. The instantaneous values of the currents are represented by the instantaneous projections of the lines as they revolve around the point O. At each instant, the total current in the main circuit is equal to the sum of the instantaneous partial currents, or to $c + c'$. Consequently, the magnitude of the effective or maximum value of the current in the main circuit is proportionally represented by the length of the line OA'', and the angular direction of OA'' gives the angular relation of the phase of the total current to the phases of the partial currents. When the divided circuit contains more than two branches, the same method may be extended, as already explained for the composition of electric pressures.

For convenience in using the graphical methods for solving alternating-current problems, it is well to distinguish between two different diagrams. The first diagram represents the magnitude and relative phase positions of the electric pressures or currents. This may be called the **Phase Diagram.** The other diagram is the polygon formed by laying off lines equal and parallel to the lines in the phase diagram. This may be called the **Vector Diagram.** Figures 68 (full lines only) and 69 are respectively phase and vector diagrams for representing five electrical pressures. The resultant pressure is represented in magnitude and phase by the line OA_4. If the closing line of the vector diagram is inserted in the phase diagram by drawing from O a line in the direction obtained by following round the vector diagram against the direction in which the lines were drawn, the line so inserted evidently represents the resultant of the component pressures or currents. If the line be drawn from *O* in the opposite direction, it represents a balancing pressure or current.

These simple propositions, which so evidently come from the ordinary graphical mechanics (statics and dynamics), give a [illegible] n that is necessary for the rapid and a [illegible] of problems relating to the flow of curr [illegible] l compound circuits containing definite [illegible] uctances, and capacities in their differe [illegible] olutions of complicated problems the [illegible] l is often preferable to the analytical, [illegible] or of analytical calculations is rapidly increased with the complication of the circuits, while the ease and accuracy of graphical calculations are not affected thereby. The graphical method also has the advantage of showing directly to the eye the relative phases of the pressures or currents in different parts of the circuit. The graphical solutions have the same limitations in regard to alternating currents or pressures which are not sinusoidal as have the analytical methods; and where the wave form is not sinusoidal, only an approximation can be arrived at by judiciously correcting the results shown by the diagrams based on sine functions.

The problems relating to alternating-current circuits which may be solved by graphical methods may be divided into three classes: (1) where the current flows through all parts of the circuit in series; (2) where the same electrical pressure is impressed upon all parts of the circuit (parallel circuits); and (3) where the first and second classes are combined. Solutions in the third class are effected by combining partial solutions of the first and second classes.

50. Series Circuits.—*First Class.* Suppose a circuit is given which has a certain resistance and self-inductance and it is desired to know what impressed pressure with a frequency f is required to pass through it a certain current C. In this case the impressed pressure is made up of two components: (1) the pressure required to pass the current through the resistance of the circuit; (2) the pressure required to balance or overcome the counter inductive pressure. The inductive pressure is 90° in phase behind the active pressure, and the phase diagram which shows the relative phases of the pressures in the circuit is therefore like that shown in Fig. 70. The active pressure OA' is equal to CR, and the inductive pressure OA'' is equal to $2\pi fLC$. The inductive component of the impressed pressure is required to balance $2\pi fLC$ and is therefore equal to and opposite to OA''. An arrowhead is therefore placed on OA'' to show that in the vector diagram its direction must be taken from A'' to O, instead of from O outwards, as is done with the other lines. The vector diagram is therefore given by drawing O_1A_1 equal and parallel to OA', A_1A_2 equal and parallel to $A''O$, and closing the polygon by the line O_1A_2 (Fig. 70). The line O_1A_2 taken in the direction from O_1 to A_2 represents the magnitude and relative phase of the impressed pressure. When inserted in the phase diagram, it is the line OA'''. The angle ϕ, by which the current lags behind the impressed pressure, is the angle $A_2O_1A_1$.

If a number of inductive circuits are connected in series, the inductive pressure line of the phase diagram is equal to $2\pi fC(L_1+L_2+L_3+\text{etc.})$, and the active pressure line is equal to $C(R_1+R_2+R_3+\text{etc.})$, where $L_1, L_2, L_3, R_1, R_2, R_3$, etc., are the self-inductances and resistances of the different parts of the circuit. If the circuit is non-inductive the phase diagram and vector diagram each become a single horizontal line equal in length to CR, while if the inductive circuit contains no resistance the diagrams each become a single vertical line which is equal in length to $2\pi fLC$.

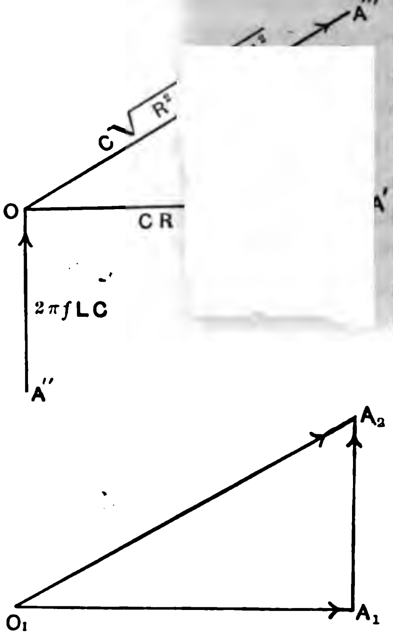


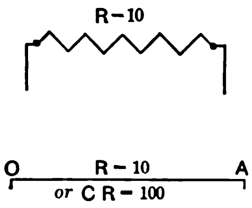
Fig. 70

Dividing the lengths of the sides of the vector polygon by the value of C gives the equivalent resistance for each. The vector polygon may therefore be used directly for determining the impedance of a circuit when the resistances and reactances of its various parts are known. A vector polygon representing pressures, and one representing equivalent resistances, may evidently be converted, one into the other, by a simpl change of scale. It is to be remembered that in a seri circuit the current has the same phase, and is equal at any given instant, in all parts of the circuit, but the phase, with reference to the current, of the pressure impressed at the terminals of different parts of the circuit depends wholly upon the relation between the individual resistances and reactances of the parts.

Examples.—In the following examples it is desired to find for each of the given circuits: (1) the impedance of the circuit; (2) the current which flows through the circuit when the impressed pressure is 100 volts; (3) the impressed pressure which is required to pass 10 amperes through the circuit; (4) the angle by which the current lags behind the impressed pressure. The frequency in each case is taken to be $127\frac{1}{2}$, whence $2\pi f$ is equal to 800.

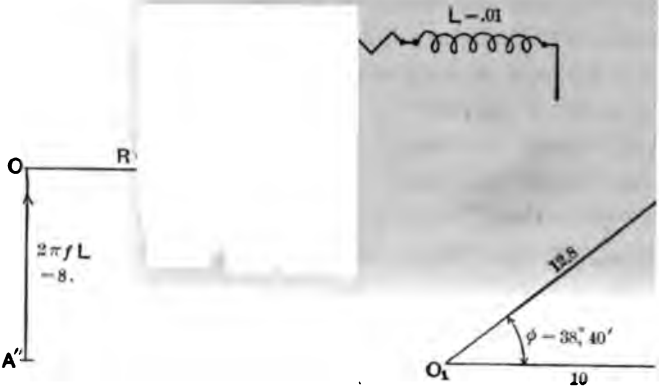
Circuits containing Resistance and Self-inductance.

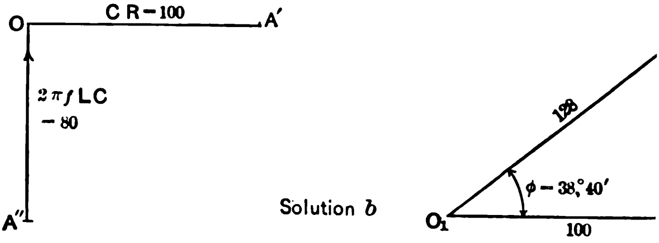
a. The circuit consists of a non-inductive resistance of 10 ohms. The phase and vector diagrams for the first two solutions are resistance diagrams, each consisting of a horizontal line 10 units in length. The impedance of the circuit is 10 ohms, and the current which flows when a pressure of 100 volts is impressed on the circuit is 10 amperes. The diagrams for the third solution are pressure diagrams, each consisting of a horizontal line CR (= 100) units in length, and the impressed pressure required to pass 10 amperes through the circuit is 100 volts. The angle ϕ is zero.



Solution *a*

b. The circuit consists of an inductive coil of
ohms resistance and .01 henry self-inductance.
phase and vector diagrams for the first two soluti
are resistance diagrams, as shown in the figure.
impedance is shown by the closing line of the diagr

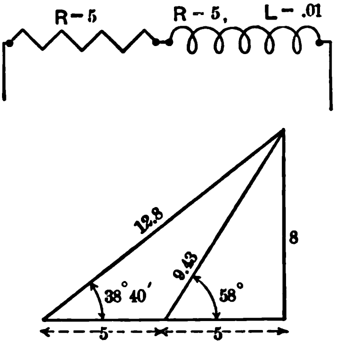




Solution *b*

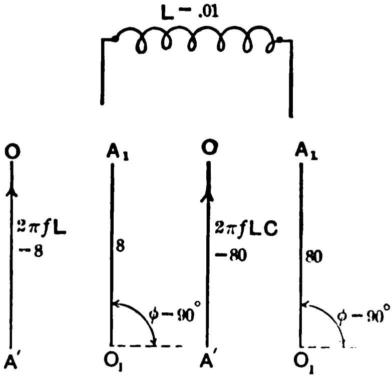
to be 12.8 ohms, whence it is seen that 7.81 ampe
will flow through the circuit when the impressed pr
ure is 100 volts. The third solution shows that
impressed pressure required to pass 10 amperes thro
the circuit is 128 volts. The angle ϕ is 38° 40′.

c. The circuit consists of a non-inductive coil of 5 ohms in series with an inductive coil of 5 ohms and .01 henry. The total phase and vector diagrams for this are exactly the same as in example *b.* The vector diagram may be laid off as shown in the figure. This shows that the pressure impressed upon the circuit as a whole is not equal to the algebraic sum of the pressures measured between the terminals of the parts of the circuits, but it is still equal to their vector sum.



Solution *c*

d. The circuit consists of an inductance of .01 henry. The phase and vector diagrams for the first two solutions are each resistance diagrams consisting of a vertical line $2\pi fL (= 8)$ units in length. The impedance of the circuit is 8 ohms, and the current which flows through the circuit under an impressed pressure of 100 volts is 12.5 amperes. The diagrams for the third solution are pressure diagrams, each consisting of a



Solution *d*

vertical line $2\pi fLC(=80)$ units in length, and the impressed pressure required to pass 10 amperes through the circuit is 80 volts. The angle ϕ is 90°, and the current therefore lags 90° behind the impressed pressure.

The effect of a condenser placed in series in a circuit may be shown [illegible] hich are very similar to those relating t [illegible] uits. The charging current of the co [illegible] h a phase position and magnitude, tha [illegible] he total current flowing in the circuit is the same as the effect of an electric pressure which is equal to $\frac{C}{2\pi fs}$ and is 90° in advance of the circuit current. This may be called the **Condenser Pressure** (see Sect. 31). The phase diagram is like that shown in Fig. 71. The pressure impressed on the circuit when current C flows, must consist of two components: (1) the pressure required to pass the current through the resistance of the circuit; (2) the press-

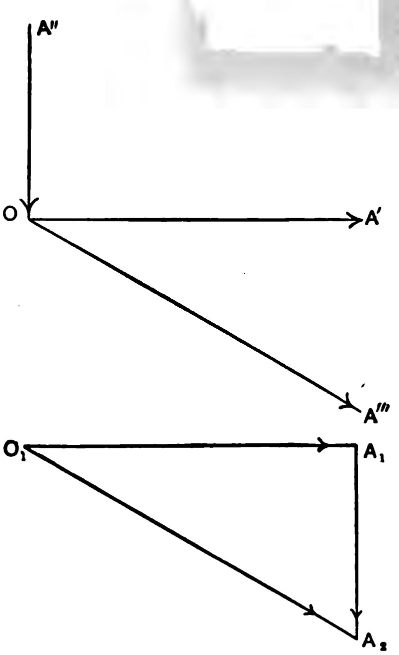


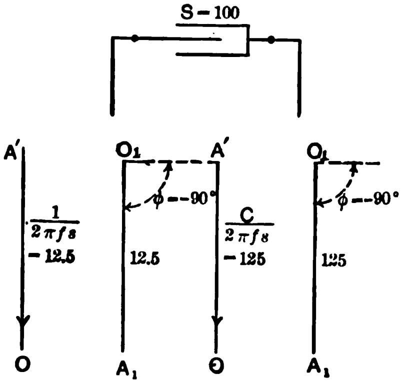
Fig. 71

ure required to balance the condenser pressure. The active pressure *OA'* in the figure is equal to *CR*, and the condenser pressure *OA''* is equal to $\frac{C}{2\pi fs}$ and is 90° in advance of the active pressure. The capacity component of the impressed pressure is required to balance $\frac{C}{2\pi fs}$, and is therefore equal and opposite to *OA''*. An arrowhead is therefore placed on *OA''* to show that in the vector diagram its direction must be taken from *A''* to *O*, instead of from *O* outwards. The vector diagram is then as shown in Fig. 71 (compare with Fig. 70).

Examples.—The following examples are to be solved for the same constants as before, the frequency being taken as $127\frac{1}{2}$.

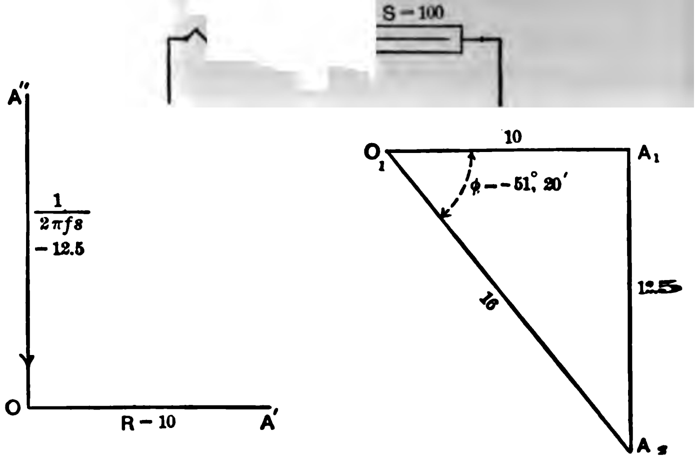
Circuits containing Resistance and Capacity.

e. The circuit contains simply a condenser having a capacity of 100 microfarads (=.000100 farad). The phase and vector diagrams for the first two solutions are each resistance diagrams consisting of a vertical line $\frac{1}{2\pi fs}$ (= 12.5) units in length. The impedance of the circuit is 12.5 ohms, and the current which flows through the circuit, when 100 volts is im-



Solution e

pressed on it, is 8 amperes. The diagrams for the third solution are pressure diagrams each consisting of a vertical line $\frac{C}{2\pi fs}$ (= 125) units in length, and the impressed pressure required to pass 10 amperes through the circuit is 125 volts. The angle ϕ is $-90°$, that is,
the current is 9
the impressed pressure.
The lines com
ıms for this example are
drawn in a dire
xactly opposite to that of
the lines in the
e example *d*.

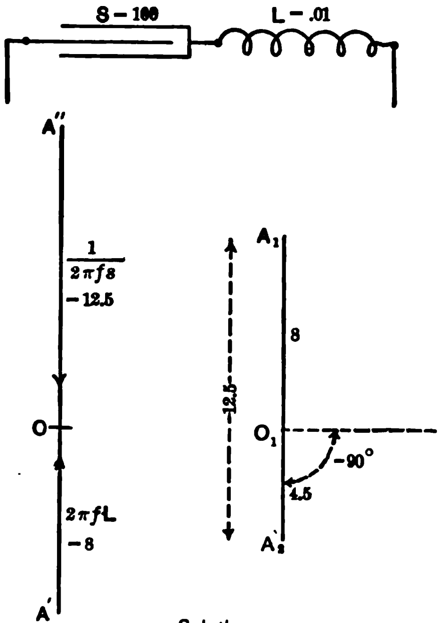


Solution *f*

f. The circuit consists of a resistance of 10 ohms and a capacity of 100 microfarads. The phase and vector diagrams are shown in the figure. The impedance of the circuit is 16 ohms and the current which flows under an impressed pressure of 100 volts is 6.25 amperes. The impressed pressure required to cause 10 eres to flow through the circuit is 160 volts. The le ϕ is $-51^\circ\ 20'$.

Circuits containing Self-inductance and Capacity.

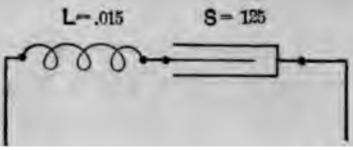
The circuit consists of a capacity of 100 microds in series with an inductance of .01 henry. The

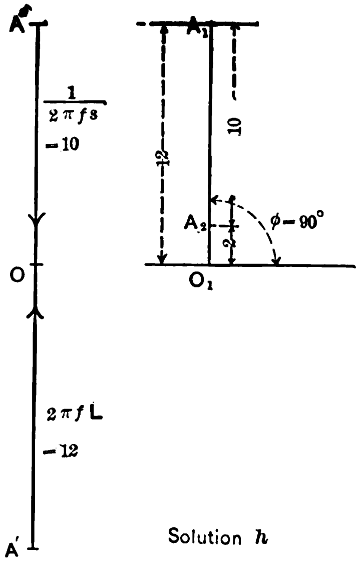


Solution *g*

grams are as shown. The impedance of the circuit 4.5 ohms. The current which flows when the imessed pressure is 100 volts is 22.2 amperes, at which ie the pressure measured between the terminals of condenser is 277.5 volts and that measured between the terminals of the inductance is 177.6 volts. impressed pressure required to pass 10 amperes thr the circuit is 45 volts. The angle ϕ is $-90°$.

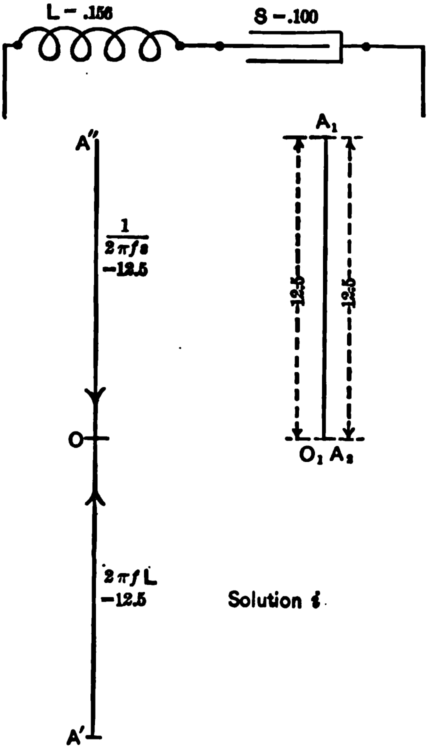
h. The circuit consists of a capacity of 125 m farads in series with an inductance of .015 henry.





Solution *h*

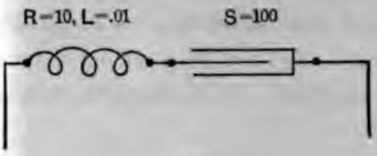
impedance of the circuit is 2 ohms and the cu which flows under 100 volts impressed pressure i amperes, at which time the pressure measured bet terminals of the condenser is 500 volts and between terminals of the inductance is 600 volts. The im- ssed pressure required to pass 10 amperes through circuit is 20 volts. With this current flowing, the

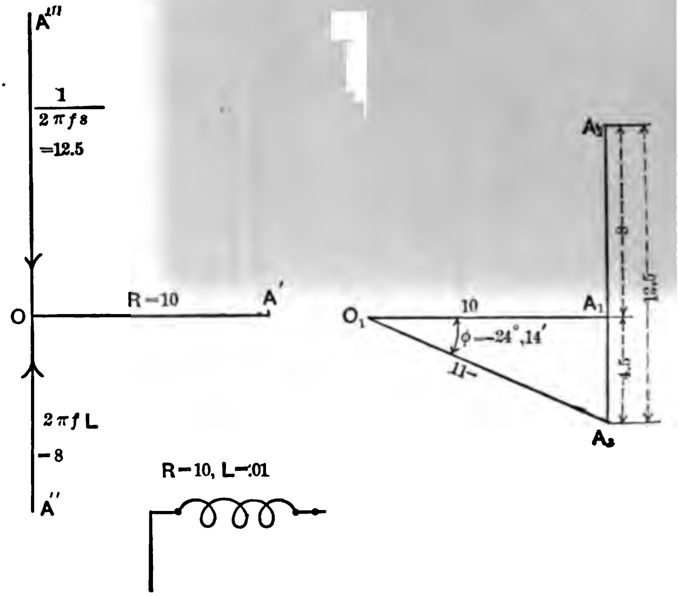


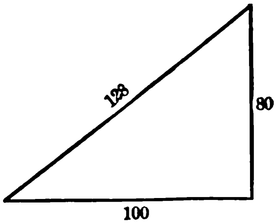
Solution 1.

ssure measured between the terminals of the con- ser is 100 volts and between the terminals of the ictance is 120 volts. The angle ϕ is 90°.

The circuit consists of a capacity of 100 micro- ls in series with an inductance of .0156 henry.


R=10, L=.01
S=100


A'''
$\frac{1}{2\pi f s}$
=12.5
O
R=10
A'
$2\pi f L$
=8
A''
O₁
10
A₁
ϕ=24°,14'
11
A₃
A₂
8
4.5
12.5
R=10, L=.01



Solution *j*

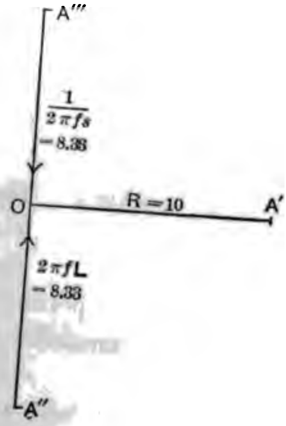
The phase and vector diagrams are shown in the figure. In this case $\frac{1}{2\pi fs} = 2\pi fL$ and the impedance of the circuit is zero.

Circuits containing Resistance, Self-inductance, and Capacity.

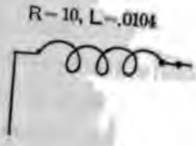
j. The circuit consists of 10 ohms in series with 100 microfarads and .01 henry. The impedance of the circuit is shown by the diagram to be 11 — ohms. The current which flows through the circuit when 100 volts are impressed at its terminals is 9.1 amperes. The pressure required to pass 10 amperes through the circuit is 110 — volts. This is the vector sum of 125 and 128, which are the pressures measured respectively between the condenser terminals and the remainder of the circuit. The angle ϕ is $-24° 14'$.

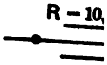
k. The circuit consists of 10 ohms in series with 150 microfarads and .01042 henry. The diagrams show that the impedance of the circuit is ten ohms. One hundred volts is therefore the impressed pressure that gives a current of 10 amperes. When 10 amperes flow in the circuit, the pressure measured between the terminals of the condenser is 83.3 volts, and that measured between the terminals of the remainder of the circuit is 130 volts. The angle ϕ is zero.

51. Conclusions in regard to Series Circuits. — The eleven examples thus given cover every fundamental arrangement of series circuits which may occur. An examination of the diagrams and of the principles involved in their construction, makes the following statements evident :


A'''
$\frac{1}{2\pi fs}$
$= 8.33$
O
R = 10
A'
$2\pi fL$
$= 8.33$
A''

O_1


R = 10, L = .0104


R = 10,

1. When *non-inductive circuits* are connected in series, the total impressed pressure equals the sum of the pressures measured between the terminals of the individual parts, and the total resistance of the circuit is equal to the sum of the resistances of the individual parts.

2. When *inductive circuits of equal time constants* are connected in series, the total impressed pressure equals the sum of the pressures upon the individual parts, measured between their individual terminals, and the total impedance of the circuit is equal to the sum of the individual impedances.

3. When *inductive and non-inductive circuits* are connected in series with each other, or when *inductive circuits of unequal time constants* are connected in series, the total impressed pressure equals the vector sum, which is always less than the algebraic sum, of the pressures measured between the terminals of the individual parts, and the individual pressures are each less than the total impressed pressure. The total impedance of the circuit is equal to the vector sum of the individual impedances, each of which is less than the total.

4. When *condensers* are connected in series *by conductors of negligible resistance*, the total impressed pressure equals the sum of the pressures measured across the individual condensers, and the total impedance of the circuit is equal to the sum of the impedances of the individual condensers.

5. When *condensers* are connected in series *with non-inductive resistances*, the total impressed pressure

equals the vector sum, which is always less than the algebraic sum, of the pressures measured between the terminals of the individual parts of the circuit, and the individual pressures are each less than the total impressed pressure. The total impedance of the circuit is equal to the vector sum of the individual impedances, each of whic
total.

6. When
connected in series *with*
inductive resi
impressed pressure equals
the vector sun
ys less than the algebraic
sum, of the p
ed between the terminals
of the individu
ircuit. Since the effects of
capacity and self-in
spectively cause the angle
ϕ to become negative and positive, *the individual pressures may be* either *greater* or less *than the total impressed pressure*, depending upon the relation between the various resistances, capacities, and inductances in the circuit. The total impedance of the circuit is equal to the vector sum of the individual impedances, each of which may be either greater or less than the total impedance.

The third, fifth, and sixth paragraphs above make the following proposition evident: When in series circuits, the angles taken between the phases of the current and the individual pressures, measured at the terminals of the parts of the circuit, are all either positive or negative, the total impressed pressure is always greater than any of the individual or partial pressures. When the angles taken between the phases of the current and the partial pressures are in part positive and in part negative, *some or all of the partial pressures may be greater than the total impressed pressure.*

Parallel Circuits.—*Second Class.* The graphical
tment of problems relating to parallel circuits is en-
y analogous to that given for series circuits. As the
plest cases of parallel circuits are those in which the
e electrical pressure is impressed upon all the parts
he circuit, these will be treated first. In this class
same general operations are used in solving prob-
s as in the first class, but alternating currents and
portional conductivities are dealt with instead of
rnating pressures and proportional resistances. Sup-
a circuit made up of two branches in parallel, each
a known resistance and reactance, and it is desired
now what impressed pressure with a frequency f is
ired to pass through it a certain current C. In this
, the total current is made up of two components,
of which flows through one of the branches and is
sely proportional to the impedance of the branch,
e phase of which has an angular retardation with
t to that of the impressed pressure which depends
he time constant of the branch. The total cur-
hich is inversely proportional to the equivalent
ce of the parallel circuit, is equal in magnitude
ition to the resultant of the branch currents.
dition is represented by Fig. 72, in which OA
are the currents in the two branches respec-
nd ϕ' being their respective lag angles. The
hase position of the impressed pressure is
the horizontal line. Then the resultant or
t in the circuit is represented in magnitude
y OA''. OA is equal to $\frac{E}{I_1}$, OA' is equal to

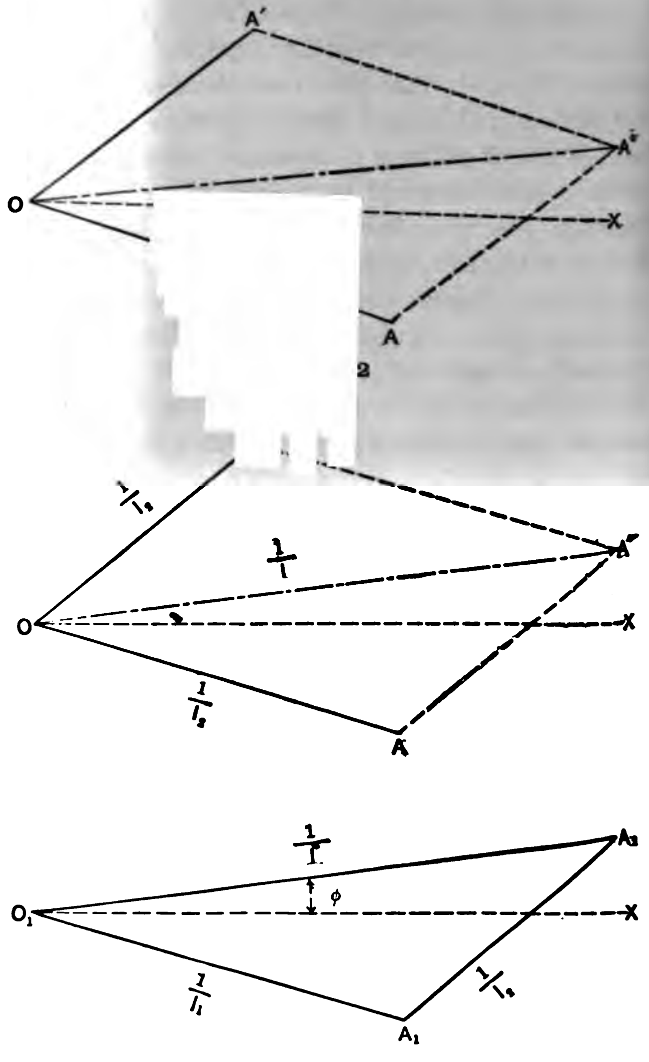
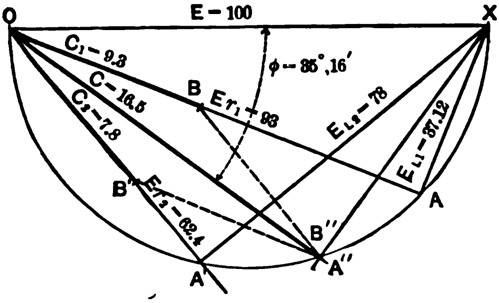


Fig. 73

and OA'' is equal to $\frac{E}{I}$, where E is the pressure im-
essed on the circuits, and the denominators of the
actions are the respective impedances of the branches
nd of the total circuit. It is therefore evident that the
eciprocal of the equivalent impedance (= the equiva-
lent apparent conductivity) on a parallel circuit may
be at once derived from the apparent conductivities of



$I = \frac{100}{16.5} = 6.06$ OHMS.

Solution *c*

Fig. 74

the branches, by taking their vector sum, as is shown in Fig. 73. The equivalent apparent conductivity, or the equivalent impedance of a circuit being known, the pressure required to pass a given current through it, or the current flowing under a given impressed pressure, may be at once derived. In dealing with series circuits, the phase of the current (or active pressure) has been assumed to be along the horizontal line, or line of reference. In dealing with parallel circuits, it is more convenient to assume the phase of the impressed pressure (or apparent conductivity) for the reference phase. It must always be remembered, however, that angles of lag are measured from lines representing current to lines representing pressure. Thus, in Fig. 74, the angle ϕ is posi[illegible]e current lags behind the pressure.

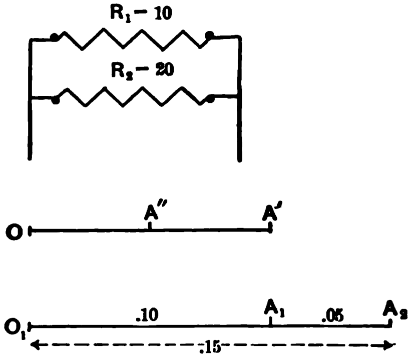
EXAMPLES. – [illegible]ng examples it is desired to find for each [illegible]rcuits: (1) the equivalent impedance of t[illegible] the current which flows through the circ[illegible]mpressed pressure is 100 volts; (3) the im[illegible]sure which is required to pass 10 amperes through t[illegible] circuit; (4) the angle by which the total current lags behind the impressed pressure. The frequency is taken as in the examples of the first class to be $127\frac{1}{2}$, whence $2\pi f$ is equal to 800.

Circuits containing Resistance and Self-inductance.

a. The circuit consists of two non-reactive[*] branches in parallel, one having 20 ohms resistance and the other 10 ohms resistance. The phase diagram for the solutions, using the apparent conductivities of the circuits as a basis of work, is two horizontal lines superposed, of lengths respectively .05 and .10 unit. The vector diam is given by drawing these consecutively, and equivalent apparent conductivity of the circuit is $_2$ in the figure. The equivalent impedance of the

---

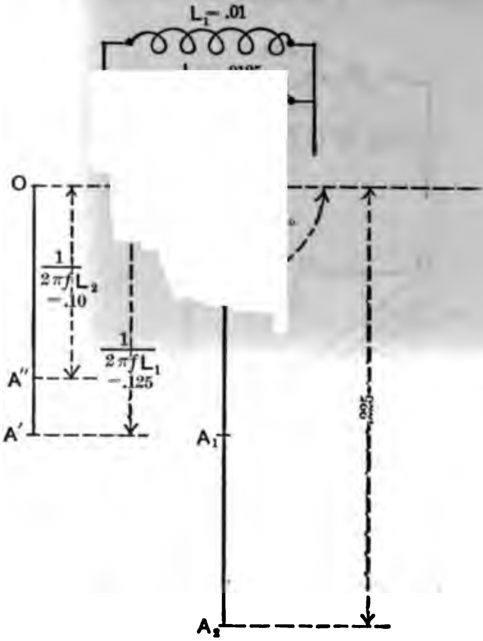
[*] The terms *inductive*, *capacity*, and *reactive* circuit, will hereafter be used with the following significations: an *inductive circuit* is one containing inductance, but not capacity; a *capacity* or *condenser circuit* is containing capacity, but not inductance; a *reactive circuit* is one cont ing either inductance or capacity or both inductance and capacity *non-reactive circuit* is, therefore, one which contains neither induct nor capacity, that is, one which contains a plain resistance only.



Solution *a*

uit is therefore 6.67 ohms. The current flowing ler 100 volts pressure is 15 amperes, and the press- required to pass 10 amperes through the circuit is 7 volts.

b. The circuit consists of a non-reactive branch of 10 ms and an inductive branch of .01 henry. The phase agram consists of two lines at right angles (one being orizontal), since the current in the non-reactive branch in phase with the impressed pressure, and that in the ductive branch lags 90° behind the impressed pressure. he lengths of the lines are respectively $\frac{1}{R}$ (= .10) its and $\frac{1}{2\pi f L}$ (= .125) units. The vector diagram is own. The equivalent conductivity of the circuit is 5 and the impedance is 6.25 ohms. The current wing under an impressed pressure of 100 volts is 16 respectively .01 and .0125 henry. The diagrams consist of vertical lines as shown. The conductivity is .225 and the impedance is 4.44 ohms. The current flowing under 100 volts pressure is 22.5 amperes, and it requires 44.4 volts to cause 10 amperes to flow. The angle of lag is 90°.

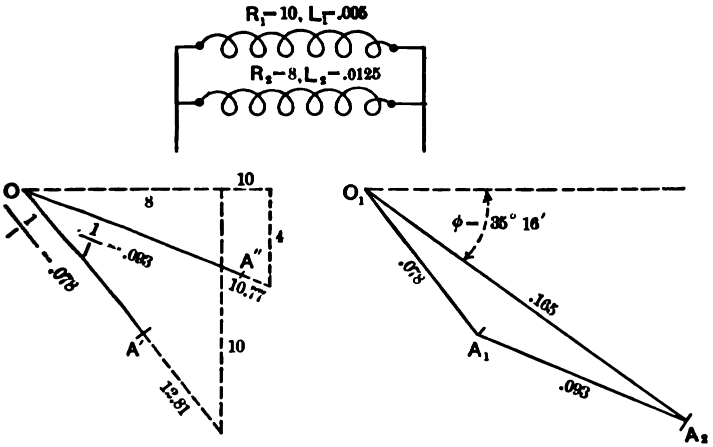


Solution *d*

e. The circuit consists of two reactive branches of respectively .005 henry and 10 ohms, and .0125 henry and 8 ohms. The diagrams are as shown. The conductivity of the circuit is .165, and the impedance is 6.06 ohms. The current flowing under 100 volts pressure is

16.5 amperes, and the pressure required to pass 10 amperes through the circuit is 60.6 volts. The angle ϕ is 35° 16′.

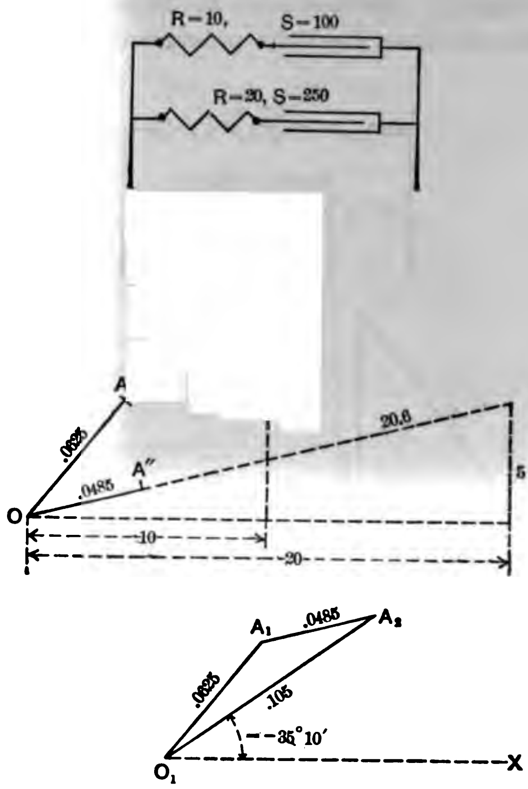
The five preceding examples cover all the fundamental combinations of resistance and inductance in parallel circuits. The following four in like manner cover the combinations of resistance and capacity. The solutions in the two cases are entirely similar, but the lag angles become negative on account of the influence of capacities.



Solution *e*

Circuits containing Resistance and Capacity.

f. When two or more condensers are connected in parallel by wires of negligible resistance, they evidently act upon the circuit exactly as though it contained one condenser with a capacity equal to the combined

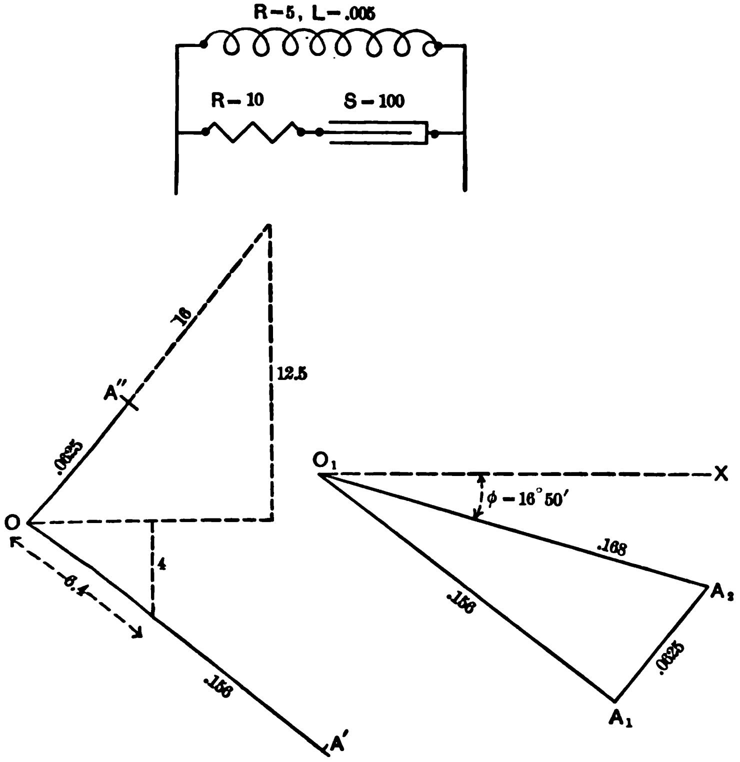


Solution *i*

The following examples cover the fundamental combinations of capacities and inductances.

j. The circuit consists of two reactive branches, respectively of 5 ohms and .005 henry, and of 10 ohms and 100 microfarads. The conductivity of the circuit is shown to be .168 and the impedance is 5.95 ohms. The current flowing under a pressure of 100 volts is

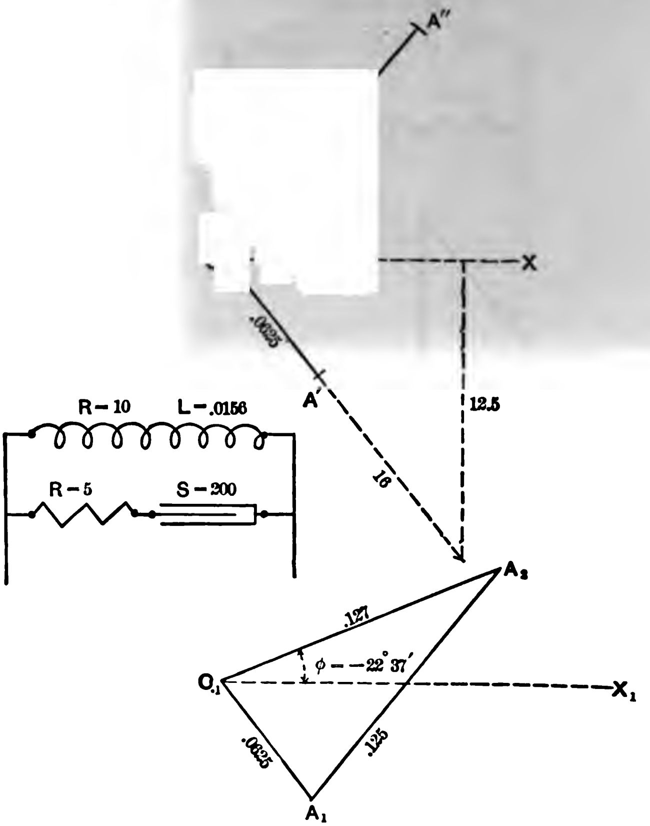
16.8 amperes, and the pressure required to cause a current of 10 amperes is 59.5 volts. The angle ϕ is 16° 50′.



Solution *j*

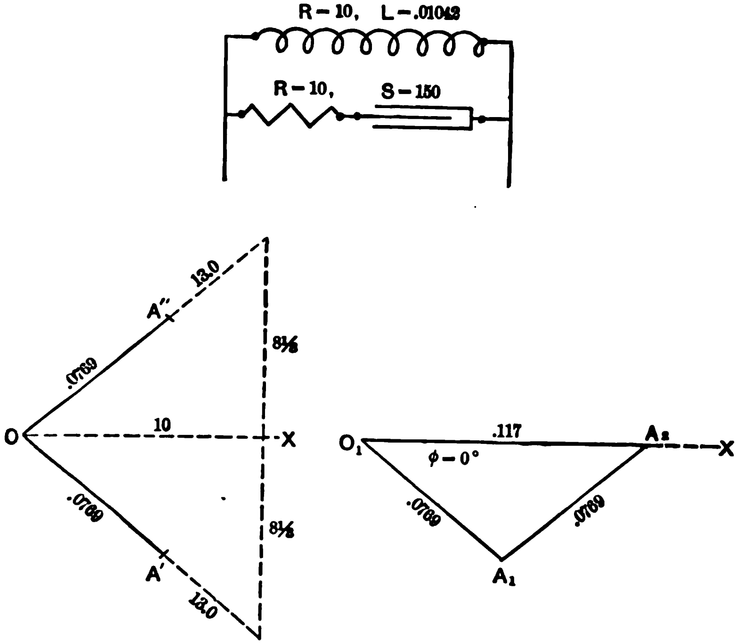
k. The circuit consists of two reactive branches, respectively of 10 ohms and .0156 henry, and of 5 ohms and 200 microfarads. The conductivity is shown to be .127 and the impedance of the circuit is 7.87 ohms.

The current flowing under a pressure of 100 volts is 12.7 amperes, and 78.7 volts are required to pass 10 amperes through the circuit. The angle ϕ is $-22° 37'$.



Solution *k*

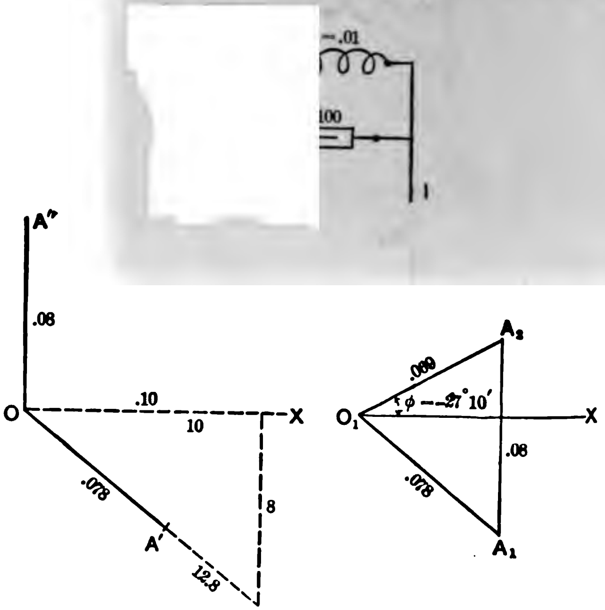
l. The circuit consists of two reactive branches of respectively 10 ohms and .01042 henry, and 10 ohms and 150 microfarads. The diagrams show that the impedance of the circuit is 8.47 ohms and the angle ϕ is equal to zero.



Solution *l*

m. The circuit consists of two reactive branches respectively of 10 ohms and .01 henry, and of 100 microfarads. The diagrams show the joint impedance to be 14.5 ohms. The impedances of the branches are respectively 12.8 and 12.5, so that when the impressed pressure is 100 volts, 6.9 amperes flow in the main circuit, while 8 and 7.8 amperes respectively flow in the branches. The angle ϕ is $-27° 10'$.

n. The circuit consists of two reactive branches respectively of .01 henry, and of 10 ohms and 100 microfarads. The diagrams show the impedance to be 11.6 ohms. The impedances of the branches are respectively 8 and 16, so that when 100 volts pressure is impressed upon the circuit 8.6 amperes flow in the main leads, while 12.5 and 6.25 amperes flow respec tively in the two branches. The angle ϕ is 62° 53′.

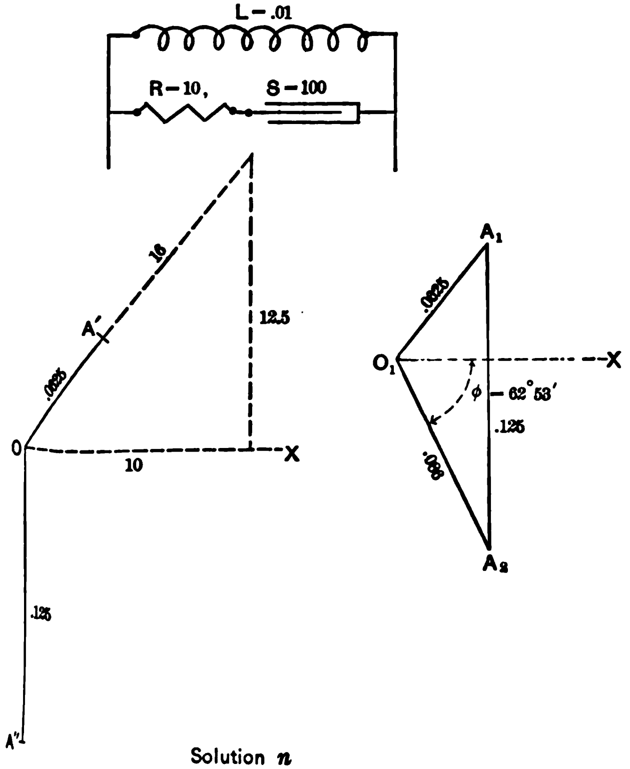


Solution *m*

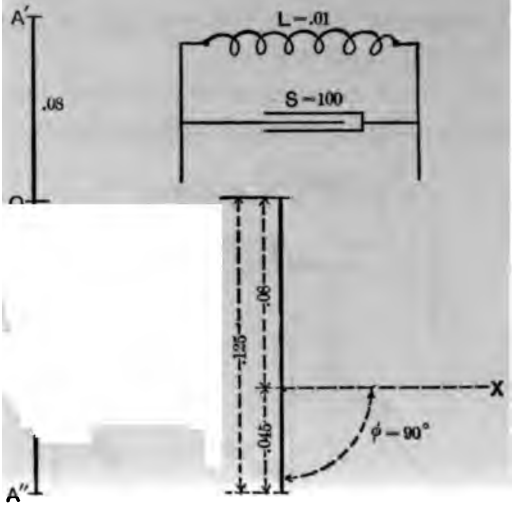
o. The circuit consists of two reactive branches re spectively of .01 henry and of 100 microfarads. Th impedance of the circuit is 22.2 and the impedance of the branches are respectively 8 and 12.5 ohms. Whe the impressed pressure is 100 volts, the main current i

45 amperes and that in the branches is 12.5 and 8 amperes. The angle ϕ is 90°.

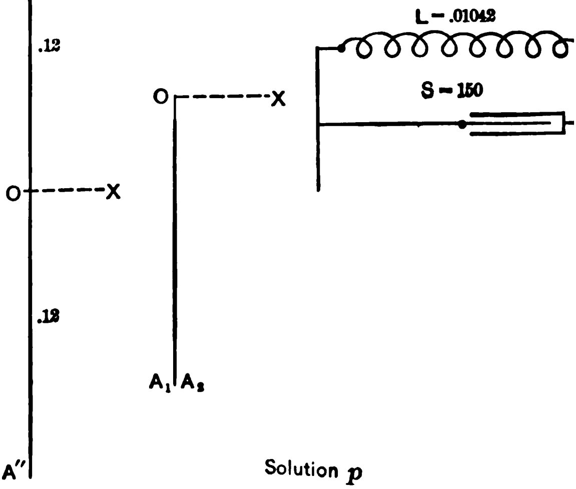
p. The circuit consists of two reactive branches of respectively .01042 henry and 150 microfarads. The diagrams show that the two branch currents are in exact opposition and of equal value and that the joint conductivity is zero, so that the main current is zero. When the impressed pressure is 100 volts the branch currents are each 12 amperes, and when 10 amperes flow in each branch the pressure is 83.3 volts.



Solution *n*



Solution *o*



Solution *p*

53. Conclusions in Regard to Parallel Circuits.—*Second Class.* The sixteen examples just presented cover every fundamental arrangement of simple parallel circuits. An examination of the diagrams and the principles involved in their construction makes evident the following statements, which are in many respects analogous to those given as applying to series circuits:

1. When *non-reactive circuits* are connected in parallel, the total current equals the algebraic sum of the currents in the branches, and the joint conductivity of the circuit is equal to the algebraic sum of the branch conductivities.

2. When *inductive circuits of equal time constants* are connected in parallel, the total current equals the algebraic sum of the currents in the branches, and the joint conductivity of the circuit is equal to the algebraic sum of the branch conductivities.

3. When *inductive and non-reactive circuits* are connected in parallel with each other, or when inductive *circuits of unequal time constants* are connected in parallel, the total current is equal to the vector sum, which is always less than the algebraic sum, of the branch currents, and the individual branch currents are each smaller than the total current. The joint conductivity of the circuit is equal to the vector sum of the branch conductivities, each of which is less than the joint total.

4. When *condensers* are connected in parallel *by wires of negligible resistance*, the total current equals the algebraic sum of the branch currents, and the joint conductivity equals the algebraic sum of the branch conductivities.

5. When *condensers* are connected in parallel *with non-reactive resistances*, the total current equals the vector sum, which is always less than the algebraic sum, of the branch currents, and the individual branch currents are each smaller than the total current. The joint conductivity of the circuit equals the vector sum of the branch conduc which is smaller than the joint total.

6. When *c* onnected in parallel *with inductive circ* urrent equals the vector sum, which is an the algebraic sum, of the currents es. Since the effects of capacity and of s t-indu ce respectively cause the angle ϕ to become negative a nd positive, *the individual branch currents may be* either *greater* or less *than the main or total current*, depending upon the relation between the various capacities and inductances in the circuit. The joint conductivity of the circuit equals the vector sum of the branch conductivities, each of which may be either greater or less than the joint conductivity.

The third, fifth, and sixth paragraphs make evident this proposition, which is similar to that given for series circuits (Sect. 51): When in parallel circuits the currents in the branches are all either lagging or leading with respect to the impressed pressure, the total or main current is always greater than the current in any one of the branches. When the currents in part of the branches lead the impressed pressure and in other branches lag behind the pressure, ***some or all the branch currents may be greater than the total or mai current.*** It is even theoretically possible for the angl such a relation that *a large current may flow in ranches while the main current is zero.*

With the methods thus set forth it is possible to any problem which may arise in regard to the imıce presented by any circuit to the flow of a sinul current. When the current is not sinusoidal the :tions do not strictly apply, but for the alternating nts which are commonly found in practice the ıximation of the deductions to the facts is fairly * In every case it is assumed that the parts of ircuits have no appreciable mutual magnetic effect. e parts are mutually inductive, the solutions beentirely different and much more complicated.

e solutions for parallel circuits may be made by er method in which pressures and impedances are ved. This method may be best exemplified by :ations. Suppose, for instance, it is desired to find ıint impedance of the branched circuit in example *e* 52). It may be assumed that an impressed pressf 100 volts acts on the circuit. Upon a line, *OX*, senting this pressure (Fig. 74) is drawn a semi. From *O* draw the line *OA* making a lag angle of th *OX*, where $\tan \phi_1 = \frac{2\pi f L_1}{R_1} = .4$. Then *OA* is to $C_1 R_1$ and *XA* is equal to $2\pi f L_1 C_1$ since the at *A* is a right angle. The current in this branch the impressed pressure is equal to *OX*, is $\frac{OA}{R_1}$, his may be laid off from *O* to *B*. The current in

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mpare Bedell on hedgehog transformer with condenser, *Trans. 'nst. E. E.*, Vol. 10, p. 515.

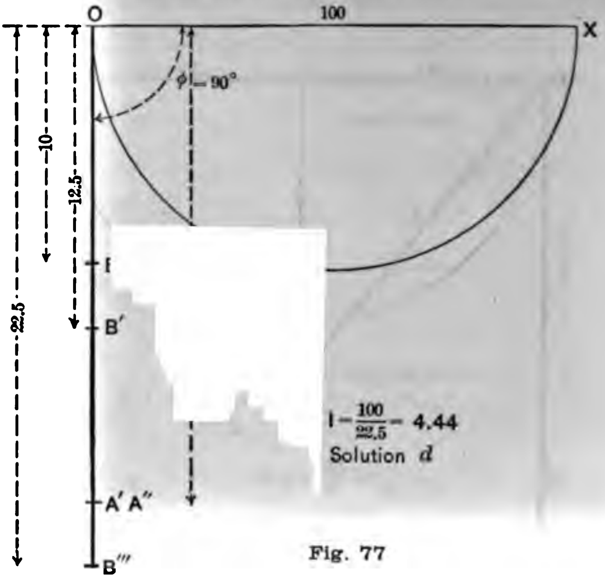

O
100
X
φ = 90°
10
12.5
22.5
B'
A' A''
B'''
I = 100/22.5 = 4.44
Solution *d*

Fig. 77

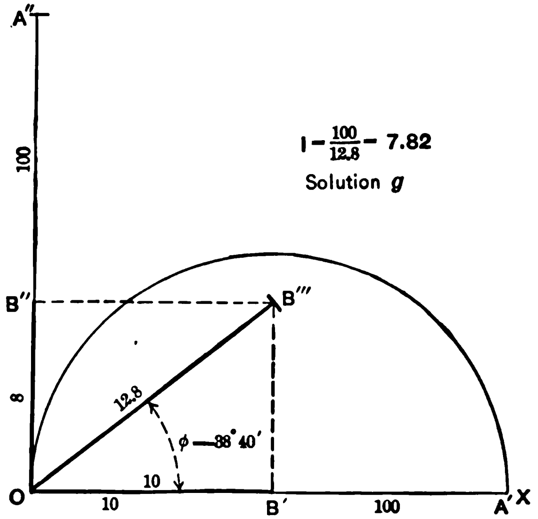
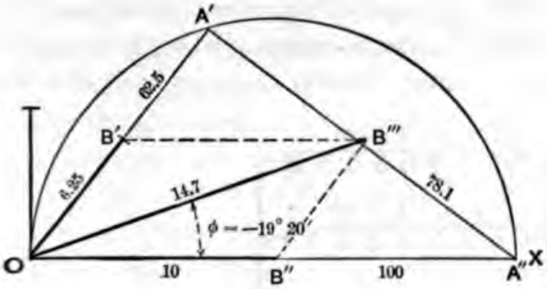

A''
100
I = 100/12.8 = 7.82
Solution *g*
B''
B'''
8
12.8
φ = 38°40'
10
O
10
B'
100
A'
X

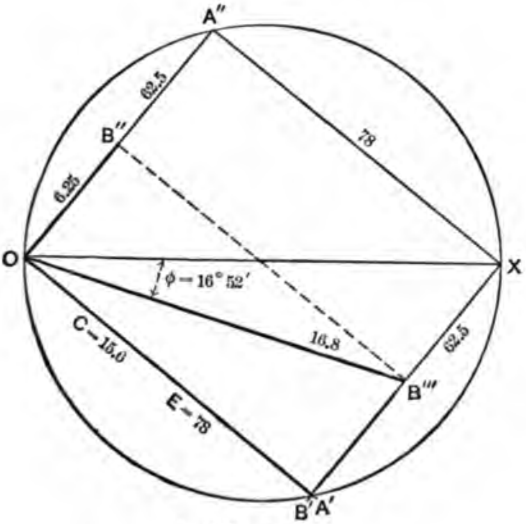
Fig. 78



$I = \frac{100}{14.7} = 6.8$

Solution *h*

Fig. 79

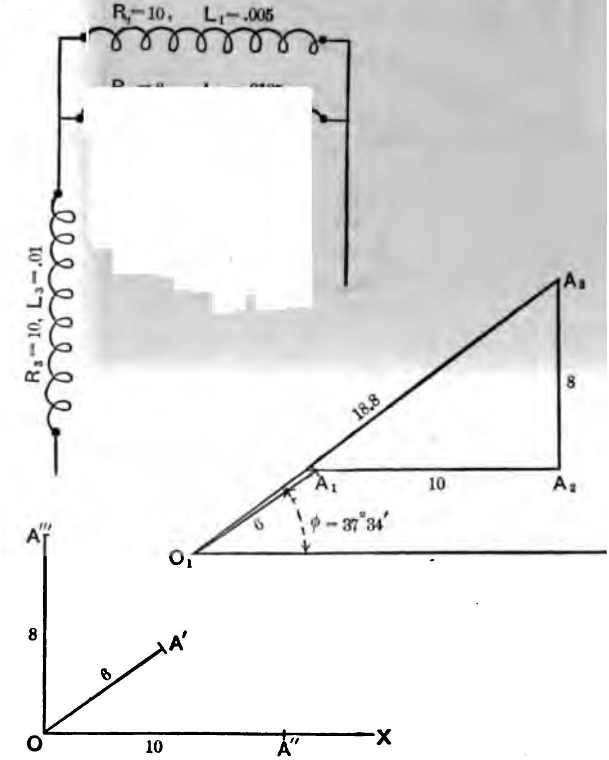


$I = \frac{100}{16.8} = 5.95$

Solution *j*

Fig. 80

and it simply requires experience to acquire con
able facility in the solutions relating to the most
plicated circuits. Several examples are given bel



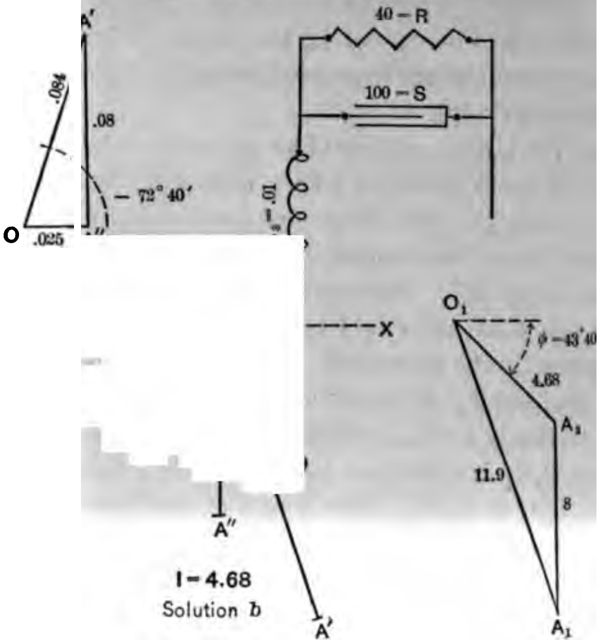
I = 18.8
Solution *a*

indicate the general procedure. In these example
desired to determine as before: (1) the total impe
of the circuit; (2) the total current flowing ur
pressure of 100 volts; (3) the pressure required to

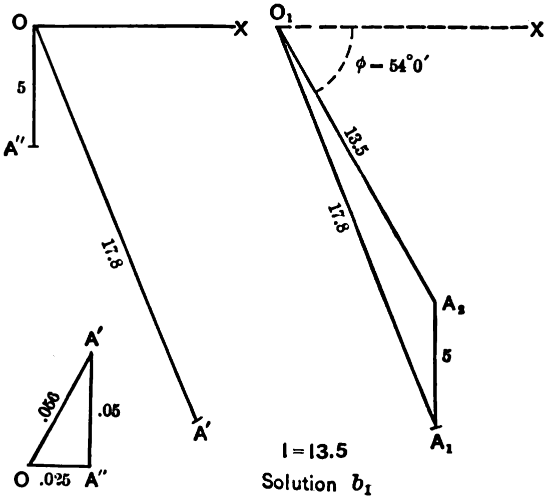
10 amperes to flow; and (4) the lag angle between the total current and the impressed pressure. The frequency is taken as 127½.

a. The circuit consists of an inductive coil of 10 ohms and .01 henry in series with a branched circuit similar to *e*, Sect. 52. We know that the parallel part of the circuit has an impedance of 6.06 ohms, and that the lag angle is 35° 16′. Therefore OA' is laid off in the phase diagram 6.06 units in length, and making the proper angle with the horizontal. The line OA'' is then laid off horizontally $R_3 (= 10)$ units long, and OA''' is laid off vertically $2\pi f L_3 (= 8)$ units long. In the vector diagram O_1A_1 is equal and parallel to OA', A_1A_2 to OA'', and A_2A_3 to OA'''. The length of the line O_1A_3 gives the impedance of the circuit, which is equal to 18.8 ohms. The current which flows under a pressure of 100 volts is 5.32 amperes, and it requires 188 volts to cause 10 amperes to flow through the circuit. The angle of lag, ϕ, is the angle A_3O_1X, and is equal to 37° 34′.

b. The circuit consists of an inductance of .01 henry in series with a branched circuit having two branches containing respectively 40 ohms and 100 microfarads. The joint impedance of the branched part of the circuit is first found in the usual manner. This is 11.9 ohms, and the lag is $-72°$ 40′. In the phase diagram, OA' is therefore laid off equal to 11.9 and making a lag angle of $-72°$ 40′, and OA'' is laid off $2\pi f L (= 8)$ units in length and making a lag angle of 90°. Laying off the vector diagram gives O_1A_2 equal to 4.68 and making a lag angle of $-43°$ 40′. The current flowing under a press-



I — 4.68
Solution *b*



I = 13.5
Solution b_1

ure of 100 volts is 21.4 amperes, and the pressure required to cause 10 amperes to flow is 46.8 volts. When 10 amperes flow in the main circuit, the pressure at the terminals of the branched circuit is 119 volts, and the currents which flow through the resistance and the condenser are respectively 3 amperes and 9.5 amperes. The pressure across the inductance L_3 is then 80 volts.

b_1. If the frequency in the preceding example is cut down to 80, the relations are materially changed. The impedance of the branched circuit becomes 17.8 ohms, and the lag angle in it is $-63° 32'$. The phase diagram, therefore, is as shown. From the vector diagram it is seen that the joint impedance of the whole circuit is 13.5 ohms, and the total current is 54° 0′ ahead of the impressed pressure. The total current flowing when the impressed pressure is 100 volts is 7.4 amperes, and it requires 135 volts to cause 10 amperes to flow. When 10 amperes are flowing, the pressure at the terminals of the branched circuit is 178 volts, and the currents which flow through the resistance and the condenser are 4.45 and 8.9 amperes respectively, while the pressure across the inductance L_3 is 50 volts. To maintain a pressure of 100 volts at the terminals of the divided circuit requires an impressed pressure of 76 volts. With this pressure 5.6 amperes flow through the circuit.

c. The circuit consists of a combination as shown in the figure on page 204. The resistances of the branches of the circuit are $R_1 = 5$ ohms, $R_2 = 10$ ohms, $R_3 = 8$ ohms; the inductances are $L_1 = .005$ henry, $L_2 = .01$ henry, $L_3 = .0125$ henry; and the capacities are $s_1 = 150$ microfarads, $s_2 = 100$ microfarads, $s_3 = 125$ microfarads.

ample b_1 made by that method. In this it is assumed that 100 volts is impressed upon the branched part of the circuit. Then lay off a length OX on the horizontal axis representing 100 volts and mark $OC_1 = \frac{100}{40} = 2.5$, which is the current in the first branch. The current in the second branch is 90° in advance of the pressure, and is represented by OC_2 which is vertical and $2\pi f s_2 E\,(=5)$ units in length. The resultant of these currents is OC, which is 5.6 units in length. The impressed pressure measured across the terminals of the entire circuit is the resultant of the 100 volts at the terminals of the branched part of the circuit, and the pressure required to pass 5.6 amperes through the inductance $L_1 = .01$ henry. The line representing the latter pressure is perpendicular to the line representing the current in the circuit. Drawing a semicircle on OX, and from the intersection of OC with the semicircle drawing a line to X gives the direction of this pressure. The magnitude of the pressure is $2\pi f L_1 C = 28$ volts. This pressure must be laid off from X to E, and the total impressed pressure is represented by OE. This shows that when 100 volts is maintained at the terminals of the branched circuit, 76 volts must be impressed on the total circuit. The resistances of the circuit may be calculated from the data thus found, as also can the pressure required to maintain a certain current through the circuit.

Figure 82 shows the solution of example c by this method. As before, the pressure at the terminals of the divided circuit is assumed to be 100 volts for the purposes of the solution. This is laid down as OX, and a semicircle is drawn upon the line as a diameter.

ı ϕ_3 is equal to zero, so that the current in the : branch is laid off on *OX* to *B*, a distance of 12.5 ts. Tan $\phi_2 = .45$, as shown by calculation, and the : *OA'* is laid off at that angle from *OX*. From *O* this line, *OB'* is laid off equal to the current in the ond branch, or $\frac{1}{I} OX$. The resultant of the lines ʔ and *OB'* is *OB''*, which represents the total cur- ıt in the circuit. *OA''* is the equivalent active press-

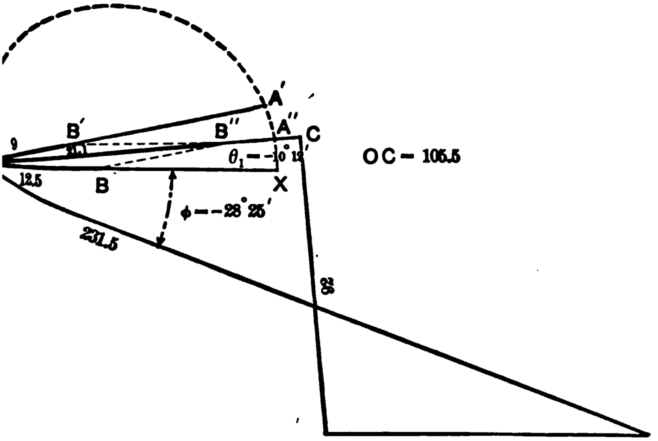


Fig. 82

e in the circuit. The total pressure impressed on the 'cuit is the resultant of the pressure impressed on : divided circuit, the active pressure due to resistance , and the reactive pressure due to L_1 and s_1. The ive pressure required to pass current *OB''* through is represented by *OC*, which is equal to CR_1. The :tive pressure is perpendicular to this and is equal to

$2\pi f L_1 C - \frac{C}{2\pi f s_1}$; it is represented by the line *CD*. The pressure impressed on the parallel circuit is represented by the line *DE*, which is equal and parallel to *OX*. The closing line, *OE*, represents the impressed pressure *E* on the circuit when the current is *C*, and the impedan[illegible] is $\frac{E}{C} = 10.97$. The angle of lag is the [illegible]8° 25′.

56. An An[illegible]—The problems just solved graphically n[illegible]y solved analytically.*

In Sect. 49 [illegible]hown that current, pressure, and impedance [illegible] determined in magnitud and relative phase by means of a polar diagram. Thus in Fig. 83, suppose *OX* to be the initial line an *OA′*, *OA″*, and *OA‴* to be pressures or impedances i series, or currents or conductances (reciprocals of impedances) in parallel, which are represented in relativ phase by the angular positions, and in magnitude b the lengths of the lines. It has just been shown th the resultant of two or more similar electrical quantitie may be found by treating their representative lines a vectors; such vectors may be combined by algebraically adding the vertical and horizontal components o the individual lines, by which means the vertical and horizontal components of the resultant are determined. If a', b'; a'', b''; and a''', b''' are the horizontal and

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* Steinmetz on Complex Quantities, *Proceedings of the International Congress held at Chicago in 1893*, p. 33; Steinmetz on Hysteresis, *Trans. Amer. Inst. E. E.*, Vol. 11, p. 576; Tait's *Quaternions*, Hardy's *Quaternions*, etc.

:rtical components of OA', OA'', and OA''', and A, B, ıe components of the resultant, then

$$A + B = (a' + a'' + a''') + (b' + b'' + b'''),$$

r $$A + B = (a' + b') + (a'' + b'') + (a''' + b''').$$

n this expression there is nothing to distinguish the ˌorizontal from the vertical components; or, in general,

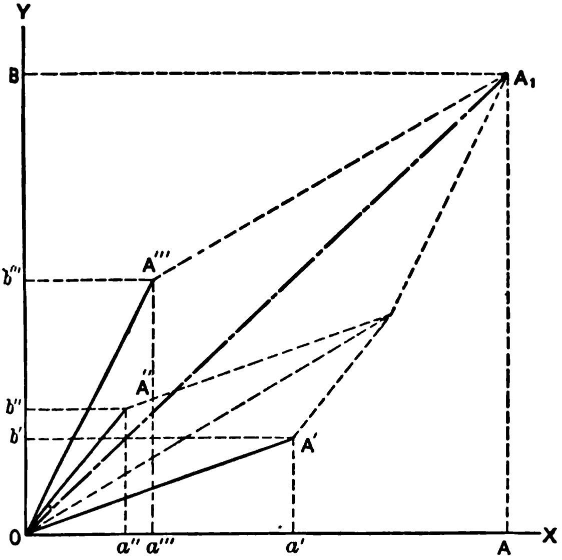

Fig. 83

here is nothing to indicate the angular positions of the omponents, or of the lines represented by them, with eference to the initial line. To fully indicate the magitude and position of a line by its rectangular comɔnents, we must abandon the methods of algebra for ɔometric processes. Therefore we may consider, for

P

the moment, that the components t and u of the vector A (Fig. 83 a) both lie on the initial line OX, but in order

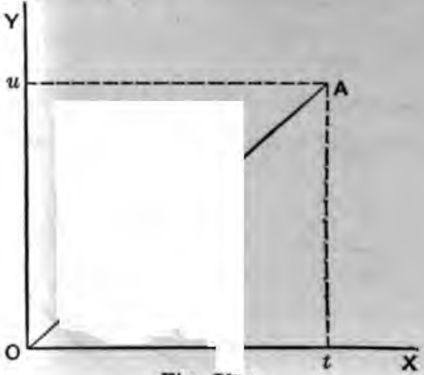

Fig. 83 a

that t and u may determine the vector A, u must be rotated 90°. To indicate such a rotation, a prefix such as i may be used. Then A will be represented in magnitude and angular position by the expression

$$t + iu,$$

where the sole function of the letter i is to indicate that the component u stands 90° from the initial line, and the addition is geometric. Inasmuch as iu is positive, u has been rotated ahead 90°; $-iu$ would indicate that u had been rotated in a negative direction 90°, or u is measured downwards (the negative direction) from the origin. If t and iu are both negative, they are both measured in the negative direction; hence, if $t + iu$ i multiplied by -1, there results $-t - iu$, t and u a both reversed in direction, and the vector line OA rotated 180° (Fig. 84); $it - u$ means that the line l been rotated forward 90°, since t is positive but stands at 90° from the initial line, and u is negative; $-it+u$ means that the line has been rotated back 90°. Finally, multiplying by i means advancing the vector line 90°,

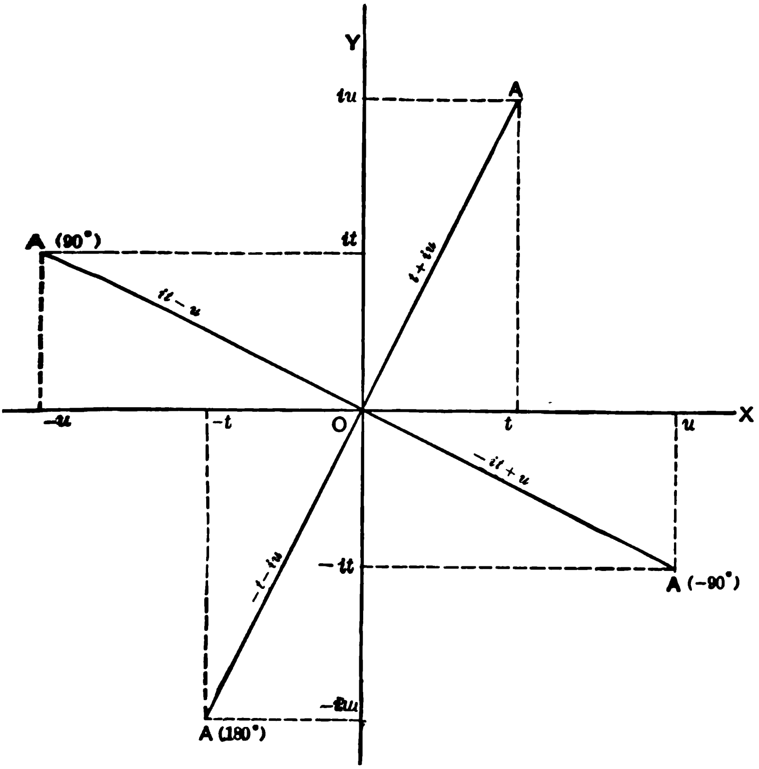

Fig. 84

for $i(t+iu) = it + (+i)(iu)$, and as i^2 indicates rotation twice forward, i^2u becomes $-u$, and therefore $i(t+iu)$ is geometrically equal to $it-u$; also multiplying by $-i$ means turning the vector back 90°, for $-i(t+iu)$

$= - it + (- i)(iu)$, and as $- i^2$ indicates rotation f ward 90° and back 90°, $- i^2 u = u$, and there resu $- it + u$.

The vector expressing a sine wave may now be repr sented in magnitude, as heretofore shown, by

$$\overline{{}^2 + u^2},$$

in phase b

$$= \frac{u}{t},$$

in phase a he complex quantity

$$i \sin \phi),$$

and also, as j. ndicat y the equivalent quantity

$$t + iu.$$

The addition of the vectors given in the first illustrat now becomes

$$A + iB = (a' + a'' + a''') + i(b' + b'' + b''').$$

Suppose it is desired to combine two impedan which are in series, as I' and I'', Fig. 85, in which r', and r'', l'' are the rectangular components (resistan and reactances). A sinusoidal pressure acting on must evidently overcome a self-inductive reactan equal and opposite to Ol' and a resistance Or', whi the pressure acting on I'' must overcome a capaci reactance equal and opposite to Ol'' and a resistan Or''. Then, if R, H, are the components of the resul ant (I'''), this equation may be written

$$I''' = R + iH = (r' + r'') + i(l' - l''),$$

from which the magnitude and phase position of the resultant impedance may be found.

If the impedances are in parallel, their reciprocals must be combined, in which case the resultant is the reciprocal of the impedance (conductivity) of the divided circuit. It is evident that the components of the conductivity (reciprocal of impedance) will not be equal to the reciprocals of the components of the impedance. The components of the conductivity must therefore be found in terms of the impedance components. If ρ, λ are the conductivity components and r, l the impedance components (resistance and reactance) of a single circuit, we may write by the principles of geometric multiplication,

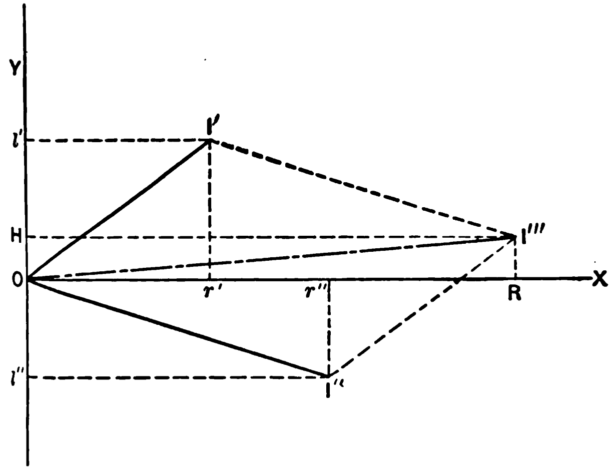

Fig. 85

$$K = \frac{1}{I} = \rho \mp i\lambda = \frac{1}{r \pm il}. \qquad (a)$$

The numerical values of the first and second terms of the right-hand member of the expression $K = \rho \mp i\lambda$ are respectively proportional to the active current and wattless current in a circuit. When a circuit contains inductive reactance only, l is essentially positive, but the wattless current *lags* 90° behind the active current, so that λ is essentially negative. When a circuit contains capacity reactance only, l is essentially negative, but the wattless curı ve current by 90°, so that λ is essentia en a circuit contains both inductive an ıce, the signs of l and λ are depende ative magnitudes of the inductance a e value of K may be written in a gene

$$K=\frac{1}{I}=\frac{1}{r+il_i-il_c}=\rho-i\lambda_i+i\lambda_c.$$

To reduce the equation

$$\rho \mp i\lambda=\frac{1}{r \pm il} \qquad (1$$

to a more convenient form, the numerator and denominator of the right-hand member may be multiplied b $r \mp il$, whence

$$\rho \mp i\lambda=\frac{r \mp il}{(r \mp il)\,(r \pm il)}=\frac{r \mp il}{r^2+l^2},$$

since i^2 indicates the operation which is equivalent to multiplying by -1.

Hence, $$\rho \mp i\lambda=\frac{r}{r^2+l^2} \mp i\frac{l}{r^2+l^2};$$

but the impedance (I) is

$$I=\sqrt{r^2+l^2}.$$

Therefore, $$\rho \mp i\lambda=\frac{r}{I^2} \mp i\frac{l}{I^2},$$

or
$$\rho = \frac{r}{I^2} \text{ and } \lambda = \frac{l}{I^2}$$

Since r, l, and I are known or can be determined from the conditions presented, problems relating to parallel circuits can now be solved.

Returning to the example, if ρ', λ', and ρ'', λ'' are the components of the conductivities of two parallel circuits having impedances I' and I'' (Fig. 85),

$$\rho' - i\lambda' = \frac{r'}{I'^2} - i\frac{l'}{I'^2}$$

and
$$\rho'' + i\lambda'' = \frac{r''}{I''^2} + i\frac{l''}{I''^2},$$

and if ρ, λ are the components of the final conductivity,

$$\rho + i\lambda = \left(\frac{r'}{I'^2} + \frac{r''}{I''^2}\right) + i\left(\frac{l''}{I''^2} - \frac{l'}{I'^2}\right).$$

The intrinsic sign of $i\lambda$ depends upon the relative signs and magnitudes of $\frac{l'}{I'^2}$ and $\frac{l''}{I''^2}$. The impedance of the circuit will be the reciprocal of the conductivity thus found.

When I is the impedance and K the conductivity of a circuit, we write $I = r \pm il$ and $K = \rho \mp i\lambda$, according to the principles of geometric addition which assert that the sum of two sides of a triangle taken in consecutive directions is equal to the third side. This is entirely opposed to the ordinary conceptions of algebra or arithmetic.

These processes which enable us to find the joint impedance of parallel or series circuits when the elements of the individual parts of the circuits are known, equally enable us to find the impedance of any combination of such circuits by computations which are almost as simple and rapid as those which would be used in dealing with a continuous-current system. Also, when the impedances of any combination of circuits have been obtained, it is the pressures in any portion when a si is flowing and to find the current when ssure is applied.

The meani s in the expression for impedance ar may be explained by: multiplying (r ent in the circuit), when it is evident th tive pressure and IC the component of pressure acti against the reactance; and also by multiplying $(\rho + i\lambda)$ by E_i (pressure impressed on the circuit), when ρE_i is the active component of the current and λE_i the wattless current.

The following is a recapitulation of the formulas for the analytical solution, by geometric processes, of problems relating to alternating-current circuits.

GEOMETRIC EQUATIONS.

$$I = r + il_L - il_s.$$

$$I_x = \Sigma I = \Sigma r + i\Sigma l_L - i\Sigma l_s$$

$$= \Sigma r + i\Sigma(l_L - l_s)$$

$$= I_x(\cos\phi_x + i\sin\phi_x).$$

$$K = \frac{1}{I} = \rho - i\lambda_L + i\lambda_s$$

$$= \frac{r}{I^2} - i\frac{l_L}{I^2} + i\frac{l_s}{I^2}.$$

$$K_x = \Sigma K = \Sigma\frac{r}{I^2} - i\Sigma\frac{l_L}{I^2} + i\Sigma\frac{l_s}{I^2}$$

$$= \Sigma\frac{r}{I^2} - i\Sigma\left(\frac{l_L}{I^2} - \frac{l_s}{I^2}\right).$$

ALGEBRAIC EQUATIONS.

$$\rho = \frac{r}{I^2} = rK^2, \quad \lambda = \frac{l}{I^2} = lK^2.$$

$$r = \rho I^2 = \frac{\rho}{K^2}, \quad l = \lambda I^2 = \frac{\lambda}{K^2}.$$

$$I = \sqrt{r^2 + l^2} = \frac{r}{\cos\phi} = \frac{l}{\sin\phi}.$$

$$K = \sqrt{\rho^2 + \lambda^2} = \frac{\rho}{\cos\phi} = \frac{\lambda}{\sin\phi}.$$

$$\tan\phi = \frac{l}{r} = \frac{\lambda}{\rho}.$$

$$C = \frac{E}{I} = EK.$$

For convenience in computations the geometric equations should be set out, for example, as follows:

$$\begin{array}{r|l|l} \underline{r} & \underline{l_L} & \underline{l_s} \\ I' = r' & + \, i l' & \\ I'' = r' & + \, i l_L'' & - \, i l_s'' \\ I''' = r''' & & - \, i l''' \\ \hline I_x = \Sigma r & + i \Sigma l_L & - i \Sigma l_s \end{array}$$

$$= r_x \pm i l_x.$$

57. Illustration of Analytical Method.—In illustration of this method, solutions to some of the problems to be found on the foregoing pages are given. *I*, *K*, and ϕ are used to represent respectively impedance, conductivity, and lag.

Series Circuits (see Sect. 50).

Forming the equations for the non-inductive and inductive coils in problems *c*, *h*, *i*, and *j* of Sect. 50.

c.

$$\begin{array}{l} I_{\text{I}} = 5 + i\,8 \\ I_{\text{II}} = 5 + i\,0 \\ \hline I = 10 + i\,8 \end{array}$$

$$C = \frac{100}{I} = \frac{100}{12.8} = 7.82.$$

$$\tan \phi = \frac{8}{10} = .8, \quad \phi = 38^\circ \, 40'.$$

$$I = \frac{r}{\cos \phi} = \frac{l}{\sin \phi} = 12.8.$$

$$E = 10\,I = 10 \times 12.8 = 128.$$

As the prefix of the reactance term in the expression for *I* is $+i$, the angle ϕ is positive.

h.

$$\begin{array}{l} I_{\text{I}} = 0 + i\,12 \\ I_{\text{II}} = 0 - i\,10 \\ \hline I = 0 + i\,2 \end{array}$$

$$C = \frac{100}{2} = 50.$$

$$\tan \phi = \frac{2}{0} = \infty, \quad \phi = 90^\circ.$$

$$I = \frac{l}{\sin \phi} = \frac{2}{1} = 2.$$

$$E = 10 \times 2 = 20.$$

i.

$$\begin{array}{l} I_{\text{I}} = 0 + i\,12.5 \\ I_{\text{II}} = 0 - i\,12.5 \\ \hline I = 0 + 0 \end{array}$$

$$I = 0.$$

j.
$$I_1 = 0 + i10$$
$$I_{11} = 0 + i8$$
$$I_{111} = 0 - i12.5$$
$$I = 0 - i\,4.5$$
$$C = \frac{100}{0.96} = 9.12.$$

$$\tan\phi = \frac{4.5}{10},\ \phi = -24^\circ\,14'.$$
$$I = \frac{r}{\cos\phi} = \frac{l}{\sin\phi} = 10.9$$
$$E = 10 \times 10.96 = 109.6.$$

Here eactance term in the e sion fo angle ϕ is negative.

its (see Sect. 52).

Appl r conductance,

b.
$$K_1 =$$
$$K_{11} = 0 \qquad - i\frac{8}{64+0}$$
$$K = .1 - i\,.125$$
$$C = \frac{100}{I} = 100\,K = 16.$$

$$\tan\phi = \frac{.125}{.1} = 1.25;\ \phi = 5$$
$$K = \frac{\rho}{\cos\phi} = \frac{\lambda}{\sin\phi} = .16.$$
$$I = \frac{1}{K} = \frac{\cos\phi}{\rho} = 6.25.$$
$$E = 10 \times 6.25 = 62.5.$$

ϕ is positive in this case, as $-i$ in the expressi K shows that the current lags behind the pressure

e.
$$K_1 = \frac{10}{100+16} - i\frac{4}{100+16}$$
$$K_{11} = \frac{8}{64+100} - i\frac{10}{64+100}$$
$$K = .1349 - i\,.0954$$
$$C = \frac{100}{I} = 100\,K = 16.5.$$

$$\tan\phi = \frac{.0954}{.1349} = .7072.$$
$$\phi = 35^\circ\,16'.$$
$$K = \frac{.1349}{\cos\phi} = .165.$$
$$I = \frac{1}{.165} = 6.06.$$
$$E = 10 \times 6.05 = 60.6.$$

h.
$$K_1 = \frac{10}{100+0} \quad - i\frac{0}{100+0}$$
$$K_{11} = \frac{10}{100+156} + i\frac{12.5}{100+156}$$
$$K = .1390 + i\,.0488$$
$$C = \frac{100}{I} = 100\,K = 14.7.$$

$$\tan\phi = \frac{.0488}{.1390} = .3509.$$
$$\phi = -19^\circ\,20'.$$
$$K = \frac{.1390}{\cos\phi} = .147.$$
$$I = \frac{1}{.147} = 6.8.$$
$$E = 10 \times 6.8 = 68.$$

ϕ is negative in this case, as $+i$ in the expression for K shows that the current leads the pressure.

j. $K_1 = \frac{5}{25+16} - i\frac{4}{25+16}$

$K_{11} = \frac{10}{100+156} + i\frac{12.5}{100+156}$

$K = .1610 - i\,.0487$

$C = \frac{100}{I} = 100\,K = 16.8.$

$\tan\phi = \frac{.0487}{.1610} = .3025,\ \phi = 16^\circ\ 50'.$

$K = \frac{.1610}{\cos\phi} = .168.$

$I = \frac{1}{168} = 5.95.$

$E = 10 \times 5.95 = 59.5.$

l. $K_1 = \frac{10}{100+69.4} - i\frac{8.33}{100+69.4}$

$K_{11} = \frac{10}{100+69.4} + i\frac{8.33}{100+69.4}$

$K = .1181 - i0$

$C = \frac{100}{I} = 100\,K = 11.8.$

$\tan\phi = \frac{0}{.1181},\quad \phi = 0.$

$K = \frac{.1181}{\cos\phi} = .1181.$

$I = \frac{1}{.1181} = 8.47.$

$E = 10 \times 8.47 = 84.7.$

The effect of the reactances in this circuit is noteworthy.

o. $K_1 = \frac{0}{64+0} - i\frac{8}{64+0}$

$K_{11} = \frac{0}{156+0} + i\frac{12.5}{156+0}$

$K = 0 - i.045$

$C = \frac{100}{I} = 100\,K = 4.5.$

$\tan\phi = \frac{.045}{0} = \infty.$

$\phi = 90^\circ.$

$K = \frac{.045}{\sin\phi} = .045.$

$I = 22.2.$

$E = 10 \times 22.2 = 222.$

Series and Parallel Circuits Combined (see Sect. 55).

a. $K_A = .0862 - i\,.0345$

$K_B = .0487 - i\,.0609$

$K_{AB} = .1349 - i\,.0954$

$I_{AB} = 6.06\,(\cos\phi_{AB} + i\sin\phi_{AB})$

$= 6.06\,(.8165 + i\,.5774)$

$= 4.95 + i\,3.50.$

$I_{AB} = 4.95 + i\,3.50$

$I_C = 10 + i\,8$

$I = 14.95 + i\,11.50$

$\tan\phi_{AB} = \frac{.0954}{.1349},\quad \phi_{AB} = 35^\circ\ 16'.$

$K_{AB} = \frac{.1349}{\cos\phi} = .165.$

$I_{AB} = \frac{1}{.165} = 6.06.$

$\tan\phi = \frac{11.50}{14.95},\quad \phi = 37^\circ\ 34'.$

$I = 18.8.$

$C = \frac{100}{18.8} = 5.32.$

$E = 10 \times 18.8 = 188.$

b_1. $K_A = .025 - i\,0$

$K_B = .0 \quad + i\,.0503$

$K_{AB} = .025 + i\,.0503$

$I_{AB} = 17.8\,(\cos\phi_{AB} + i\sin\phi_{AB})$

$\quad = 17.8\,(.4$

$I_{AB} = 7.937 -$

$I_C = 0 \quad +$

$I \quad = 7.937 -$

$$\tan\phi_{AB} = \frac{.0503}{.025},\ \phi_{AB} = -63^\circ\ 32'.$$

$$K_{AB} = \frac{.025}{\cos\phi} = .0561.$$

$$I_{AB} = \frac{1}{.0561} = 17.8.$$

$$\tan\phi = \frac{10.922}{7.937},\ \phi = -54^\circ\ 0'.$$

$$I = \frac{7.94}{\cos\phi} = 13.5.$$

$$C = \frac{100}{13.5} = 7.4.$$

$$E = 10 \times 13.5 = 135.$$

CHAPTER V.

THE MAGNETIC CIRCUIT OF ALTERNATORS.

58. Losses in an Alternator. — The principles which enter into the design of alternators have already been thoroughly set forth in Vol. I. and in the first chapter of these notes. There are, however, certain peculiar features in the magnetic circuits and the methods of applying the windings to alternators that require considerable modifications of the deductions found in Vol. I. These will now receive examination in detail. As in continuous-current dynamos (Vol. I., p. 248), the internal losses of alternators are caused by:

1. C^2R *loss* in the conductors on armature and field.
2. *Foucault* or *eddy currents* in armature cores and field.
3. *Foucault* or *eddy currents* in armature conductors.
4. *Hysteresis* in armature cores.
5. *Friction* of bearings and brushes, and air friction.

In well-designed continuous-current dynamos, the pole pieces usually cover not less than two-thirds of the armature surface (Vol. I., pp. 167 and 272). In alternators, the poles usually cover about one-half of the armature surface, or even less (Sect. 5). This would make it appear, upon a superficial examination, that the field ampere-turns, and therefore the field losses, must be much greater in the alternator. However, since alternator armatures are made proportionally larger in diameter, in order to give space for the windings and to avoid excessive magnetic leakage, the proportional excitation really required need not be much increased when the magnetic designed. In the same way, while n an one-half of the armature surface i ire, the surface for winding is made increasing the diameter, while the nun ns is not much reduced. Consequently, oduced in a given length of conductor mensurable in the two classes of machines. Thi llustrated by the table on page 14, and by the following machines of three excellent makers:

1. Two-pole continuous-current dynamo of 60 K.W. output; armature core, 15″ diameter, 15″ long; winding requires 185 lbs. wire; C^2R_a loss, 2.4 per cent; speed, 900 revolutions per minute; fields with 15,000 ampere-turns at full load; field wire, 470 lbs.; total C^2R_f loss, 1.7 per cent; total weight of the machine, 10,000 lbs.

2. Four-pole continuous-current dynamo of 75 K.W. output; armature core, 22″ diameter, 17¼″ long; winding requires 235 lbs. wire; C^2R_a loss, 3.0 per cent; fields with 15,000 ampere-turns at full load; field winding requires 756 lbs. wire; total C^2R_f loss, 2.3 per cent; total weight of machine, 11,000 lbs.

3. Alternating-current dynamo of 70 K.W. outpu armature core, 24″ diameter, 18½″ long; winding requir 70 lbs. wire; C^2R_a loss, 1.6 per cent; speed, 1050 revol

tions per minute; fields with 25,000 ampere-turns at full load; field winding requires 725 lbs. wire; total C^2R, loss, 2.4 per cent; total weight of the machine, 9500 lbs.

The table on pages 224 and 225 gives data of two English alternators built by the firm of Elwell & Parker,* and of five American machines of about equal capacity. All but one of these have drum armatures, but in the English machines they are stationary and surround the revolving poles, while in the American machines the poles surround the revolving armature. The armature of one of the American machines is of the disc type. All of the American machines were built about 1890, but all except one have since been superseded by a more substantial construction.

59. Copper Losses. — We may safely say that the percentage C^2R losses given upon pages 108 and 138 of Vol. I. can be taken as a limit towards which practice in the design of alternators is tending. The fact that the copper is divided among many cores increases the length of wire on alternator fields for a given magnetizing power as compared with continuous-current fields, and the C^2R losses allowed are usually somewhat greater than the tabular values and sometimes reach more than twice those values. (Examples: Kapp 30 K.W. alternator with 5.0 per cent loss in the field windings and 2.8 per cent in the armature conductors; Ferranti 112 K.W. alternator with 2.75 per cent loss in the field windings; and General Electric 300 K.W. alternator with 2.0 per cent loss in the field windings.) On the other hand, the losses may be brought by careful designing to approach the average values given in the tables. (Examples: Stanley 40 K.W. alternator with equal losse of 1.2 per cent in fields and armature, and 180 K.W alternator with .7 per cent loss in fields and 2.0 pe cent loss in armature; Mordey 75 K.W. alternato with 1.5 per [illegible] loss in fields and 2.3 per cent i armature; H[illegible] alternator with 1.6 pe cent loss in a[illegible] r cent in fields.) These examples ma[illegible] indefinite number, but those given [illegible] ditions. The following table may th[illegible] showing the percentage C^2R losses, [illegible] he best present practice, and towards [illegible] practice in the design of alternators must eventu[illegible]ly [illegible] nd. At the present time it is not unusual to build alternators of greater capacity than 75 K.W. with upwards of double the field loss give in this table, notwithstanding the fact that the tab gives values for the loss in the fields of alternato which are considerably greater than the tabular val for continuous-current machines (Vol. I., p. 138).

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* See Thompson's *Dynamo-Electric Machinery*, 4th ed., p. 668.

Kilowatts.	Per Cent in Armature.	Per Cent in Fiel
30	2.4	2.8
35	2.3	2.6
40	2.25	2.4
50	2.2	2.2
60	2.15	2.0
75	2.1	1.8
00	2.0	1.7
150	1.9	1.6
200	1.9	1.6

Table based on cold resistances at about 25° C. (75° F.

60. Foucault Current Losses. — Foucault currents in lternator cores must cause much greater loss than in ordinary continuous-current machines. Thus, the armatures of two-pole continuous-current machines of considerable capacity seldom exceed 1500 revolutions per minute, which makes twenty-five complete magnetic cycles in the core per second. In alternators, the commercial frequency commonly lies between 40 and 140, while the greater number of machines are built for frequencies between 60 and 135. The number of magnetic cycles of the armature core is evidently equal to the frequency. Since the heating in the core discs which is caused by foucault currents is proportional to the square of the number of cycles per second, it becomes particularly important that the discs composing alternator armatures be well insulated from each other. It is therefore poor policy to depend upon oxide alone for insulation, and tissue paper should always be placed between the discs, and a cardboard be inserted at intervals. All burrs caused by punching the discs or truing the surface of the core should be carefully avoided or removed.

Foucault currents in the pole pieces are felt quite severely in some types of alternators, but the loss caused by them can always be brought within reasonable limits, in well-designed machines, by making the pole faces of laminated iron which is cast into the frame. (Compare Vol. I., p. 154.) Machines having smooth armature cores and surface windings do not ordinarily require this precaution, but in machines having toothed armature cores, laminated poles are quite important.

The loss due to foucault currents in armature conductors of a fixed size is much greater for alternating than for continuous-current dynamos, on account of the more frequent and sudden changes in the strength of the field through which they pass. However, since alternators are, in general, built for considerably higher pressures than co [illegible] nachines designed for a similar duty, t [illegible] the alternator armatures are of proporti [illegible] ss-section. This reduces the eddy curr [illegible] extent that they are not particularly n [illegible] in very large or special machines. Th [illegible] ding armature coils with copper ribbon [illegible] broad side parallel with the lines of fo [illegible] so [illegible] decrease eddy currents. In very large machines built to generate a pressure not exceeding 2000 volts, armature conductors of a large cross-section become essential, but the uniform practice of building large alternator armatures with imbedded conductors avoids all difficulty from eddy currents. (Compare Vol. I., p. 153.)

61. Hysteresis Losses. — The effect of hysteresis in iron core armatures is proportional to the number of magnetic cycles per second, and is therefore much greater in alternator armatures, for a given magnetic density, than in those of continuous-current machines. There are only two ways of decreasing the hysteresis loss per cycle and per unit volume: (1) by reducing the induction; (2) by improving the quality of the iron used in the core. Reducing the induction serves to decrease the foucault current loss also, and is therefore doubly advantageous. As a consequence, the induction in alternaatures is sometimes made as low as 4000 lines of er square centimeter, and seldom exceeds 7000 er square centimeter. A fair average value is 000 lines per square centimeter. This is just ne-half the average value for continuous-current res (Vol. I., p. 111). Halving the induction tends ter the foucault-current loss for an equal number es per second. But, as already shown, the num- cycles in alternator armatures commonly varies ree to six times the maximum number occurring inuous-current machines of the same capacity, so e special precautions for insulating the discs can- neglected. This is rendered more necessary by that reducing the induction requires the arma- be increased in size. In the same way, the sis loss varies nearly in the proportion of B to power, or in other words, halving the induction es the hysteresis loss per unit volume to one- On the other hand, since the cross-section of ust be increased to decrease the induction per centimeter, it is not possible to bring the hyste- ss to the limit attainable in continuous-current es. This makes the careful selection and hand- the iron designed to enter alternator cores of importance. Some manufacturers of electrical ery not only select their armature iron with great this account, but anneal all armature discs after ve been assembled and the surface of the core has rned up. For this purpose, the cores are carefully own after all machine work on the discs has been ed and the discs are annealed, after which the cores are again assembled. This method of handling the iron removes the hardening effect of the tooling consequent upon turning up the cores (Vol. I., pp. 52 and 73).

62. Armature Ventilation. — Since the hysteresis and foucault-current losses are greater in alternator armatures, it is necessary that more opportunity be given for cooling than [illegible] tinuous-current machines. The real effe[illegible] of rotation upon cooling has never be[illegible] termined, but the experiments of Re[illegible] to show a cooling effect that is rougl[illegible] to the velocity. Consequently, on a[illegible] igh surface velocity, alternator armatu[illegible] ficiently cooled than are continuous-curre[illegible] Internal ventilation is also rendered more effective by reason of the high surface velocity. For the purpose of internal ventilation a revolving armature is virtually converted into a centrifugal blower, which sucks in air at its centre, along the shaft, and ejects it from the surface. For this purpose air ducts are made, which run through the core near the shaft, and which communicate with the surface through radial ducts. The rotation of the armature then causes a continuous circulation of air through the ducts, which is proportional in volume to the surface velocity of the armature.[†] Since the conductors cover only a portion of the surface and do not cross the ends of drum armatures, the cores may be easily arranged for internal ventilation, and advantage is usually taken of

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[*] *Bulletin de la Société Internationale des Électriciens*, 1892; *Electrical World*, Vol. 19, p. 336; also *Textbook*, Vol. 1, p. 109.

[†] Kent's *Mechanical Engineer's Pocket Book*, p. 516.

this. In the case of revolving disc armatures, wings may be so placed as to blow the air across the face of the disc. Ring armatures and pole armatures are more difficult to arrange for effective internal ventilation, and therefore they are frequently left unventilated. When the fields revolve, the poles cause a vigorous circulation of air, which serves in lieu of the blower action of a revolving armature.

63. Armature Radiating Surface. — With all precautions to avoid excessive heating in alternator cores they still tend to heat to a higher temperature than the cores of continuous-current armatures. It then becomes a matter of some concern to determine the possible amount of energy which may be expended in the armature conductors without placing an excessive additional burden upon the cooling surface. There is no valid reason for admitting a higher temperature limit in alternator armatures than is allowed in continuous-current machines (Vol. I., p. 105). As this fact becomes entirely admitted by all manufacturers of alternating-current machinery, the large C^2R losses in alternator armatures that are now common will rapidly disappear. On account of the large amount of heat caused by core losses which must be radiated, it is not safe to allow an armature C^2R loss of one watt for each square inch of cooling surface. In common practice, for each watt of C^2R_a loss an average of from $1\frac{1}{2}$ to 2 square inches of outside *winding surface* is allowed. This constant is sometimes made much larger, but seldom smaller except in disc armatures. In the latter the core losses need not be provided for, the whole capacity of the radiating surface may be utilized to carry off the C^2R_a loss, and it is possible to decrease the radiation surface to considerably less than one square inch per watt.

64. Density of Current in Armature Conductors.—The density of current in conductors wound upon iron-core armatures in good practice should usually not much exceed [illegible] from 500 to 600 circular mils. (Comp[illegible] urrent dynamos, Vol. I., p. 110.) Mar[illegible] tures are wound with wire having not m[illegible] ne-half this cross-section per ampere, b[illegible] llow an excessive rise of temperature i[illegible] during working. In disc armatures wit[illegible] nstant may be greatly reduced without [illegible] r. [illegible] he Mordey 75 K.W. alternators have a current density in the armatures of one ampere to about 380 circular mils, and the current density in Ferranti alternator armatures is about one ampere to 335 circular mils. Even with such a great current density, these machines are comparatively cool in operation.

65. Field Radiating Surface.—Since the field cores of alternators are usually quite thin, the windings are often of a depth equal to one-half the thickness of the cores. At the same time this depth is generally no greater than that on many continuous-current dynamo fields. The same radiating constant can therefore be safely used, that is, from .35 to .40 watt per square inch of the outer surface of the winding (Vol. I., p. 142). Since the depth of the winding generally bears a large ratio to the thickness of the core, the energy radiated per square inch of core surface is much greater. Th ratio of winding depth to thickness of core is also wide ariable, and the radiation constant must be based upon he external surface of the windings. The field cores hould therefore be made of such a length that the area f the external surface of the windings in square inches nay be from two to three times the C^2R_f loss in watts. he form of the fields, and therefore the effect of the ron in conducting away the heat in the field windings, as a considerable effect on the allowable value of the onstant, as is also the case in continuous-current mahines. The area of the field cores is given by the formula, $A = N_a\frac{v}{B_f}$, where N_a is the useful armature flux required from one pole, v is the leakage coefficient, and B_f the induction desired in the field core. The pole width is determined by the mechanical and electrical conditions which fix the pitch, and only the length of the pole faces may be altered to vary B_f. If the winding depth exceeds two inches, it is likely to cause injurious heating in the lower layers.

66. Leakage Coefficient.—The value of the leakage coefficient is quite large in most types of alternators. It probably averages 1.5 in alternators with surface wound drum or ring armatures and with poles set in a circle either without or within the armature. In pole armatures it is doubtless equally as great, but in machines having well-designed, toothed, ring or drum armatures, and small air spaces, the leakage coefficient may be less. In machines with ring or disc armatures in which poles of opposite sign are ranged alternately on either side of the armature, the value of the coefficient may be as great as 2.0. In machines of the Mordey type, where the poles on each side of a disc armature are of th same sign, or of the Stanley so-called inductor type, th leakage coefficient becomes unity; that is, practicall all of the lines of force set up in the magnetic circui cut the armature conductors, and are therefore usefu in developing electric pressure.

The calc kage coefficient of alterna tors may be the same methods as thos used for cor nachines.*

67. Deter **Number of Armature Con- ductors.**—T $\frac{2KSNVp}{10^8 \times 60}$ (Sect. 5) may be put into the Vf —, since $\frac{Vp}{60}$ is equal to the frequency, f. Taking per value for K, as already explained, gives the effective value of E. In a well-designed machine of the usual American types, the value of K is about 1.1, which is the value for a machine which gives a sinusoidal electric pressure and in which the differential action is negligible. The con ditions of service usually fix the values of E and f i any particular case, and the equation then contains tw dependent variables, N and S. The ratio of these i determined from the form and dimensions of the arma ture which it is desired to make. The number of poles is limited by constructive considerations and the impor tance of keeping the leakage within reasonable limits. On the latter account the poles must not come too near together. With this in view the frequency cannot be increased beyond a certain limit by increasing the n ber of poles, without carrying the periphery velocity of the armature beyond the safe limit. Thus, suppose a machine be designed with a ten-pole stationary field, and its drum armature be designed to drive at the safe limit of velocity. If it be desired to increase the frequency by 20 per cent, two poles must be added to the field and the armature revolutions kept constant, or the armature must be speeded up 20 per cent. The latter is not permissible on account of mechanical safety. If the poles are already as close together as economy admits, in order to increase the number of poles the pitch circle must be increased in diameter, which requires a proportional increase of the armature diameter. This again calls for an increase of the surface velocity, the revolutions per minute remaining constant. Hence in any type of machine a limit of frequency may be reached which cannot be safely exceeded. The limiting frequencies in the ordinary types of machines designed for commercial service are from 100 to 150. In the Mordey type the limit is much higher, since the poles may be very close together and cause no leakage, and structural considerations, only, set the limit. Commercial frequencies are all less than 150 and many are less than 100, so that all practical types of alternators may be used in commercial service.

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* *Textbook*, Vol. 1, p. 133; also *Electrical Engineer*, Vol. 18, p. *et seq.*

Fixing the frequency of a machine and the periphery velocity and revolutions of the revolving part, fixes both the diameter of the armature and the number of poles. The diameter of the armature fixes the space for armature coils. With the winding space fixed, the value of S is determined from the number of insulated conduct-

ors of requisite area which can be properly placed
the armature. It is quite usual to wind alternator a
tures with only two layers of wire, but this must be
termined in any particular case by many condit
affecting the design. The value of S being d
mined, the length of the armature must be made
that the agnetization may be set
in the mature without forcing
magnetic ch a value. Finally the
of radiat , loss should be checked.

It is ere the effect upon the
put of a king a change in the num
of armat The electric pressure d
oped in the coils due to their cutting lines of for
proportional to the number of turns in the coils.
electric pressure of self-induction, on the other h
is proportional to the square of the number of t
Hence, increasing the number of armature turns be
a certain limit may actually decrease the output of
machine. It is desirable to determine the numb
turns that will give maximum output. If L_1 repres
the effective or working self-inductance for each a
ture conductor, then the total effective self-induct
of the armature is $L = S^2L_1$. In the same wa
E_1 represents the electric pressure developed per
ductor, the total pressure developed in the arma
is $E = SE_1$. From the fundamental formula

$$C = \frac{E}{\sqrt{R^2 + 4\pi^2 f^2 L^2}},$$

we get

$$C^2R^2 = E^2 - 4\pi^2 f^2 C^2 L^2,$$

or
$$CR = \sqrt{E^2 - 4\pi^2 f^2 C^2 L^2}.$$

This may be put into the form
$$CR = \sqrt{S^2 E_1^2 - 4\pi^2 f^2 C^2 S^4 L_1^2},$$

if the external circuit to which the alternator is connected is non-inductive. The term, CR, is the active pressure in the alternator circuit, and it is desirable for this to be a maximum for a given armature. Differentiating the equation with respect to S and solving for a maximum, we get the following:
$$\frac{d(CR)}{dS} = \frac{S(E_1^2 - 8\pi^2 f^2 C^2 S^2 L_1^2)}{\sqrt{S^2 E_1^2 - 4\pi^2 f^2 C^2 S^4 L_1^2}} = 0,$$

hence $E_1^2 - 8\pi^2 f^2 C^2 L_1^2 S^2 = 0$. Or CR is a maximum when
$$S = \frac{E_1}{2\sqrt{2}\,\pi f C L_1}.$$

In this it is assumed that the armature resistance is small compared with that of the external circuit, which is always the case in efficient working. For E_1 may be substituted its value
$$E_1 = \frac{2KNf}{10^8},$$

and the expression for S at maximum output becomes
$$S = \frac{KN}{10^8\sqrt{2}\,\pi C L_1} = \frac{N}{CL_1}A,$$

where A is a constant depending upon the type and dimensions of the machine. If K has a value of 1.1, the value of A is practically 25×10^{-10}. The final form of the variable portion of the expression giving the maximum economical value of S is striking. Its numerator is the total useful magnetization due to the fields, passes through an armature coil, and its denomi is the magnetization passing through the coil due t current in each of its own conductors.* This crit shows that the armatures of some old-style ma alternators had too much wire for economy; th they would have supplied a larger pressure to a inductive circuit if fewer conductors had been on their armatures.

When the external circuit is inductive, as is u the case, the number of armature turns which gi maximum active pressure is smaller than when th ternal circuit is non-inductive. If S' be the numb conductors giving the maximum active pressure the external circuit has self-inductance L_e, and S b number of conductors giving the maximum pre when the external circuit is non-inductive, then

$$S^2L_1 = S'^2L_1 + L_e \quad \text{or} \quad \frac{S'^2}{S^2} = 1 - \frac{L_e}{S^2L_1},$$

or

$$\frac{S'}{S} = \left(1 - \frac{L_e}{S^2L_1}\right)^{\frac{1}{2}}.$$

In the latter expression S^2L_1 is the self-inducta the armature when wound with the proper numb turns to give a maximum active pressure whe external circuit is non-inductive. When L_e is g than S^2L_1, the right-hand member of the expressi $\frac{S'}{S}$ becomes imaginary. It is impossible to put so turns on commercial alternator armatures as the cri shows would give the greatest output, since the tion of regulation in constant-pressure alternators demands that the fall of pressure in the armature due to resistance and inductance shall be as small as possible.*

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* Picou, *Machines Dynamo-Électriques*, p. 271.

68. Example of Armature Calculation. — Suppose it is desired to design a 50 K.W. alternator of the American type, for 1000 volts terminal pressure and 50 amperes, at a frequency of 60, using a smooth core armature with surface windings. Assume K to be 1.1; then, adding 10 per cent to the terminal pressure to allow for loss of pressure in the armature due to C^2R loss and inductance, we have

$$SN = \frac{1100 \times 10^8}{2.2 \times 60} = 833,000,000.$$

We may take 1000 revolutions as a satisfactory maximum speed at which to aim. One thousand revolutions and a frequency of 60 gives a fractional number of poles while the number must be an even integer. Taking 900 revolutions gives exactly eight poles, which is satisfactory. Then taking the safe periphery velocity as 6000 feet per minute makes the diameter of the armature a trifle over two feet. Call the diameter of the core two feet. The periphery of this is 75.4 inches. Somewhat more than one-half of this winding space should be occupied by wire, say 46 inches. Each coil must generate one-eighth of the pressure, or $137\frac{1}{2}$ volts, if the armature is connected in series. The diameter of the pitch circle for the poles may be set approximately as 25 inches, and its

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* Compare Kapp's *Dynamos, Alternators, and Transformers*, p. 377; teinmetz, *Trans. Amer. Inst. E. E.*, Vol. 12, p. 326.

circumference is then 78.5 inches. The combined width of the poles should be somewhat less than half of this, or say 38 inches. This makes each pole $4\frac{3}{4}$ inches or 12.1 centimeters in width. Since 46 inches of the armature circumference are occupied by wire, about 27.4 inches are left for the openings in the
centres of t inch is allowed for insu-
lation betw This makes the openings
approximate de. The cross-section of
the armatur owing 525 circular mils to
the ampere, circular mils. This is the
cross-section & S. Ga. wire which has a
diameter of 1 uble cotton-covered. Two
hundred and fifty-eigh hese will go into 46 inches
in one layer, but the number of armature conductors must be a multiple of twice the number of coils, or 16. Two hundred and fifty-six is the multiple which is nearest to 258. This makes 32 conductors or 16 turns per coil, and gives N the value of 3,254,000 lines of force. Allowing an average induction of 5500 under the pole faces makes the length of the pole faces, practically, $19\frac{1}{4}$ inches. This is too great a length to be practical in a machine of the capacity under consideration, and two layers of wire must be put on the armature, thus reducing N; or by rolling the wire into a rectangular form and placing it on the armature surface on its edge, it may be made to occupy less surface and more conductors may be put on the armature; in which case either the air gap induction or the dim sions of the armature may be reduced, or these may reduced together. It is therefore quite common to v

ıator armatures with special rectangular wire or n, and we will take a ribbon which is 250 × 80 mils ›ss-section, which gives an area equivalent to about o circular mils. When this is triple cotton-covered, mensions may be taken to be 270 × 100 mils, and ›f these will wind into a space 46.4 inches wide. makes 58 conductors or 29 turns per coil. This N the value of, practically, 1,795,000 lines of force, nakes the length of the pole faces approximately ches when the average induction in the air gap ›o. It remains then to fix the exact dimensions of rmature and pole pieces. Taking the diameter e armature core as 24 inches, and adding double hickness of insulation gives, say, 24.25 inches. makes allowance for two layers of japanned canvas ı layer of mica under the coils. The winding eter is 24.25 inches, and the circumference is inches. From this is subtracted 46.40 inches, : inches, which is the space occupied by the con- rs and the insulation between the coils, and there ns 27.78 inches for the spaces within the coils. coils, made so as to turn down at the ends of the therefore have the following approximate dimen- (also Fig. 86): outside length, $A = 19\frac{7}{8}$ inches; : length, $B = 14$ inches; outside width, $C = 9\frac{1}{4}$ s; inside width, $D = 3\frac{1}{2}$ inches. This leaves $\frac{1}{4}$ between the coils which may be filled by a strip ılcanized fibre or paraffined wood. The total ı of wire is approximately 950 feet, which has proximate weight, insulated, of 80 pounds and a esistance of .40 ohm. The C^2R_a loss is therefore

1000 watts, or 2.0 per cent, based on the cold re
ance, which is not far from the tabular value (page 2
the total winding surface is $76.2 \times 19.8 = 1509$ sq
inches, and this gives more than $1\frac{1}{2}$ square inches
watt C^2R_a loss, which is satisfactory. Since the a
ture conductors number 464, it is required that A
equal to f force. One-half this n

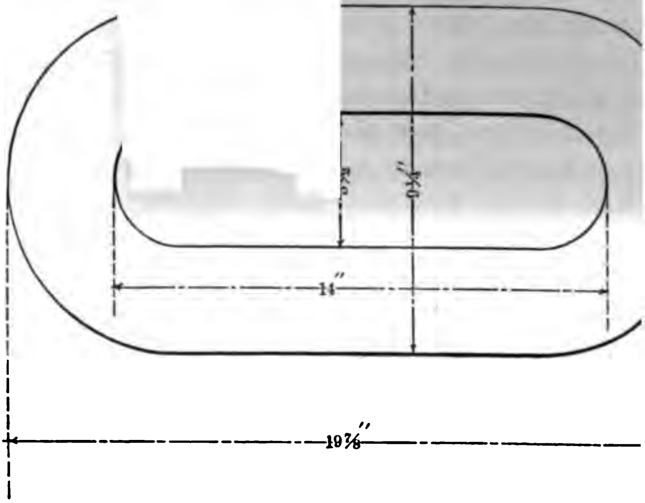

Fig. 86

ber of lines passes through each magnetic circuit i
armature. Putting B_a as 4000 makes the cross-se
of the armature core about 225 square centimete
35 square inches. The length of iron in the core
be assumed to be $12 \times .80$, or 9.6 inches. The
of the core discs must therefore be about $3\frac{3}{4}$ in
or the inner diameter of the discs is $16\frac{1}{2}$ inches.

The external finished diameter of the armatu

$24.25 + .540 + .164 = 24.95$. This allows 31 mils for the thickness of insulation under the bands, and 51 mils for the wire in the bands. Wire of 51 mils diameter, or 16 B. & S. gauge, is used on account of the high periphery velocity of the armature. Allowing a little under $\frac{1}{4}$ inch (210 mils) for mechanical clearance makes the diameter of the polar circle 25.37 inches, or $25\frac{3}{8}$ inches. The circumference of the polar circle is therefore 79.7 inches. The pitch of the poles is 9.95 inches, and the distance between their tips is 5.2 inches.

69. Armature Self-Inductance. — The self-inductance of a smooth-core alternator armature may be approximately estimated from the magnetic and electric data of the machine. The reluctance of each magnetic circuit nust be calculated exactly as in the case of a continuus-current multipolar dynamo, in order to determine he field windings. In the American type of alternator, he reluctance to be overcome by the magnetic pressre of each field core belongs to that part of the circuit hich lies between the lines AA' and BB' in Fig. 87. alling that reluctance P, the ampere-turns for each ld core are in number $\overline{nc} = \frac{NP}{1.25}$. The reluctance the different parts of the magnetic circuit met by es of force which are set up by the armature turns hen the fields are excited, may be assumed to be equal the reluctance in the same parts of the circuit met the lines set up by the field coils. The number of es of force set up in the portion of the magnetic cirit between the lines AA' and BB' by a unit current one armature coil, the centre of which is directly under a pole face, is $\frac{1.25\, S_1}{2\, P}$, where S_1 is the number of conductors in the coil, and is equal to twice the number of turns in the coil. Each one of these lines of force in completing its circuit must link another armature coil, so that we may say that the number of lines of force set up by each pair of coils is $\frac{1.25\, S_1}{2\, P}$. The self-induction of the pair of coils is therefore

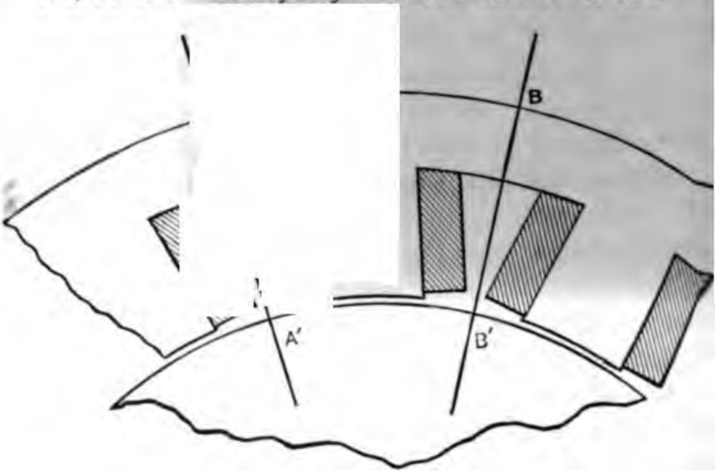

Fig. 87

$$L_1 = \frac{1.25\, S_1^2}{2 \times P \times 10^8},$$

since S_1 is equal to the number of turns in two coils. The self-induction of the whole armature is equal to L_1 multiplied by the number of pairs of coils, when the armature is connected in series, or

$$L = \frac{1.25\, S_1^2 p}{2 \times P \times 10^8}.$$

When the armature is connected with the halves in parallel, the inductance is one-fourth as great as is given by this formula; but in the case of two similar armatures built for the same pressure and output, the one connected with the halves in parallel has twice as many conductors in each coil as has the other armature, and their self-inductances are equal. If the value of P, in the preceding example, is taken as .004, the self-inductance is shown by substitution to be $L = .021$ henry.

The effect of the ampere-turns of the armature coil, within the ordinary load limits of a smooth-core armature, will not greatly alter the permeability of the highly magnetized fields, so that L may be taken to be approximately constant with varying loads, provided the armature pressure be kept constant.

The path of the lines of force set up by the armature coils has been assumed to be the same as the path of those set up by the field magnets. This is approximately true for machines with smooth drum armature cores, or with coreless armatures, and the real effect of the armature turns upon the number of lines of force in the magnetic circuits, is to increase or decrease the number that would exist were the armature turns absent, rather than to set up an independent magnetization. The effects of armature reactions and of self-induction are therefore closely related. In the case of machines with toothed armature cores, the reluctance in the path of the magnetization due to the field is materially smaller than when the cores are smooth, and hence it is to be expected that the self-inductance of toothed-core armatures will be large. If the teeth are T-shaped as in Fig. 88, the reluctance measured around the path of the lines of force set up by the armature coils may be materially smaller than the reluctance measured along the path of the magnetization due to the field coils. This is due to the effect of the leakage from tooth to tootl the self-inductance of an armature ha teeth which are close together maj be very large. In some

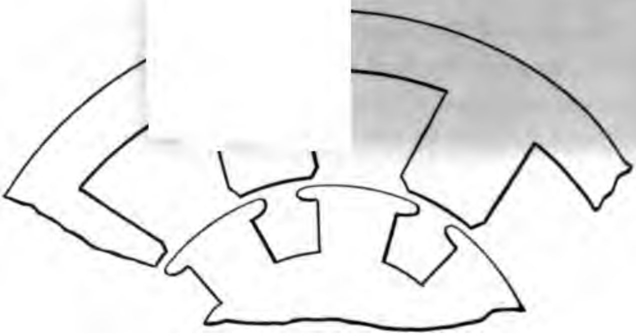

Fig. 88

such machines which are arranged to have a specially large armature self-inductance in order to obtain a self-regulating constant-current machine, as in the Stanley arc light alternator, the inductance may be as much as two or three henrys.

70. Armature Reactions. — The armature reactions of alternators by no means cause as serious consequences as those of continuous-current machines. When the current of the armature is in exact phase with the impressed pressure, the armature current has comparaively little opportunity to affect the field magnetism. When the armature conductors are directly between the pole pieces, the instantaneous current is zero, and therefore at this point the armature has no effect upon the field magnetism. When the coils have moved through one-half the pitch, a sheet of current at its maximum value flows directly under the pole faces. This current has such a direction that its magnetic effect tends to crowd the lines of force of the field into the trailing tips of the poles (Fig. 89). Hence the field is weakened on account of the increased reluctance of the magnetic circuit. This effect is probably not very marked in the usual forms of alternators, since the reluctance of the path occupied by the armature or cross-lines of force is quite large. The distortion and consequent weakening of the field may be reduced by cutting a slot longitudinally across the pole faces, or by some of the methods described in Vol. I., Chap. VI.

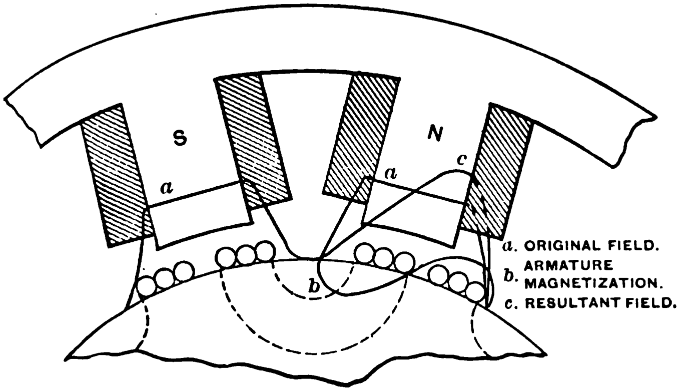

Fig. 89

zero, but has an instantaneous va
the amount of retardation. This
is in such a direction that its m
that of the field (Fig. 90). As
opposing effect merges into th
indicated. When the machine
operating as a motor, the curren

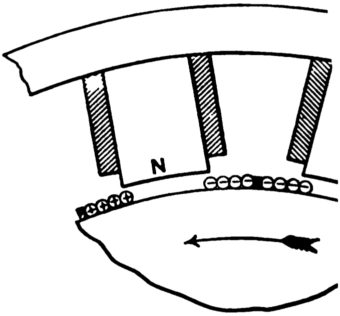

Fig. 90

dently tends to strengthen the
weaken th

of the impressed pressure, to strengthen the fields, may be taken advantage of to make an alternator completely self-regulating, or even self-exciting, through the action of its armature current. This, however, requires the use of a condenser attached across the armature terminals to give the proper lead to the current, which is undesirable. The opposing and cross-magnetic effects of the retarded armature currents of alternating generators, when operating under usual conditions, cause the external characteristic to slope toward the horizontal axis. This effect must be added to the slope of the characteristic caused by true and inductive resistance in the armature. It is often difficult to distinguish between the effects of armature reactions proper and of self-induction, and they are sometimes treated as alike.*

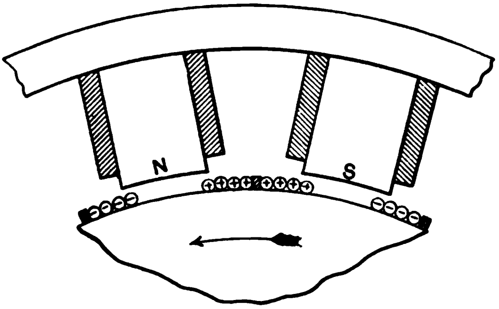

Fig. 91

The quantitative effect of the current, and of the angle of lag, on armature reactions is not readily determined. It is evident that the effect is a periodic one which depends for its relative instantaneous values upon the instantaneous positions of the coils with reference to the poles; and which depends further for its actual instantaneous and average values on the curr[illegible] angle of lag, and the shape of th[illegible] Doubtless the relative shapes of a[illegible]l pole pieces also enter the relation. [illegible] of the reactions is periodic, it is di[illegible]e its exact result in any particular c[illegible]s except that of experiment. The [illegible]irly large masses of iron, and they do [illegible]idly to changes in their magnetic surroundings. [illegible] inertia is caused by the effect of foucault currents and the considerable inductance of the field windings, which tend to suppress sudden magnetic changes. It is therefore safe to assume in general that the discernible effect of armature reactions is an average of the instantaneous values. The instantaneous value of the back turns of each coil at any moment is $\sqrt{2}\, nC \sin(a - \phi) \cos a$, where n is the number of turns of each coil, C is the effective value of the current which is assumed to be sinusoidal, and ϕ is the angle of lag. This expression may be averaged between the limits $a = 0°$ and $a = \pi$, with the result that the back turns appear to be approximately equal to $2.22\, nC \sin\phi$.* This formula purports to give the number of ampere-turns to be added to each pole on account of back turns, and the result is positive or negative as the current lags or leads; but it does not include the ffect of cross-magnetization, which is sometimes considerable but is difficult to predetermine. The method f figuring the effect of inductance has already been indicated (Sect. 69), and all the corrections necessary can now be made in computing field windings. This is carried out as explained in Vol. I. (p. 143 *et seq.*), due attention being given to modifying conditions already explained.

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* Kapp's *Dynamos, Alternators, and Transformers*, p. 394; and Hawkins and Wallis' *The Dynamo.*

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* Compare Kapp's *Dynamos, Alternators, and Transformers*, p. 412.

71. Field Excitation of Alternators.—The windings of the field magnets of alternators are usually classified

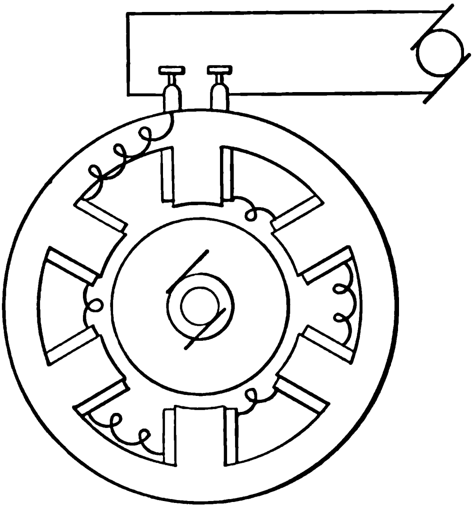

Fig. 92

according to their arrangement in circuit. The principal divisions are three: **separately excited, self-excited, compositely excited;** so-called, respectively, when the magnetizing current is supplied from an external source (Fig. 92), when it is supplied through a rectifying commutator from the armature of the machine under consideration (Fig. 93), or when these two arrangements are combined (Fig. 94).* Self-excited alternators may again be divided into series-wound and shunt-wound, depending upon, first, whether the whole current is rectified and led through a comparatively small number of turns [illegible] magnets (Fig. 95), or, second, wheth[illegible] of the current is rectified and led th[illegible]rcuit many times around the magnets ([illegible]ple: Zipernowsky alternator.) Of th[illegible] self-excited alternators, the shunt-wou[illegible]ommon. In this, either the whole pres[illegible]ature, or that of one or more coils, may [illegible]irectly upon the rectifying commutator, by means of a transformer attached to the armature (Fig. 97). (Examples: Westinghouse and Zipernowsky alternators.) Figure 97 illustrates the arrangement when the fields rotate.

Evidently a third division might be added to these, which would be a combination of the other two, or a compound winding in which both the shunt and series field currents are supplied by rectification. This, however, would require two rectifying commutators, which at the best are unsatisfactory, and for other reasons would not prove practical. To gain the result for which compounding is used in continuous-current dynamos, the composite winding is used. That is, the alternator is externally excited to its normal pressure on open circuit and the internal losses are compensated by series ampere-turns from self-excitation. The self-exciting

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* Compare *Text-book*, Vol. I., p. 136.

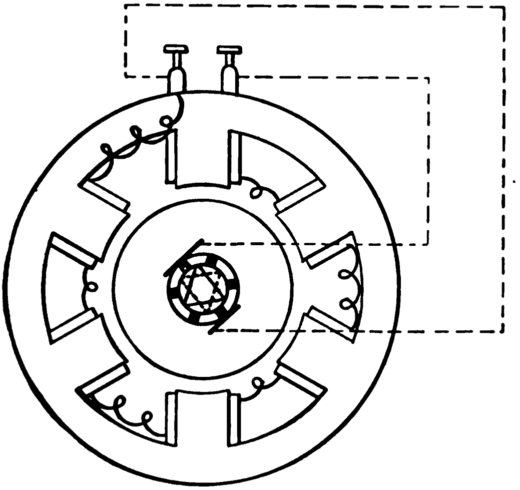


Fig. 93

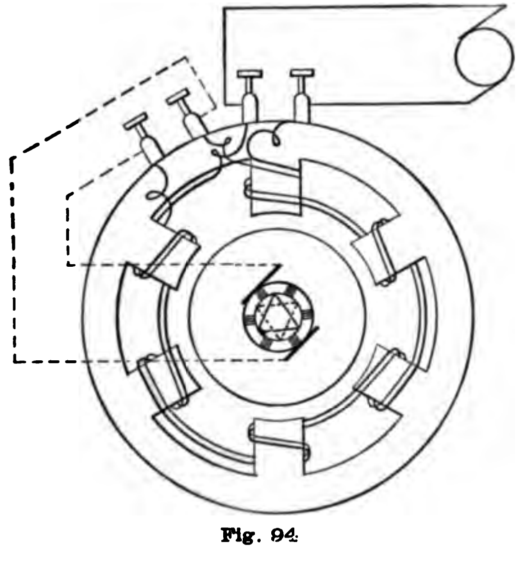


Fig. 94

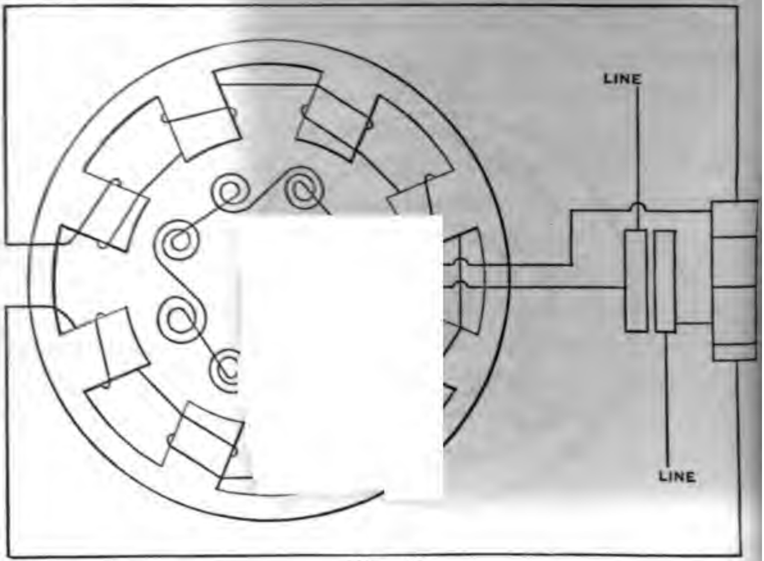


Fig. 95

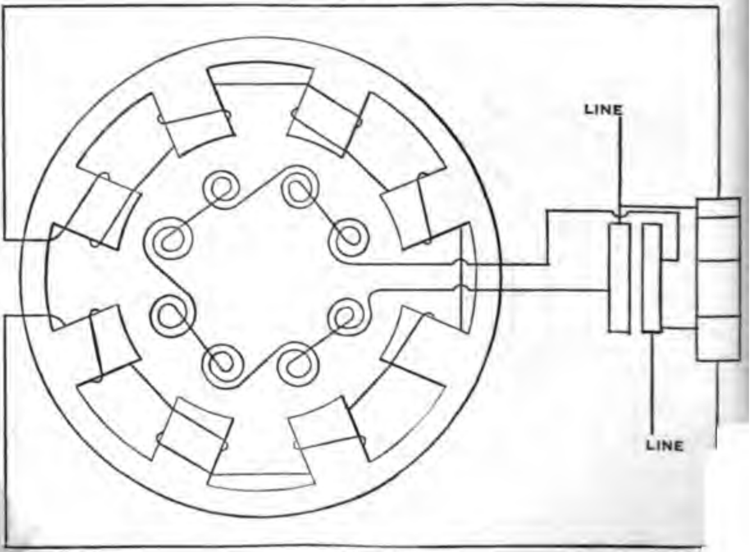


Fig. 96

ircuit of the composite winding may be arranged in arious ways. Thus the armature current may all be

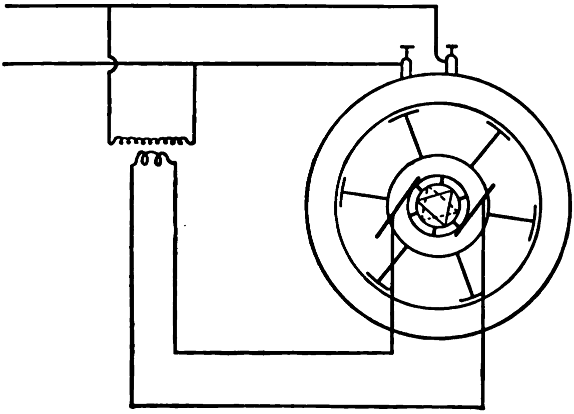


Fig. 97 a

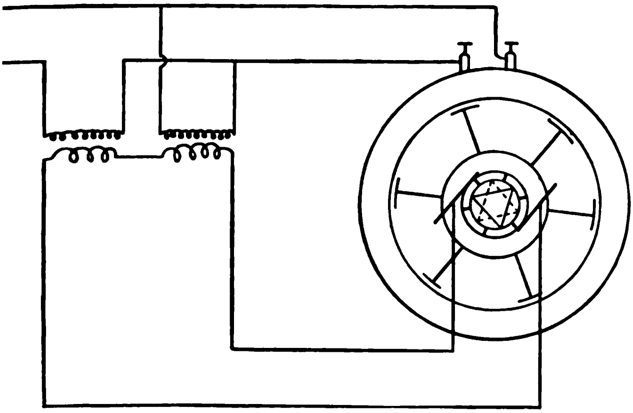


Fig. 97 b

ectified for use in excitation (Fig. 98) (example: Thomson-Houston alternator), or the armature curent may pass through a special transformer attached one or two poles (Figs. 98 and 101). (Examples: Westinghouse, Thomson-Houston, and General Electric alternators.) In the latter case, the series turns must always be equally divided between two poles with symmetrical positions when the armature is connected with its halves in parallel (Fig. 102). Composite windings may be arranged with th in a shunt circuit, but no

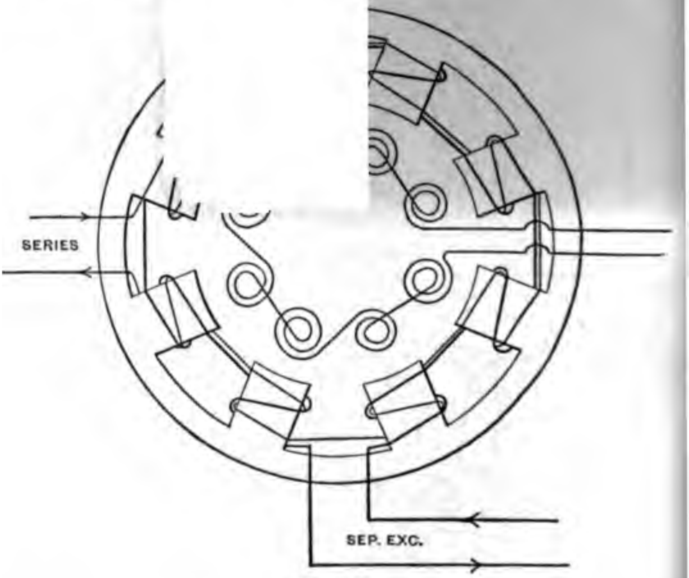


Fig. 100

advantage is gained by this arrangement over complete self-excitation in shunt or by separate excitation. This arrangement is, therefore, not used. In some self-exciting alternators, a separate set of exciting coils wound on the armature and connected to a rectify commutator. These may be wound directly with n armature coils or across a chord of the armature (Fig. 103). (Example: old-style Thomson-Houston nator.) The compounding may be effected in self- ed alternators by means of shunt and series trans- ers combined as in Fig. 97 *b*, which shows a machine stationary armature. (Example: Ganz alternators.)

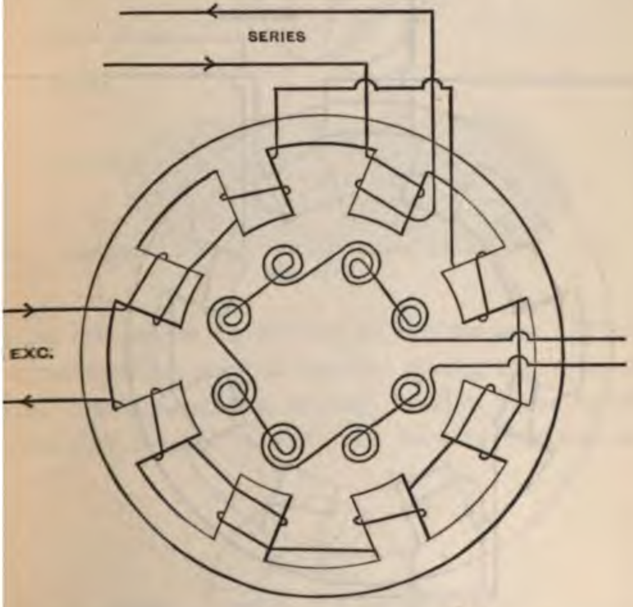


Fig. 101

e rectifying commutator in every case has as many ents as there are poles on the alternator, and alter- segments are connected together, making two sets 104). To each of these sets one of the alter- g-current terminals is attached. Brushes bearing the commutator at opposite non-sparking points then collect a rectified current. Various devices been employed to avoid sparking at the rectifying mutator, but in American machines no special pr tions are taken. (Examples: Westinghouse, Thon Houston, and General Electric alternators.) In

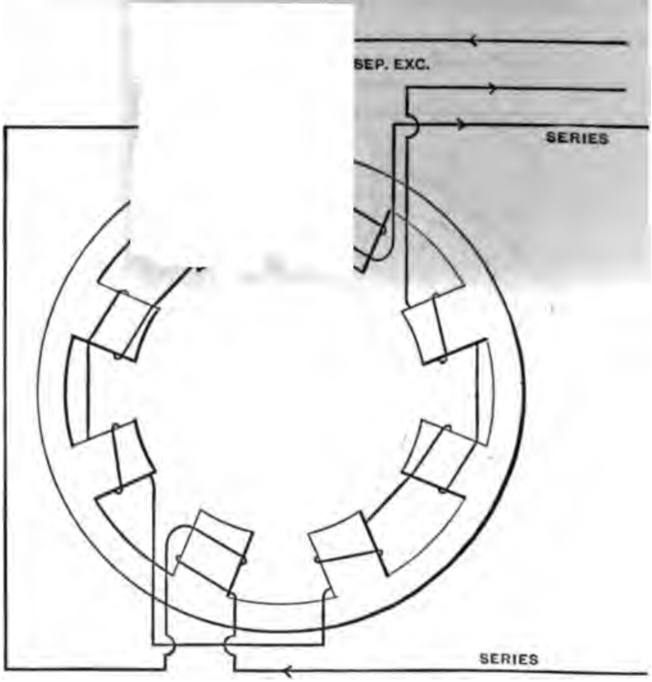


Fig. 102

Zipernowsky alternator, built by Ganz & Co. of B Pesth, the following arrangement of the commutat employed: Between the commutator divisions ar serted narrow metallic sectors which are conne together. Four brushes are used, two on each sid the commutator. One brush of each pair is set a little in the lead of the other, and the pair is connected together through a small resistance. The leading brushes are connected directly to the circuits. When the commutating point is reached during the rotation of the commutator, the trailing brushes move on to intermediate segments, while the forward brushes are still on main segments. Hence both the field circuit and supply circuit are short-circuited for an instant through the resistances connecting the brushes (Figs.

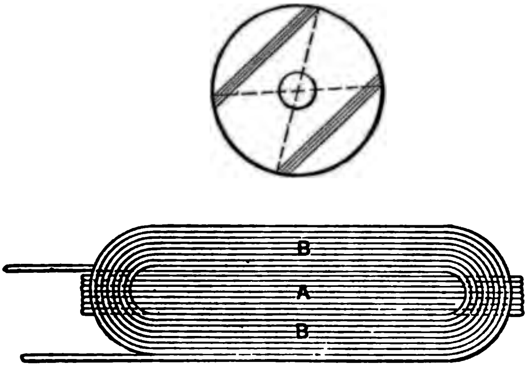


Fig. 103

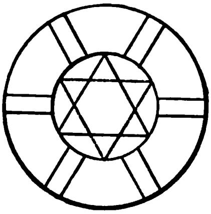


Fig. 104

105 and 106). Short-circuiting the supply circuit has
a disadvantage, but if a transformer is used for excitation it may be so designed that no harm results. Th
short-circuiting of the field circuit is claimed to giv
two points of advantage: First, it allows the con
mutation to be effected with little sparking; secon
upon sho fields, their self-inductic
tends to u in the windings, and thi

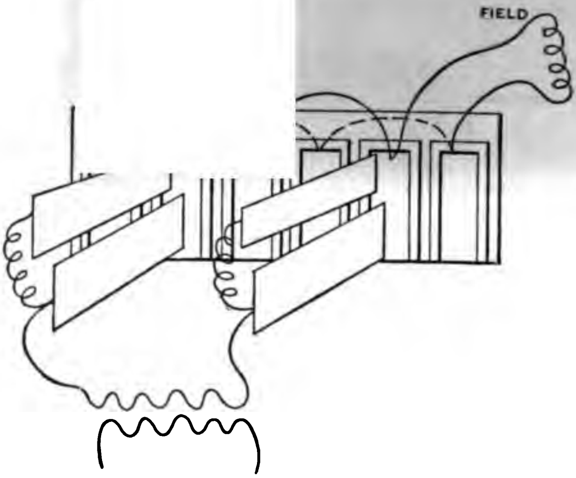


Fig. 105

therefore, does not fall to zero at each commutatic
as is shown in Fig. 107, but the current curve b
comes a wavy line more like that of Fig. 108. Pic
says * that it is preferable to place the brushes ahe
of the point of least sparking. In this case the spa
is due to a decreasing current, and is thin and wea

---

* *Machines Dynamo-Électriques*, p. 99.

ith the brushes behind the point of least sparking, e spark is due to a rising current and it is of great agnitude. The method here outlined to avoid sparkg does not seem to have any marked advantages ver direct commutation, which is ordinarily used in merican self-exciting machines. The advantage of a avy current in the field, instead of a discontinuous ne, is doubtless equally well gained in the American

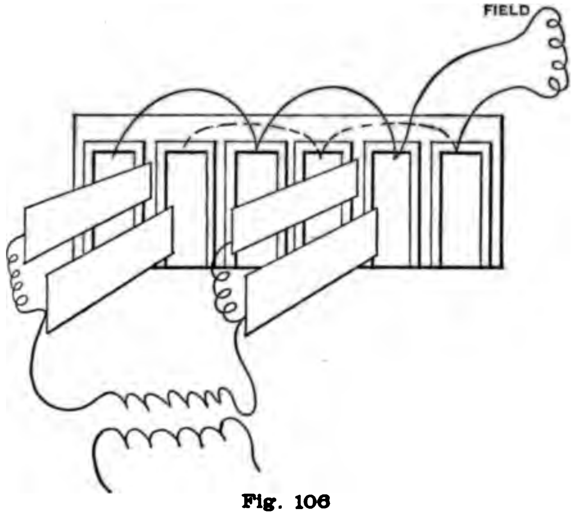


Fig. 106

achines by the use of copper brushes of considerable hickness on the rectifying commutator, which shortircuit the supply circuit and field circuit at the inant when they bridge over the insulation between two gments. In composite-wound machines the shortcuiting of the series field supply, which occurs for an stant at each commutation, is a matter of no moment, since cutting the small resistance of the series fields in and out of the main circuit cannot have an appreciable effect on the operation of the machine. For shunt-wound self-exciting machines, the current for rectification must either be supplied through a trans-

former or by n ate exciting coil on the armature, to av the external circuit by short-circuiting at the rectifier.

For the purpose of varying the magnetizing effect of the series turns, a variable shunt is often connected

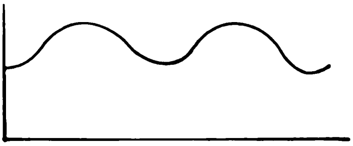


Fig. 108

across their terminals (Fig. 109) and a shunt is sometimes placed across the rectifier terminals in such a way that only a fixed proportion of the total current is rectified and passes through the series field winding. This is for the purpose of reducing the difficulties caused by sparking, by reducing the current to be rectified.

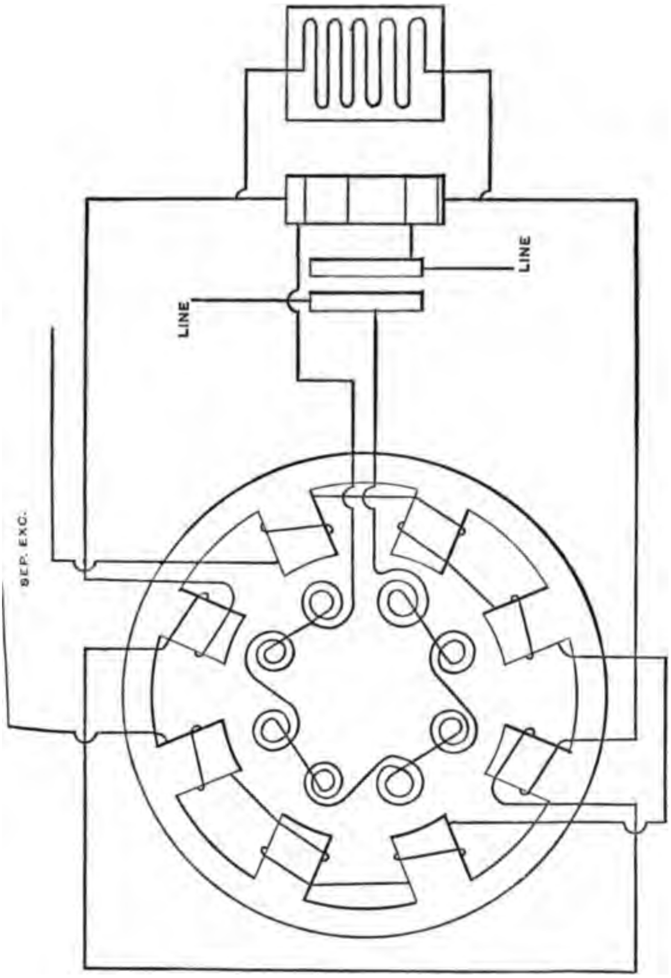


Fig. 109

CHAPTER VI.

CHARACTERISTICS, REGULATION, ETC.

72. Alternator Characteristics. — As in contin current machines (Vol. I., p. 195), there are four c which exhibit particularly important relations bet the functions of alternators. These curves, may be called characteristics, may be enumerat follows:

1. *The Curve of Magnetization.*
2. *The External Characteristics.*
3. *The Loss Line.*
4. *The Magnetic Distribution Curve and Pr Curve.*

73. Curve of Magnetization. — The curve of ma zation shows the relation between the total el pressure developed in the armature and the am turns on the field. From the total electric pressu the armature, the value of N_a may be deduced by n of the formula $E = \frac{2KSN_af}{10^8}$, provided the value of known. The value of K cannot be determined ex by calculation, but may be ascertained by means o fourth curve. The experimental determination o curve of magnetization is carried out exactly as in the case of continuous-current machines, substituting for the plain voltmeter an instrument which is capable of measuring alternating pressures. It is desirable that the instrument used shall indicate the effective value of the pressure; hence, the measurements must be made by either a hot wire instrument such as the Cardew voltmeter, an electrostatic instrument modelled after the quadrant electrometer, or a non-inductive form of high resistance electrodynamometer. All instruments used in alternating-current measurements which depend for their indications upon electrodynamic action, must be constructed with no masses of conducting metal about them, or their constants will depend upon the frequency of the current measured. This is due to the dynamic effect which foucault currents, circulating in metallic masses, must have on the currents in the moving parts of the instrument. If a voltmeter has an appreciable inductance, its reading will also depend upon the frequency, since the current flowing through it is inversely proportional to $R^2 + 4\pi^2 f^2 L^2$. From this it is readily seen that if L is not negligible in comparison with R, the current flowing through the voltmeter when it is connected to a circuit, and hence its indication, will be dependent upon the frequency. The indications of an inductive voltmeter will always be less when it is connected to an alternating circuit than when it is connected to a continuous-current circuit of equal effective pressure. The self-inductance of electrodynamometers intended for use as amperemeters is usually quite small, but in some cases may reach a millihenry. Electrodynamometers which are intended to be used as voltmeters, and have a great many turns of wire in their coils, sometimes have a self-inductance as large as several hundredths of a henry; but the commercial voltmeters that are built on the principle of electrodynamometers have a considerable non-inductive resistance in series with th , so that their time constant is small.

If the altern nination was designed to be a self-excit is some question of the comparative m s of continuous and rectified currents. vever, as we have already seen (Sect. 71), rent in a self-excited field is doubtless always a wavy one. The effective value of this, as indicated by an electrodynamometer, is very nearly the same as the average value indicated by an ordinary amperemeter. The average magnetizing effect of the current is also practically equal to that of a continuous current which gives the same indications on the instruments. A wavy current tends to set up foucault currents in the iron of the magnetic circuit and thus cause heating, but this result is not marked.

The magnetizing current of a separately excited alternator may be caused to become wavy if the armature reactions are very large. It has already been shown that the effect of armature reactions is a periodic one, and when the periodic effect becomes of sufficient magnitude it causes fluctuations in the field magnetism, which react upon the windings and throw the magnetizing current into waves. Curve *I*, Fig. 110, shows th current curve of a Stanley arc-light alternator of hig

-inductance when the machine is on short circuit. self-inductance of this machine is so great that current lag is nearly 90° when the machine is short-uited, and the current therefore has its maximum ue when the centres of the coils are almost directly ler the centres of the pole pieces. The armature rent therefore has a maximum effect upon the field

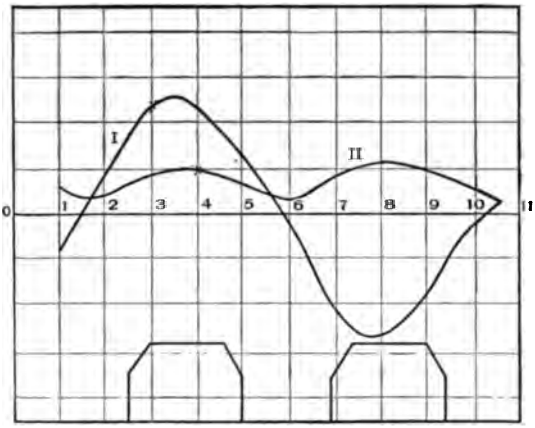
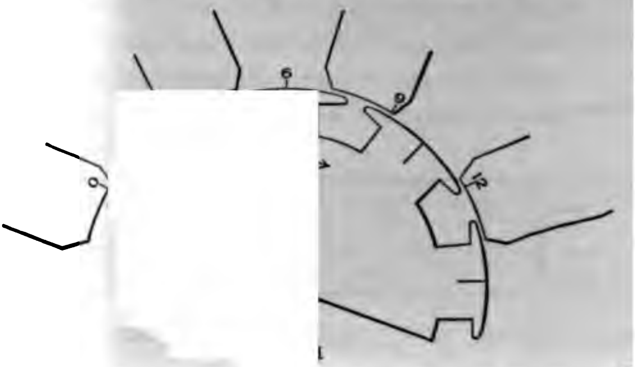


Fig. 110

agnetism, and it causes such a variation that the field irrent, which is furnished by a separate continuous-irrent dynamo, is thrown into waves as shown by rve *II* of the figure. The relative location of the les is shown in the figure, and the forms of the poles d the armature teeth with their relative location at instant the current is zero are shown in Fig. 111. ures 112 and 113 show the same features when the chine is worked upon a full load of 40 arc lamps, in

which case the current lag is not quite so great.
effect has also been found, but to a less deg
smooth-core machines with surface windings.



The general form of the curve of magnetizatio
an alternator is similar to the form of the curve

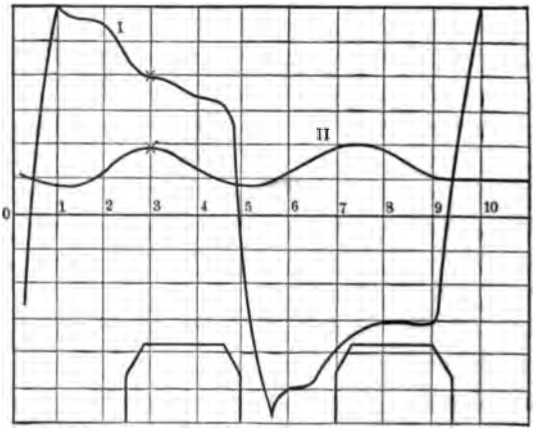


Fig. 112

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* Tobey and Walbridge, Stanley Alternate-Current Arc Dynamo, *Trans. Amer. Inst. E. E.*, Vol. 7, p. 367.

uous-current dynamo. As it is not uncommon
ernators to have a somewhat larger reluctance in
space than have continuous-current dynamos of
ne size, the knee in the alternator curve is some-
nes not so abrupt as it is in the case of continuous-
rrent machines (Fig. 114 *a* and *b*). For studying the

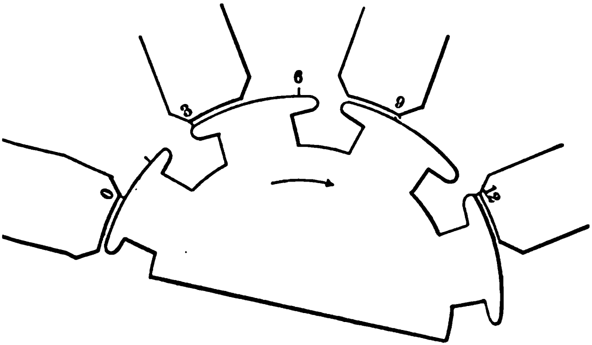

Fig. 113

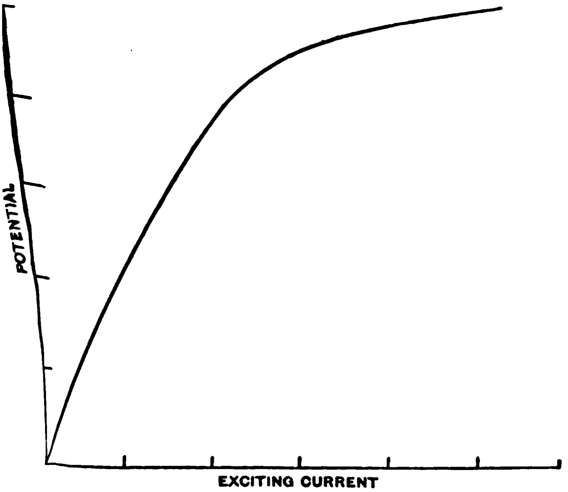

Fig. 114a

details f the design of the magnetic circuit, the may b resolved into component curves repres

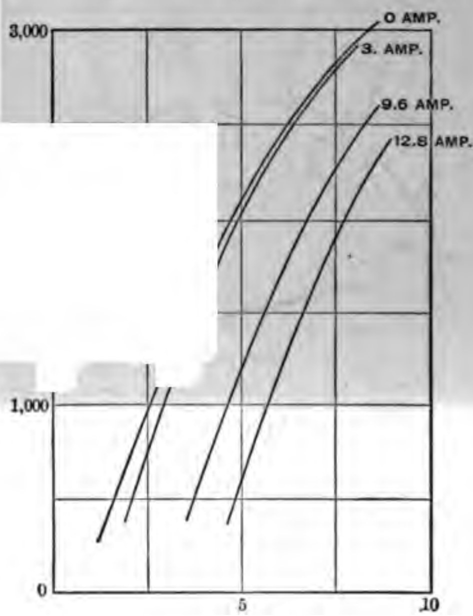

Fig. 114 b

Relation of pressure to exciting current with different current armature.

the air space, frame, and armature, exactly as exp in Vol. I., p. 197.

74. External Characteristic.—The external c teristic has different forms which depend upo method of exciting the fields. To experimentally mine the external characteristic of an alternato excited by the method for which it is designed, s give its normal pressure on open circuit. The v its terminals, and the current in the external c are measured with various resistances in the external circuit. The observations may be plotted in a curve, using volts as ordinates and amperes as abscissas. In separately excited alternators, the curve cuts the vertical axis at the highest point, and then gradually falls; the decrease of the ordinates (drop in pressure) being caused by the effects of armature resistance, armature self-inductance, and armature reactions. The measurements should be made with the alternator connected to a non-inductive circuit, since the magnitude of the armature reactions is increased, on account of the greater lag of the current, when there is self-inductance in the external circuit. If it is desired to quantitively determine the effect of lag on armature reactions, curves may be taken with different values of inductance inserted in the external circuit. The portion of the drop which is caused by self-inductance and armature reactions may be separated from that caused by resistance by the formula, $E_t^2 = E_a^2 + E_s^2$. In this case E_t is the open-circuit pressure, and E_a is equal to the terminal pressure, for the load C, plus CR_a. By taking the characteristics of an alternator when worked on circuits of different known resistances and self-inductances, the effect of armature reactions may be determined for different values of the angle of lag.* The self-inductance of some alternators is so great that the external characteristic droops to the X axis at a current not greatly exceeding full load. Figure 115 *a* shows the characteristics of two alternators, one having a small, and the other a large, self-inductance. The maxi-

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*Compare **Kapp's** *Dynamos, Alternators, and Transformers*, p. 383 *et seq.*

mum output is given by the latter when the armat current has the value, $C=\frac{N}{SL_1}A=\frac{NS}{L_a}A$, accordin the formula on page 237.

The external characteristic of a self-excited sh wound alternator is shown in Fig. 115*b*. The droo

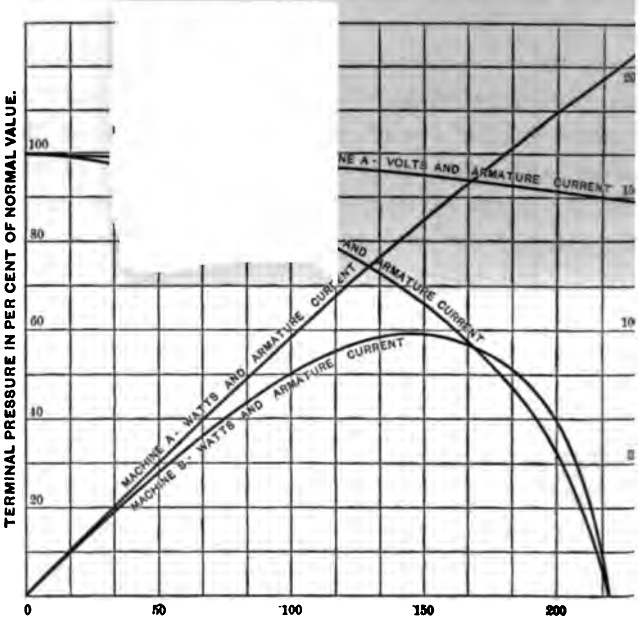

Fig. 115a

the curve is a little greater than it would be for same machine separately excited. This is due to the l of magnetizing current as the armature losses increa and the terminal pressure decreases accordingly (co pare Figs. 110 and 117, Vol. I.). The external char teristic of a shunt-wound self-excited alternator may structed from the curve of magnetization and loss :, by the process that has already been explained for tinuous-current dynamos (Vol. I., p. 205).

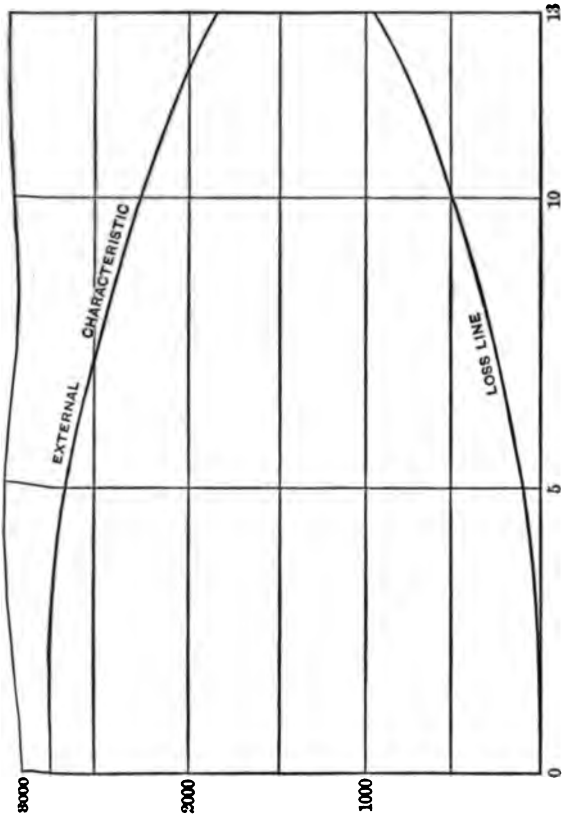

Fig. 115 b

75. Loss Line. — The differences between the ordinates of the external characteristic of a separately xcited alternator and its terminal pressure on open ircuit are the ordinates of the **Loss Line.** This difference in the ordinates is caused, as already stated, by the loss of pressure due to armature resistance, counter electric pressure due to self-induction, and the effect of armature reactions. Since the effect of armature reactions depends upon the current lag, the slope of the loss line [illegible] on the kind of circuit upon which a n[illegible]ed, and the slope is increased as the s[illegible]f the circuit is greater. When the alter[illegible]ll armature self-inductance and is wo[illegible]it of zero or negligible inductance, the [illegible] general be expected to be fairly straight [illegible]ps due to resistance, inductance, and ar[illegible]s are all, within narrow limits, directly proportiona[illegible] the armature current, provided the armature inductance is constant. In some machines (especially those with toothed armatures), these provisions are not fulfilled on account of the large value of the inductive pressure which is in quadrature with the active pressure, and the loss line may be considerably curved, so as to be convex towards the axis of current or horizontal axis. The amount of the curvature is dependent upon the degree of saturation to which the armature core is subjected, and upon the effect of the air space in the circuit of the magnetic lines which are set up by the ampere-turns of the armature. Unless the magnetic induction in the armature is denser than other conditions will admit in good practice (Sect. 61), the self-inductance is not likely to decrease as the current rises. Hence the loss line of alternators worked on non-inductive circuits is likely to be nearly straight, or else convex towards the horizontal axis (Fig. 115 *b*).

76. Instruments. — In all the measurements required in determining alternator characteristics, the instruments which are used must be carefully selected so that they may properly measure alternating currents. Thus the voltmeters must be of the types described in Section 73, and the amperemeters must be of one of the following types:

Electrodynamometers or current balances which have no metal in their frames in which disturbing foucault currents may be set up.

Known non-inductive resistances through which the current to be measured passes, and the pressure at the terminals of which may be measured by means of an electrostatic or hot wire voltmeter, or by means of an electrodynamic voltmeter having a negligible time constant.

Or finally, amperemeters dependent for their indications upon the heating effect of the current.

These instruments may be standardized and calibrated in the usual manner with continuous currents, after which they will give correct indications of the effective values of alternating currents for any frequency within the usual commercial limits. If other types of amperemeters are used, they must be calibrated by comparison with instruments of one of the types described above, using an alternating current, for the calibrating, which has exactly the same frequency as that which is to be measured in the test. Amperemeters which depend for their indications upon the attraction of a solenoid upon a laminated iron core, are likely to give widely different indications for equal currents of different frequencies. This is principally due to the effects foucault currents and hysteresis.

77. The Magnetic Distribution Curve, and Curves Pressure and Current.—These curves may be exper mentally determined by various methods. They consis of a series of curves which are closely interrelated, bu may be anc y to be, of quite dissimila forms. Th rve representing the wa of impress tal pressure developed the armatu tor, is directly depende upon the d magnetism over the po faces, and ngement of the armatu windings. g, either of these may b given a controlling influence to the exclusion of th other. The two-pole machine with Gramme armature discussed in Section 5, gives an excellent instance. The differential action, which occurs in the coils of this armature, makes its curve of pressure almost independent of the distribution of the magnetism over the pole faces, provided the same total number of lines of force is cut by the conductors per revolution; and the maximum value of the pressure is entirely independent of the magnetic distribution. This is shown by Fig. 116, where the full line in one cut shows the curve of pressure developed in the armature with a uniform distribution of the field, and the full line in the other shows the curve of pressure with the same total field greatly distorted. The dotted lines in the cuts show the distribution of the magnetism over the pole faces. The continuous pressure developed in the armature when th machine is operated as a continuous-current dynam s not affected by the distortion, provided the brushes re always placed on the neutral plane and the total magnetism passing through the armature remains contant. When the machine is converted into an alterator by the addition of collecting rings, the maximum nstantaneous pressure is equal to the pressure developed in the continuous-current machine, and is therefore ndependent of the distribution of the magnetism, but he form of the curve representing the wave of pressure is slightly altered, as shown in Fig. 116. Now sup-

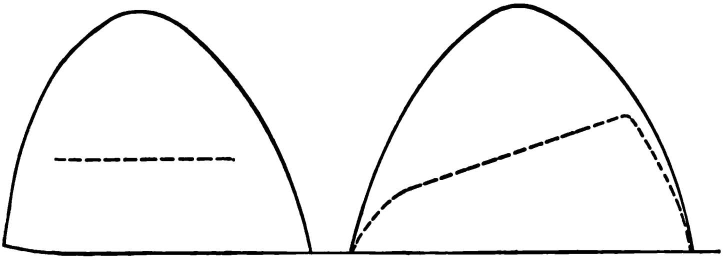

Fig. 116

pose the same machine to be arranged with a single narrow coil on the armature. The change in the magnetic distribution now not only changes the form of the pressure curve proportionally, but also changes in a marked manner the maximum value of the instantaneous pressure. The difference in the curves of pressure developed by the broad and narrow coil armatures is due to the effect of differential action in the broad coil armature. It has already been shown that differential action occurs to some degree in commercial alternators, but it does not occur to a sufficient degree to make the form

of the pressure wave independent of the magnetic distribution. It is therefore true that the magnetic distribution largely influences the form of the pressure wave, and the distribution should therefore be carefully studied during the development of a type of alternators. A
proper stud ic distribution and of the
arrangemen e windings, makes it pos-
sible to so iator that it will produce
any desired f vave. This is an important
point which ional attention later.

The ang en the curves representing
the magnet the impressed pressure, is
interesting. the curve of pressure at
any point, is proportional to the rate at which lines of force are cut by the armature conductors at that point, the rate being taken algebraically. Consequently, the pressure is zero when the magnetization is all symmetrically threaded through the coils; that is, when the algebraic rate of cutting lines by the conductors is zero. The pressure is a maximum when the rate of cutting lines is the greatest; that is, when the algebraic summation of the number of lines threaded through the coils is a minimum. A curve which shows the algebraic summation of the number of lines threaded through the coils at each instant, therefore, has angular position which is 90° from that of the pressure

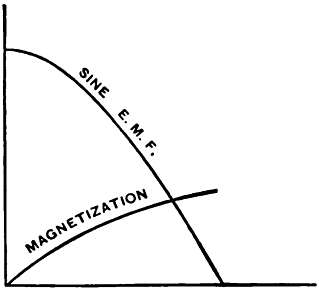

Fig. 117

(Fig. 117). The form and dimensions of this evidently depend upon the actual distribution of iagnetism and the arrangement of the armature ngs. When the pressure curve is irregular, and igular relation is not made evident from the curve, Fig. 118, the point at which the curve of mag- n cuts the X-axis may yet be easily found, since it ectly under the centre of gravity of the pressure . That is, since as many lines of force must be rawn from the coils as are inserted, for each loop e curve, the summation of the ordinates of the ire curve on each side of the crossing must be

s curve, which shows the algebraic number of of force which are threaded at each instant through oils, may be easily deduced from the curve of ire. Erect an ordinate to the pressure curve bisects the area; then by means of ordinates the half areas into a number of small areas. The etism threaded through the armature coils is alge- lly equal to zero at the instant represented by the ing ordinate, and the algebraic value of the mag- 1 threaded through the armature coils at any instant is proportional to the area enclosed by essure curve between the corresponding ordinate ie bisecting ordinate, since $e = \frac{dN}{dt}$ and $N = \Sigma edt$. fore, the ordinates of the curve representing the etism threaded through the coils are, at the in- represented by the ordinates which divide the ire curve into small areas, proportional to the area

between the corresponding instantaneous pressur
nate and the bisecting ordinate. The full proc
therefore, as follows: lay off on each ordinate a
from the X-axis proportional to the area enclos
the pressure curve between the respective or
and the bisecting ordinate. The points thus four
points on the desired curve. It is evident tha
maximum ordinate of the curve comes at the i

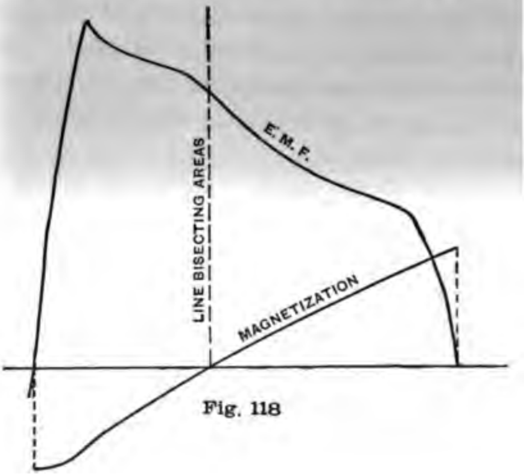

Fig. 118

when E is equal to zero. The length of this or
is equal to N_a, and the scale of the curve may th
conveniently fixed. The curve may not be symme
but, with a fixed value of N_a and a fixed armature
ing, the successive loops must always be exactly
though they may be looked upon as alternately
tive and negative, since the magnetism is alter
threaded through the coils in opposite directions.
corresponding curves of pressure and magnetis

s forms of pressure curves are shown in the panying figures. The construction of the second from the first is by the dotted In Fig. 118 the re curve is one mentally deter- from a Stanley ight alternator working on a id of arc lights.* 119 the press- irve is an equi- triangle, in Fig. is a sinusoid, Fig. 121 it is a gle. The press- rves of Figs. 122 23 are respec- a flat-topped and a parabola.† e the electrical ure is propor- to the rate of change of the number of lines of hreaded through the armature coils, the ordinates pressure curve are proportional to the tangents curve representing the number of lines enclosed

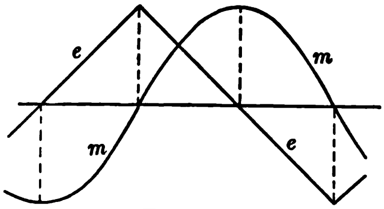

Fig. 119

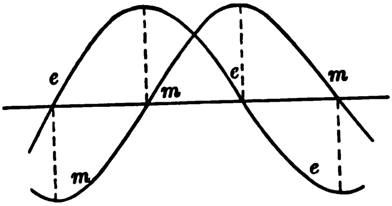

Fig. 120

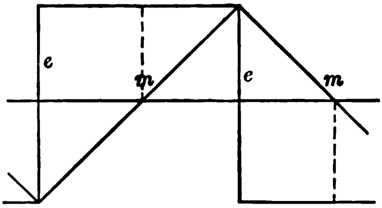

Fig. 121

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ey and Walbridge, Stanley Alternate-Current Arc Dynamo, *Amer. Inst. E. E.*, Vol. 7, p. 367.

ery, Alternating Current Curves, *Trans. Amer. Inst. E. E.*, p. 433.

by the armature coils. Figure 124 shows a graphical construction for determining the pressure curve from

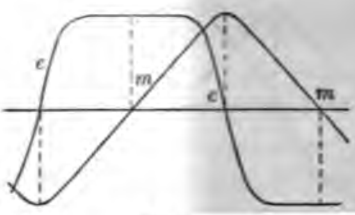

Fig. 1

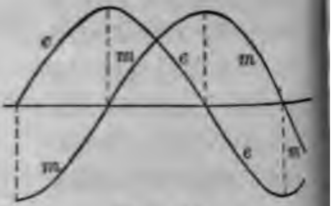

Fig. 123

this magnetic , and OA are, by construction, proporti nts of the angles with the X-axis made to the magnetic curve at p_1, p_2, and O. , and $O'a'$, $O'b'$, $O'A$ are drawn parallel aa, bb, and the tangent

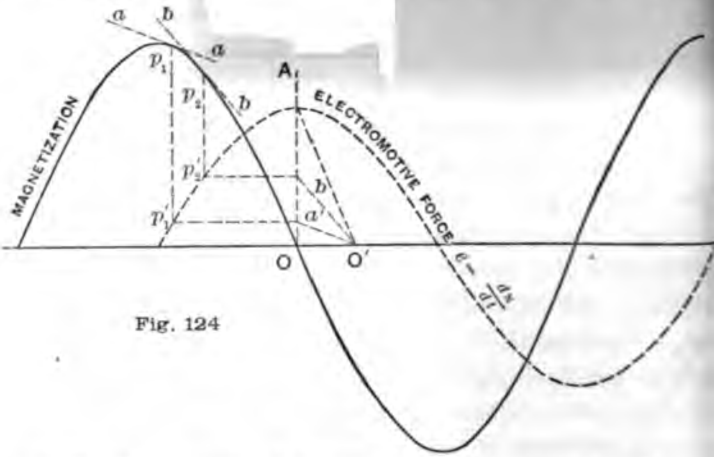

Fig. 124

O. The points of intersection of horizontals drawn n a', b', etc., and verticals drawn from p_1, p_2, etc., points on the required pressure curve.

directly useful magnetic curve is one showing
ution of the lines of force over the pole faces.
e is analogous to the magnetic distribution
continuous-current machines (Vol. I., p. 208).
experimentally determined by the fourth or
ethod given in Vol. I., p. 211. It is prob-
he magnitude of the periodic effect of arma-
ions may also be experimentally determined
wo test coils, one of which is placed in a posi-
dent with the armature coils, and the other of
laced so that its phase is 90° in advance. If

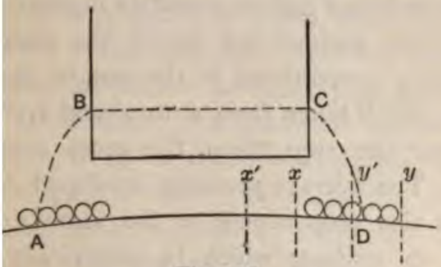

Fig. 125

e distribution of magnetism over the pole
machine is known, it is evidently possible to
tely determine the form of the pressure curve
be produced by any particular arrangement
lings. It is also equally possible to determine
ement of the windings required to give any
m of pressure curve. Again, if a particular
nding is desirable, the magnetic distribution
ecessary to give a desired pressure curve may
ned. This distribution may then be used as
designing the width and shape of the pole

faces. The application of the magnetic distribution curve is illustrated in Fig. 125. The dimensions and form of the pole pieces and of an armature coil belonging to an alternator, are indicated in the figure. The ordinates of the line *ABCD* represent the magnetic density in the air space. When the coil is in the instantaneous position represented, the value of the electric pressure is zero. As the coil moves, each conductor cuts li ppose that in one twenty-fourth of a p moved from the position indicated by o x', y'. During this motion each co ertain number of lines of force, and t y all the conductors is approximatel the sum of the areas of the curve *AB* to x' and from y to y'. The shorter the more accurate this becomes. The re developed during this interval is also proportional to the same area. Consequently an ordinate which is numerically equal to the area, may be erected at the point $a = 7\frac{1}{2}°$ (one forty-eighth of a cycle) to approximately represent the pressure at that point in the revolution of the armature. This proceeding may be repeated through the half period taking the algebraic summation of the pressures developed in the two halves of the coil, and the outline of the pressure curve is thus determined.*

The curve representing the current wave also represents, when taken to the proper scale, the curve of active electric pressure. From preceding pages it is evident that the curves of active and impressed pressures have the same forms and are superposed, if the circuit in which they act is non-inductive and without capacity. They have the same form, also, when the circuit is inductive, if the impressed pressure is sinusoidal, provided the inductance is independent of the instantaneous value of the current and the armature reactions are approximately uniform. The condition of a uniform inductance can only hold when no iron is enclosed in any portion of the circuit. In general this condition is not found in commercial service. Even with a uniform inductance the curves of impressed and active pressures will not coincide in phase, since phase coincidence between them can occur only when the circuit is without either inductance or capacity, or these exactly neutralize each other. As the latter is also a condition not often found in commercial service, it may be said that in general curves of impressed and active pressures are neither similar in form nor coincident in phase; but they are always, perforce, of the same frequency.

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* Kapp's *Dynamos, Alternators, and Transformers*, p. 364 *et seq.*

The curves are usually plotted to rectangular coördinates; inductions, pressures, or instantaneous currents being plotted as ordinates, and angular degrees or time as abscissas. To more definitely locate the phases it is not unusual to indicate the position of the pole pieces by laying them off at the top or the bottom of the plot (Fig. 126.)* No systematic study has been made of the distribution of magnetism over the pole faces of alternators. Such a study, as already intimated, would be of much value in determini most satisfactory form for pole pieces and the a ment for armature windings. To make the worl plete, it should cover alternators with various of armatures when worked at various loads und ferent conditions of current lag. Since, at a stant, the effect of the armature current on the tization of the pole pieces, depends upon the posi the armature coils as well as the strength of tl rent, this effect is evidently a variable, and conseq

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* Compare *Trans. Amer. Inst. E. E.*, Vol. 7, pp. 1, 311, 324, 367; also *London Electrician*, Vol. 28, p. 90; and *Elect. World*, Vol. 18, p. 368.

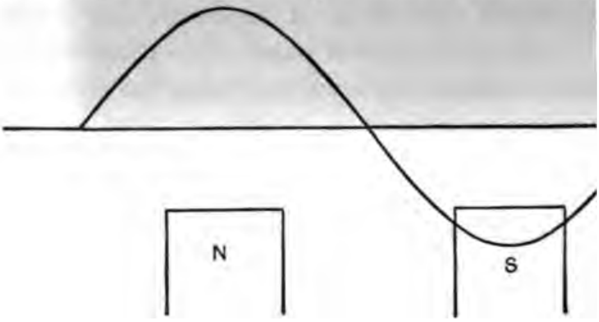

Fig. 126

the distribution of the magnetism will not be co In other words, both the cross turns and back tu any load vary continuously during each perio therefore the magnetic distribution varies with th tion of the armature. The magnitude of the va of the magnetic distribution has never been fully mined. It probably is not very great in machin smooth-core armatures, and an average distributi be assumed as satisfactorily representing workir ditions. In machines with toothed or pole arm the effect of armature reactions, and the moven

:eth across the pole faces, is often sufficient to regular pulsations in the field magnetism and nely marked distortions in the pressure curve. ›ulsations are sometimes sufficient to materially the magnetizing current. Figure 112, curve *II*, an experimentally determined curve of the field ıt of an alternator having a toothed armature core ι large self-inductance in the armature. The ne was excited by a small shunt-wound exciter.* : most satisfactory method of studying the mag- distribution is by means of a test wire laid on the e of the armature. This is similar to the method ıt coil on armature explained on p. 211 of Vol. I. ıdy the effect of the armature upon the magnetic ›ution, the test wire must be successively located ferent points on the armature within an angular equal to the pitch of the poles.

Methods of tracing Pressure and Current Waves. ·ious methods may be used for experimentally deter- g the form of electric pressure or current curves.

Ballistic Method. A ballistic galvanometer is ed to the terminals of the alternator to be . The fields are excited in the usual manner : degree desired. The armature is then quickly ced through a small arc of revolution. The of the galvanometer is read. The armature is advanced through an equal arc and the throw This is continued until the armature has been :ed through a distance equal to twice the pitch, one complete period of the pressure curve will

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* See *Trans. Amer. Inst. E. E.*, Vol. 7, p. 374.

the effect of armature reactions l
curves taken by later methods.

This method may be modified by
current when the armature is in t.
tions, instead of quickly moving tl
throws of the galvanometer, which
proportional to the number of lines c
through the armature coils when t.
the corresponding positions. The ex
mined curve therefore shows the n
force which pass through the armat
point, and from this curve the forn
curve is easily derived, as already exp
Figure 124 shows the curves thus det

2. *Gerard's Method.* This is an
which the curve of pressure of an al
current flows in the armature, may l
special apparatus. The alternator tc
tated at a very slow speed, the field be

ous pressure. By moving a sheet of sensitized paper before the galvanometer mirror, which throws upon it a beam of light, the curve of pressure may be permanently recorded.*

Each of the following methods requires the use of a revolving contact maker of some kind, and they therefore have much in common. The principal differences in the methods relate to the type of instruments used to give the indications, and the convenience with which the manipulations may be made. Whether current or pressure curves are to be obtained, instantaneous pressure measurements, only, are made. For the former, the instantaneous pressures are taken at the terminals of a non-inductive resistance, and the instantaneous currents are readily deduced.

3. *Joubert's Method* (1880). The terminals of the alternator armature, or of one bobbin of the armature, are connected to a condenser in the following manner: One armature terminal is connected permanently to one terminal of the condenser; the other armature terminal is connected to a rotating point which may be put in connection with the free terminal of the condenser, when the armature is at any desired point in its rotation. At the instant this contact is made, the condenser receives a charge which is proportional to the instantaneous pressure in the armature, and which may be measured by discharging the condenser through a ballistic galvanometer. By setting the contact to correspond with various points in the revolution of the armature, the corresponding instantaneous pressures

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* See Gerard's *Leçons sur l'Électricité*, Vol. I., p. 565, 3d ed.

may thus be measured and the curve of pressure may be plotted (Fig. 127). The contact maker used by Joubert was an insulated pin set in the armature shaft, against which a brush could be made to bear at any point in the revolution, as explained on p. 211, Vol. I. A quadrant electrometer may be used in place of the condenser and ballistic galvanometer; in which case it is desirable to introduce a condenser permanently in parallel with the electrometer, to neutralize the effect of leakage in the test circuit.*

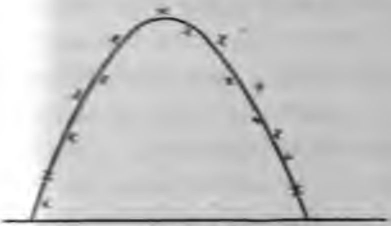

Fig. 127

Joubert's investigations made in 1880 resulted in the first determination of the curve of pressure of an alternator (see page 28). The investigations of Duncan, Hutchinson, and Wilkes probably produced the earliest series of experimental curves showing the relations between the waves of pressure and current in circuits of different kinds. The investigations of Searing and

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* See Joubert, *Comptes Rendus*, Vol. 91, 1880, p. 161; Joubert, *Journal de Physique*, 1881; Duncan, Hutchinson, and Wilkes, *Electrical World*, Vol. 11, 1888, p. 160; Searing and Hoffman, *Jour. Franklin Institute*, Vol. 128, 1889, p. 93; Ryan, *Trans. Amer. Inst. E. E.*, Vol. 7, 1890, 1; Tobey and Walbridge, *Trans. Amer. Inst. E. E.*, Vol. 7, p. 367; ndel, *La Lumière Électrique*, Vol. 41, pp. 401 and 507; Hopkinson's *amo Machinery and Allied Subjects*, p. 187; etc.

man were the first made upon an alternator with
in the armature core. Their results showed the
ɜ of pressure developed in a smooth-core drum
.ture to approach a sinusoid (Fig. 128).

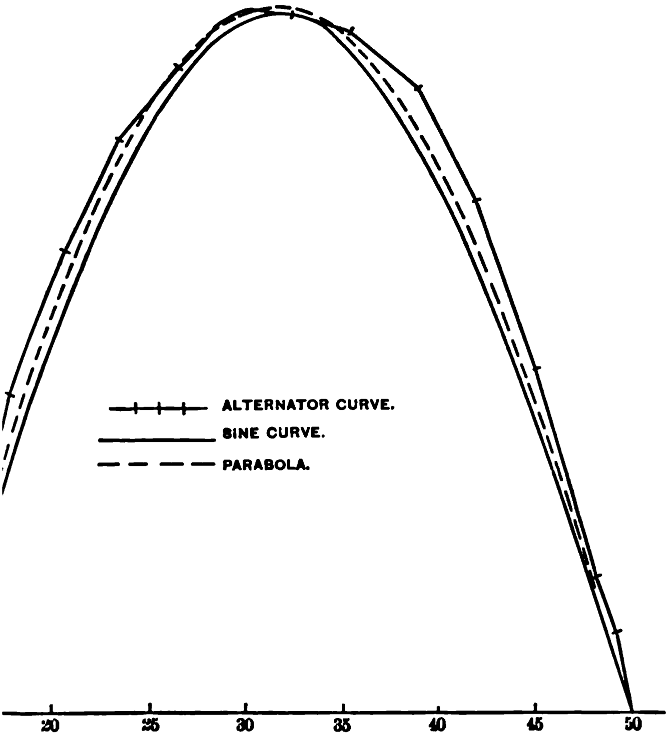

Fig. 128

Ryan's Method (1889). Professor Ryan of Cornell
'ersity, in conjunction with Professor Merritt,
ed out a series of investigations in 1889, in which
:ntirely different and original arrangement of the
uring instruments was used. The use of a con-

denser and ballistic galvanometer in determining points of the pressure and current curves involves long and laborious manipulation, and introduces various elements of inaccuracy. On the other hand, a quadrant electrometer is fairly reliable, is direct reading, and is approximately dead beat, but has a limited range. A satisfactory electrometer for use in a long investigation of the kind under consideration should have a wide range, throughout which the indications should be uniformly accurate. It is also desirable that the instrument have a simple law and an invariable constant. To fulfil these conditions, Professor Ryan designed a special zero reading electrometer which consists essentially of a cylindrical electrometer needle *A*, and four quadrants *Q, Q* (Fig. 129). On the upper side of the electrometer needle is hung a magnetized steel mirror *C*, which serves both as a magnetic needle and as a mirror. T needle is suspended by a silk fibre, and is put in m tallic contact with the case by means of a loop of ve

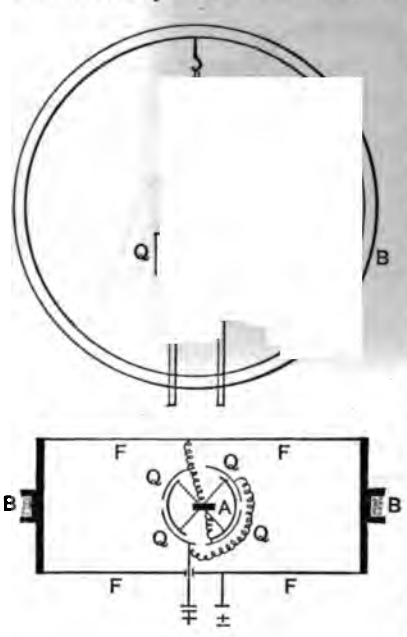

Fig. 129

ine silver wire *S*. The electrometer case is circular, nd the magnetic needle is arranged to be in its centre. round the case is wound a coil *BB* of fine insulated ire. When the plane of this coil stands in the magetic meridian, the coils and magnetic needle make a angent galvanometer. On the other hand, when the lectrometer needle is connected to the case and one air of quadrants, it makes, in combination with the ther pair of quadrants, a quadrant electrometer conected for idiostatic use. If the terminals of the elecrometer are connected to a circuit, the pressure of hich is to be measured, the needle experiences a deecting couple which is proportional to the square of he pressure. For, we have seen in Vol. I., p. 19, that he attractive force between two charged plates is

$$F = \frac{V^2 A}{8\pi D^2},$$

here *V* is the difference of potential between the lates, *A* is their area, and *D* is their distance apart. n this case both *A* and *D* are unknown. Their effecive values are constant, however, since the instrument designed to be used as a zero instrument; that is, the eedle always has a fixed position with respect to the uadrants when its indications are read. To hold the eedle at zero when the needle and quadrants are harged, a current is passed through the coils *BB* in ch a direction and of such a strength that its deflectg couple on the magnetized mirror, is opposite and ual to the couple exerted on the electrometer needle the pressure which is to be measured. The latter is

use Electric Company, replaced the galvanometer a telephone receiver (Fig. 131). Whenever contact made by the contact maker, a sharp click is heard the telephone, unless a balance of pressure exists.

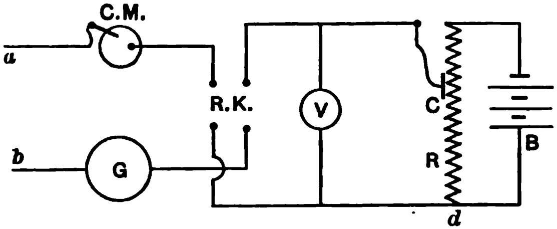

Fig. 130

order to get the balance with great exactness, it is ıally well to find the value of the balancing pressure en it is increased from a smaller value, and also when s decreased from a larger value. The mean value

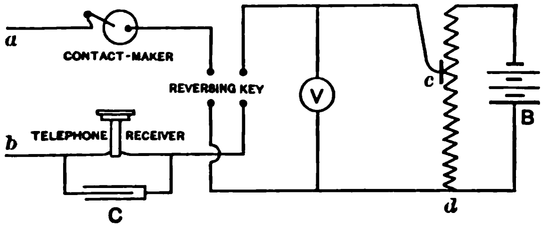

Fig. 131

:n by the two balancing points may be taken to resent the true balance. It is best to place a con- ser in parallel with either the galvanometer or tele- ne when this arrangement is used.*

Electrical World, Vol. 18, p. 140; Hopkinson's *Dynamo Machinery Allied Subjects*, p. 189. This has been modified by Duncan for espe- accurate work (*Electrical Engineer*, Vol. 19, p. 192).

6. *Duncan's Method* (1891). It is frequently desirable to make simultaneous determinations of several pressure and current curves. In this case, if one of the methods is used in which the indications are gained by the intervention of either a condenser or an electrometer, a contact maker is required for each curve. Dr. Louis Duncan of Johns Hopkins University, assisted by Mr. Carichoff and others, devised a method which avoids this multiplication of contact makers. The readings are made upon special electrodynamometers. One of these is provided for each curve which is to be traced, and the fixed coil of each is connected to the circuit to which its curve belongs. The movable coils are all wound alike of fine wire, and are connected in series. In circuit with them are connected a few cells of storage battery and a contact maker (Fig. 132).

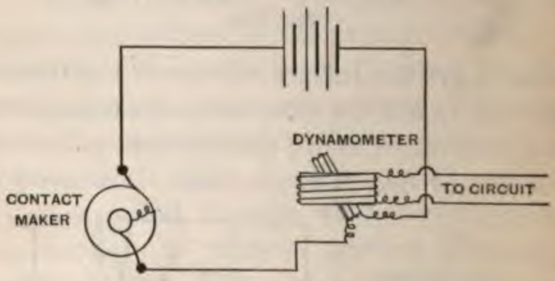

Fig. 132

It is evident that if alternating currents are passed through the fixed coils of the electrodynamometers and at a certain moment an instantaneous current be passed through the movable coils, each will receive an impulse that is proportional to the instantaneous value e alternating current in its fixed coil. If the in-aneous current be passed through the movable at recurring intervals of the same frequency as urrents under test, the movable coils will all take anent deflections which are proportional to the sponding instantaneous values of the alternating nts. By changing the instant of contact at the ct maker, the point at which the instantaneous nt passes through the movable coils may be made ident with any point on the alternating current s. Thus various points on the waves may be ltaneously determined, and the curves may be ed.

e electrodynamometers must be calibrated, but nay be readily accomplished by passing known con-us currents through the fixed coils of the instru-s while the regular interrupted test current is d through the movable coils. A calibration curve be plotted from these vations. To assure onstancy of the inter-d test current during es of observations, a onval galvanometer e inserted in the cir- In Dr. Duncan's it was found neces-to make the resist-of the circuit of the movable coils quite large (1000 in order to eliminate the effect of the variable ct resistance at the contact maker. In order that

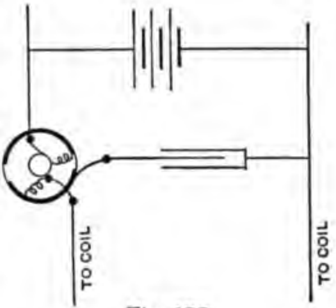

Fig. 133

the current through the coils should be brief and well defined a condenser discharge was found advantageous (Fig. 133).*

7. *Bedell's Method* (1893). Dr. Frederick Bedell of Cornell University, with others, has lately made a disposition of the instruments which is advantageous in many cases. Each of the methods thus far described depends upon the use of a special instrument or of instruments [illegible] to handle satisfactorily. On the oth[illegible]static voltmeters, which might be ma[illegible] usual instruments and are portable, gen[illegible]e which may be read over only a limite[illegible]nstrument reading up to 150 volts, for instance, is likely to give very poor indications below 60 volts. In order that such an instrument may be used, Dr. Bedell arranges it with a condenser as in Fig. 134. This condenser is kept charged to a known potential which is sufficient to bring the needle of the voltmeter to a satisfactory position on the scale. That is to say, the condenser serves to displace the zero of the voltmeter scale a known amount. The values of instantaneous pressures read on the voltmeter are then equal to the indications minus the initial readings.†

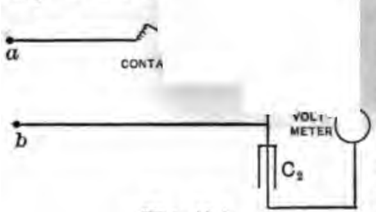

Fig. 134

8. *Pupin's Resonance Analysis* (1893). Dr. M. I. Pupin of Columbia College has devised and experi-

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* *Trans. Amer. Inst. E. E.*, Vol. 9, p. 179.

† *Ibid.*, Vol. 10, p. 503.

mented with a method for determining by resonance the various harmonics which enter into alternating-current curves. If the sinusoidal harmonics are fully known, the principal curve may be drawn (Sect. 30).*

Professor Ayrton several years ago proposed a plan for determining the sinusoidal components of a current curve by means of the vibrations of a stretched wire.

79. Contact Makers. — The earliest and simplest conact maker was, as already pointed out, simply an insuated pin set in the shaft of the alternator furnishng the current for the est. With this was a brush so arranged as to make contact with the pin at any desired point in he revolution. This arrangement is often inconvenient of application, and is likely to give rather irregular esults. The contact is likely to be variable in resistance and as the brush wears, the duration of contact varies. Each of these points introduces errors of greater or less magnitude, depending upon the condiions of the test. Various refinements of construction have been introduced by experimenters in order that he defects of the contact makers may be eliminated. Figures 135 to 139 show the contact makers used by Joubert, Searing and Hoffman, Ryan, Duncan, and Blondel.†

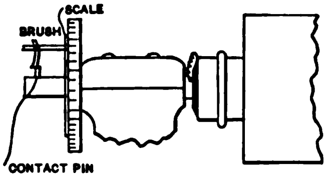

Fig. 135

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* Pupin, *Trans. Amer. Inst. E. E.*, Vol. 11, p. 523.

† *Comptes Rendus*, Vol. 91, p. 161; *Jour. Franklin Institute*, Vol. 128, 93; *Trans. Amer. Inst. E. E.*, Vol. 7, p. 3; *Ibid.*, Vol. 9, p. 181; a *Lumière Électrique*, Vol. 41, p. 512.

The contact makers used by each of these experimenters depend upon the mechanical contact between a point and a brush or spring, and therefore do not entirely avoid the difficulties from poor or variable contacts. If a contact of absolute uniformity were assured, special instruments would not be necessary for taking the indicatio ng pressure and current

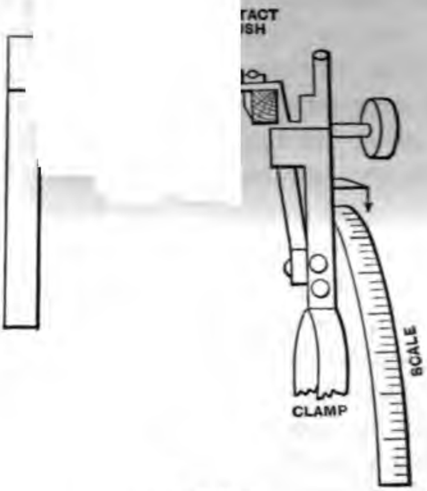
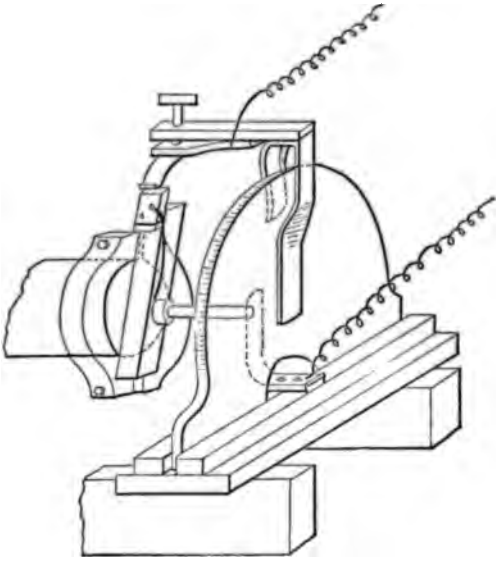


Fig. 136

curves, because the indications of a sensitive electrodynamometer might then be directly used. Professor Ryan and Dr. Bedell have lately made an ingenious arrangement by which the duration and resistance of the contact are made quite uniform. The arrangement is shown in Fig. 140. It consists essentially of a revolving disc, *D*, attached to the dynamo shaft, and a tationary graduated head, *H*. From the revolving disc ıeedle, *N*, projects. To this one dynamo terminal is ached. Upon the graduated head an insulated brass



'Fig. 137

zle, *T*, is mounted. The nozzle has a fine hole in ınd is so mounted that a thin jet of water flowing

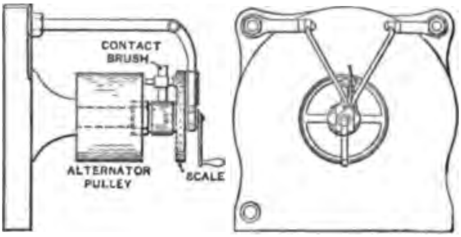


Fig. 138

m it is cut once in a revolution by the needle. A ınection from the nozzle to the indicating instrument completes the contact maker. By means of the graduated head the contact may be made at any desired point of the revolution. It is found that the jet may be satisfactorily maintained from a jar of water a few feet above the contact maker. The nozzle is radial, the jet keeps its direction for some little distance before being broken up, an [illegible] the jet quite near to the

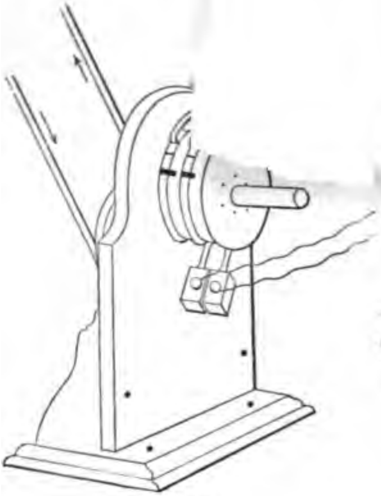


Fig. 139

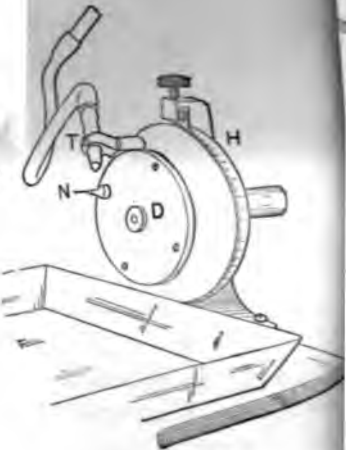


Fig. 140

nozzle where it is fairly stiff. Water with a little salt in it is used, as pure water has too high a resistance, and acidulated water corrodes the apparatus.*

The contact makers described thus far have been arranged for a single contact, but it is frequently desirable to make simultaneous observations of several curves.

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* *Trans. Amer. Inst. E. E.*, Vol. 10, p. 500.

ɜn Duncan's method is not available, this may be ily accomplished by using a contact maker with appropriate number of contact discs on the same dle (Fig. 139).* Then a satisfactory instrument, ı as an electrostatic voltmeter, may be used in each ıit. Sometimes it is not convenient to have the act maker attached to the dynamo shaft, in which it may be attached to a short length of flexible t (Fig. 141), which may in turn be attached to the ımo shaft. When connection to the alternator can-

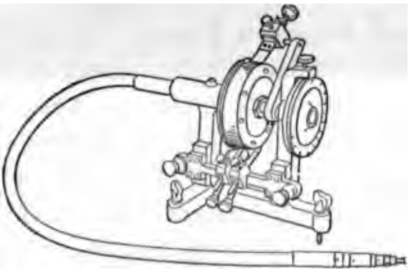

Fig. 141

ɔe conveniently made, the contact maker may be n by a synchronous motor as has been done by lel.† Siemens and Halske, and Fleming.‡

ıy of the methods in which a reflecting instrument ed may be made continuously self-recording by a ·r disposition of the apparatus. In this case a of light is thrown upon the mirror, and its devi- is recorded by means of a moving photographic

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* See Blondel, *La Lumière Électrique*, Vol. 41, p. 512.

† *La Lumière Électrique*, Vol. 50, p. 476.

‡ *London Electrician*, Vol. 34, p. 460.

x

film. In order that the complete curve may be thus recorded, the contact points must be caused to rotate continuously around the spindle of the contact maker. A form of contact maker designed for this purpose is shown in Fig. 139. Since the needle of the galvanometer or electrometer which is used with the contact maker must rigidly follow the intensity of the current impulses, the instrument must be truly deadbeat and have little inertia. The vibrations of a telephone diaphragm have been used to replace the deviations of a galvanometer or electrometer needle.*

80. Areas of Successive Curves. — In general, observations which cover one complete period entirely define the curves of current and pressure. Since there is no continuous transference of electricity in one direction, the areas of successive loops of the curves should be equal. In the pressure curves produced by an alternator, for instance, $e = \frac{dN}{dt}$, and $N = \int e\,dt$, where N is the total number of lines of force passing into the armature core and $\int dt$ is the length of the period. If N and T are constant, as would be the case for an alternator with fixed field magnetism and a rigid armature shaft which is driven at a uniform speed, the values of the successive areas must be equal. On account o various irregularities in the construction and workin of alternators, experimentally determined curves are n always uniform. In fairly large commercial machin the differences are usually not greater than might

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* See Blondel, *La Lumière Électrique*, Vol. 41, p. 401; Frœlich, *El*
otechnische Zeitschrift, Vol. 10, p. 345.

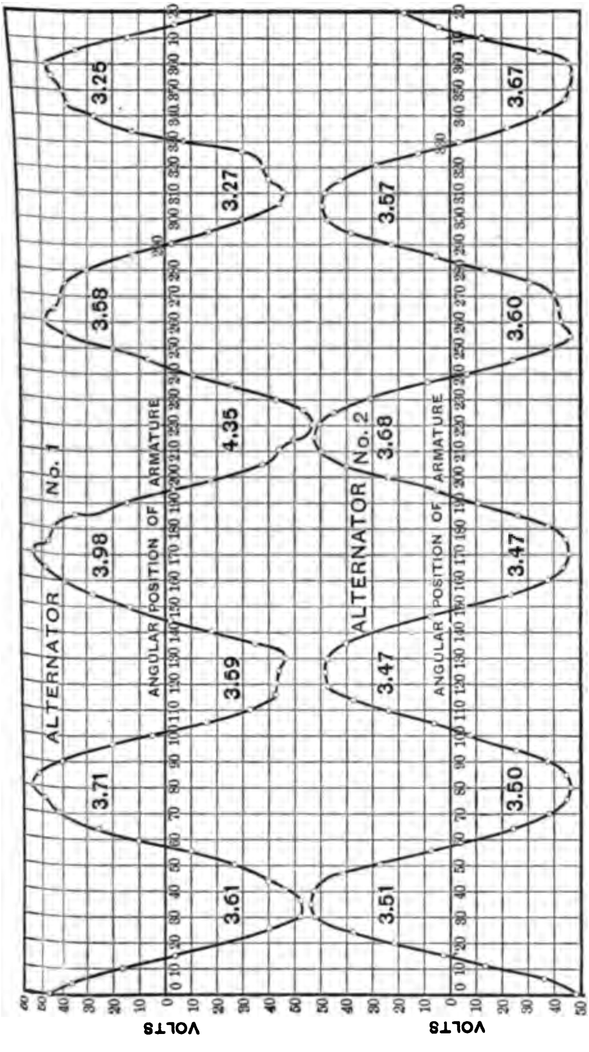
ALTERNATOR No. 1
ANGULAR POSITION OF ARMATURE
VOLTS
3.61
3.71
3.59
3.98
4.35
3.58
3.27
3.25
ALTERNATOR No. 2
ANGULAR POSITION OF ARMATURE
VOLTS
3.51
3.50
3.47
3.47
3.68
3.60
3.57
3.67

Fig. 142

caused by the errors of observation due to the experimental determination, and appreciable differences in the areas of successive loops of the curves produced by mechanically rigid machines driven at a uniform angular velocity are not to be expected, except possibly when the machines have armatures with their halves connected in parallel, and then only when the magnetic circuits lack symmetry to a considerable degree. In the case of certain small eight-pole alternators, Dr. Bedell found differences in the areas of the consecutive loops which are scarcely explainable upon the ground of errors of observation or of variable speed.* The curves given by two of these machines in one complete revolution (four complete periods) are shown in Fig. 142. The individual areas of the loops are marked upon the figure. While these differ as much as 25 per cent amongst themselves, the sums of the positive and negative areas differ by no more than might be caused by experimental errors. This apparently shows that irregularities in the magnetic circuits and in the armature windings may in some cases cause differences in the successive loops of the curve developed in one revolution, but the algebraic summation of the areas due to each revolution is zero. The latter must be true, or there would be a continuous flow of electricity in one direction. The fact that the machines tested by Dr. Bedell had notable structural weaknesses, leads to the probability that the springing of the shaft or other parts of the machine may have caused the unusual result which he found.

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* *Physical Review*, Vol. 1, p. 218.

81. Determination of the Effective Values of Current or Pressure from their Curves. — It is often desirable to determine effective values of current or pressure from the experimentally determined curves. In this case, a second curve may be plotted, the ordinates of which are equal to the square of the respective ordinates of the primary curve. The square root of the mean ordinate of the second curve is the effective value of the ordinates of the primary curve. The mean ordinate of any curve is readily determined by measuring its area by planimeter and dividing the area by the length of the base. The effective value may be directly derived from the primary curve, as originally shown by Steinmetz,* if it is plotted on polar coördinates, taking 360° to a complete period. This gives a symmetrical curve which crosses the origin at 0°, 180°, 360°, etc. For an exact sinusoid the curve is of the form shown in Fig. 143, and has its maximum value positive and negative, at 90° and 270°.

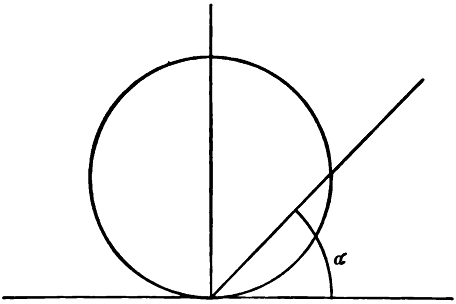

Fig. 143

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* *Trans. Amer. Inst. E. E.*, Vol. 10, p. 527; *Elektrotechnische Zeitschrift*, June 20, 1890.

Each loop is a circle with the pole on its circumference and the initial line tangent to the circumference, the maximum ordinate, a, being equal to the diameter. The area of the curve in this form may be shown to be directly proportional to the effective value of the ordinates as follows: In the case of a sinusoidal curve, the polar curve has sin a, where e is the instantaneous pr ng to an angular advance a. In plotting s of e are laid off on the radius vect al angles equal to the corresponding a line is drawn through the points thus). Each loop of this curve, that is, the pa taken between $a = 0°$ and $a = 180°$, or $a = 180°$ and $a = 360°$ is a circle, and it area is $A = \frac{1}{4}\pi d^2$, where d is the diameter of the circl By the construction, d is equal to a of the formu $e = a \sin a$, and the area of a loop of the curve is ther fore $A = \frac{1}{4}\pi a^2$. The effective ordinate of a sinusoid h already (Vol. I., p. 83) been shown to be

$$E = \frac{1}{\sqrt{2}} e_{max} = \frac{a}{\sqrt{2}}.$$

Consequently
$$E = \sqrt{\frac{2A}{\pi}} = .798\sqrt{A}.$$

This may be taken for most purposes as $E = .8\sqrt{A}$.

In the case of any single-valued function

$$e = a \sin a + b \sin 2a + c \sin 3a + \text{etc.}$$
$$+ a' \cos a + b' \cos 2a + c' \cos 3a + \text{etc.},$$

d the area of one loop of the polar curve representi

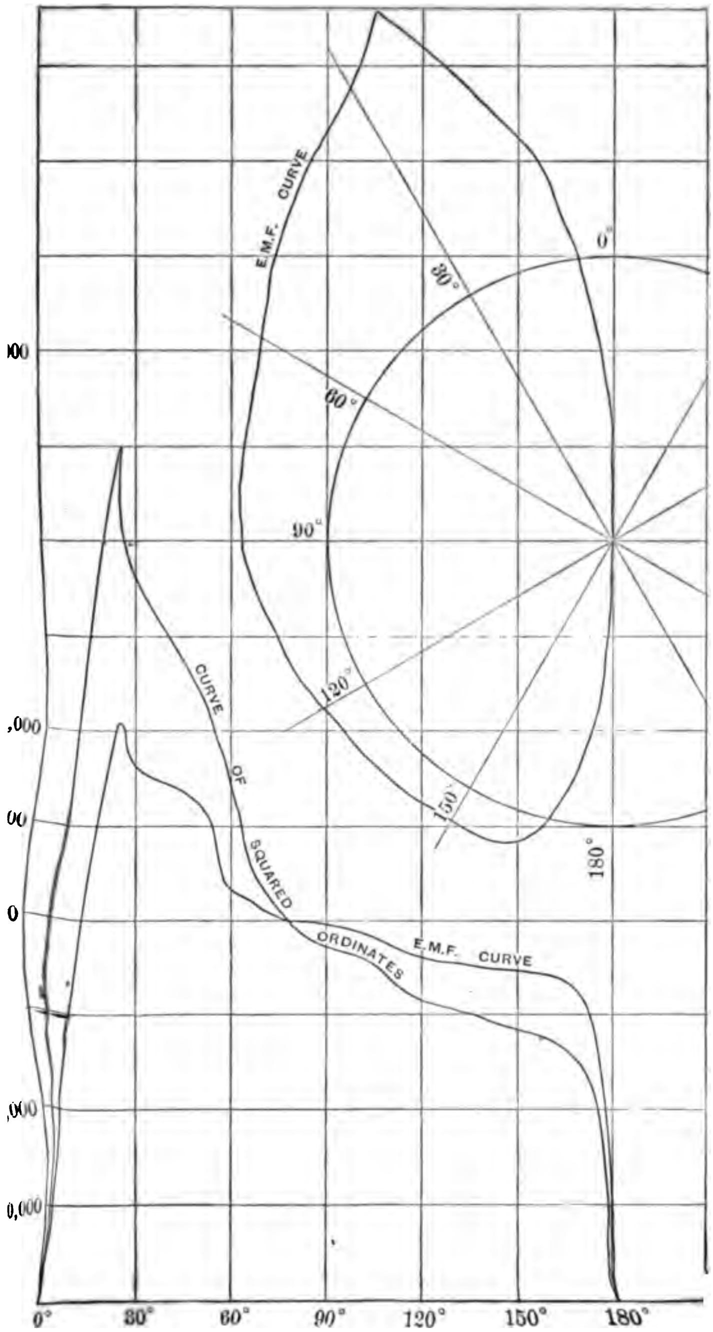

E.M.F. CURVE
0°
30°
60°
90°
120°
150°
180°
000
,000
00
0
,000
0,000
CURVE OF SQUARED ORDINATES E.M.F. CURVE
0°
30°
60°
90°
120°
150°
180°

Fig. 144.

this is $A=\int_0^\pi e^2da$. The mean of the squared ordinates of the function is av. $e^2=\frac{1}{\pi}\int_0^\pi e^2da$. The effective ordinate is

$$E=\sqrt{\text{av. }e^2}=\sqrt{\frac{1}{\pi}\int_0^\pi e^2da}=\sqrt{\frac{2A}{\pi}}=.798\sqrt{A}.$$

As before, [illegible] n as $E=.8\sqrt{A}$.

Figure [illegible] ve of squared ordinates and the polar [illegible] ssure wave of the Stanley alternator, [illegible] e has already been made.

CHAPTER VII.

REGULATION AND COMBINED OUTPUT.

82. Regulation for Constant Pressure. — *A. Separately Excited Alternator.* A separately excited alternator has, as already intimated, no inherent tendency towards regulation. The regulation is usually effected by hand, either by means of a hand regulator in the field circuit of the shunt-wound exciter or a hand regulator directly in series with the alternator fields. The adjustment of these regulators may be performed through devices actuated by a relay placed as a shunt to the main circuit, but this is considered inadvisable in this country and the use of automatic regulators with alternators is entirely unknown; but in Great Britain and Europe automatic devices are used in many large plants.

An ingenious device which seems to give satisfaction is made by the firm of Ganz & Company of Buda Pesth. The essential parts of this regulator are a solenoid which is connected as a shunt to the main circuit. This solenoid attracts an iron core which carries a mercury cup at its top. Since the current which circulates in the solenoid depends upon the pressure of the main circuit, the position of the core with its mercury cup depends upon the pressure. A series of wires of graduated engths dip into the mercury cup in such a way that

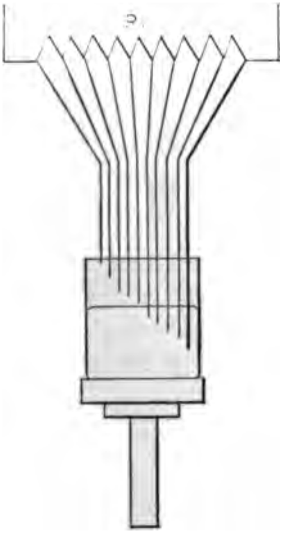
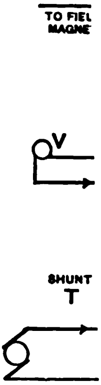


Fig. 145



of consumption rather than at the
Company have succeeded in arran
to do this without the inconvenienc

ncreases, while at the same time the dynamo pressure increases by a sufficient amount to compensate for the fall of pressure in the feeders which run to the centre of distribution. In other words, $E - CR$ must be kept constant, E being the dynamo pressure, C the current, and R the resistance of the feeders. This is effected as follows (Fig. 146): The regulator is connected to the secondary of a special transformer T, which is connected in parallel across the feeders. The pressure of the secondary of this transformer is proportional to the dynamo pressure E. Another transformer, T', is connected with its primary in series with the feeders. The pressure developed in the secondary of this transformer can be adjusted so as to be practically equal to CR for all values of the current. The secondary of this is connected in series with the secondary of the first transformer, and in such a way that their pressures are in opposition. Hence a voltmeter, V, connected across the terminals of the two secondaries indicates a pressure which is proportional to the pressure at the terminals of the feeder, or $E - CR$. If the automatic regulator is also connected across the terminals of the two secondaries, it will adjust the excitation of the alternator so that $E - CR$ is kept constant regardless of the value of C. In the figure, S is the solenoid f the regulator, R_v is the resistance automatically conolled by the solenoid to vary the excitation of the ternator, and R_1, R_2, R_3 are resistances in circuit with e regulator which are used for adjusting it to give oper indications for various values of C and R.*

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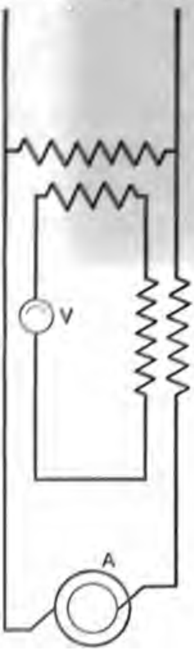
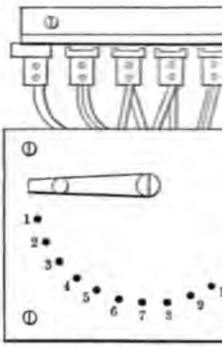
* Fleming's *Alternate Current Transformer*, Vol. II., p. 137.

The device here used for obtaining at the te
of the regulator a pressure which is proporti
$E - CR$, is exactly similar in operation to the We
house, so-called, "compensated voltmeter" which i
in this country. In this case hand regulation is
sively adopted, but it is desirable to give a co
pressure at the point of consum
To avoid the expense and anno
of "pressure wires" the statio
meter is connected in series wi
secondaries of a parallel and a
transformer which act in opp
(Fig. 147). The transformers





HAND REGULATOR

Fig. 147

properly adjusted, the voltmeter shows at all tin
lamp pressure at the centre of consumption.*
Westinghouse apparatus, several terminals are b
out from the secondary of the series transforme

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* Compare Fleming's *Alternate Current Transformer*, Vol. II.,

like that shown in Fig. 147, and the adjustment pparatus to compensate for any line drop is made nging the secondary connections and so changing e number of effective secondary turns in the or circuit. In some cases, the secondary of the transformer is not connected into the circuit of ular voltmeter coil, but is connected to an auxoil which is wound de of or over the il. Westinghouse Comlso manufacture a regulator for use in where several cire fed from one alter- r set of 'bus bars. essentially a special rmer (Fig. 148) with condary, *CD*, conin series with one re, and the primary, nnected across the the pressure in- in *CD* may be made to either aid or oppose the tor pressure by means of a reversing switch, *X*. rength of the induced pressure in *CD* may be by changing the number of effective turns in *CD* or *AB* by means of movable contacts. 149 gives a view of the transformer with the or switches. Similar devices, in which the regus effected by varying the position of the primary

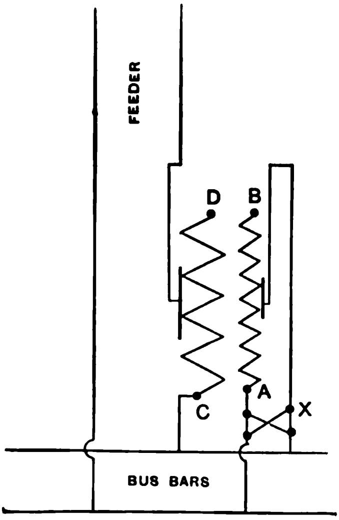

Fig. 148

and secondary coils with respect to each other, or of the core with respect to both, are manufactured by the General Electric Company.

In an English plant, for which the machinery was constructed by the Electric Construction Corporation, another plan is used in regulating separately excited alternators. In this case series-wound exciters are used, the regulation of which is effected by shunting their fields. The shunt is composed of a liquid resistance into which dip two plates which are connected across the terminals of the exciter fields. These plates are raised and lowered in the liquid, to vary the resistance of the shunt, by means of a solenoid. This in turn is actuated by an ingenious thermal relay, which consists of two stretched wires connected to the secondary of a transformer. The primary of the transformer is connected across the main circuit of the alternator or across the terminals of one coil of its stationary armature. The pressure developed in the transformer

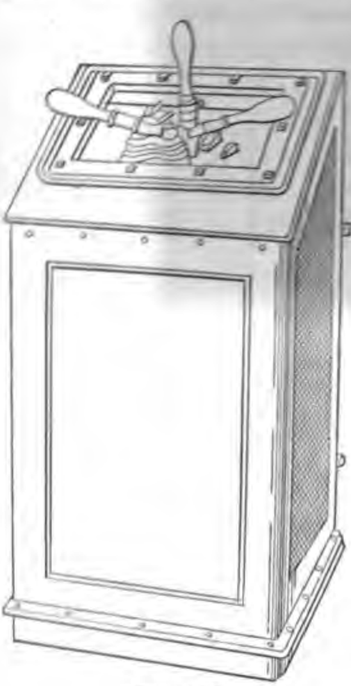
Fig. 149

ndary is therefore proportional to the alternator
sure. When the latter falls below normal, less
the normal current flows through the relay wires,
ch contract enough to actuate a switch which causes
solenoid to lift the electrolytic plates and thus
ease the resistance across the fields of the exciter.
en the alternator pressure rises above the normal,
e current flows through the relay wires, which, by
ging, actuate the regulating apparatus so that the
es are lowered further into the liquid. In this
ner the fields of the exciter are regulated so that
alternator pressure is kept constant.

n this country self-regulation of alternators is pre-
ed to automatic regulation by external devices.*
s is effected by means of composite windings (Sect.

Composite-wound alternators may be best treated
eparately excited alternators with a certain number
elf-excited series turns on the field-magnets. The
excited field turns are usually of sufficient number
nake the external characteristic a nearly straight
zontal, or slightly rising line. The predetermina-
of the number of series turns required to give
t compounding, or a desired degree of over-com-
nding, is not readily accomplished when no experi-
tal data of the machine is at hand, on account of
complex effect of self-induction and armature reac-
s upon the pressure of the machine (compare Sect.

The compounding that gives regulation on an in-
ionless load evidently may fail for an inductive
. The ratio of series ampere-turns per pole to

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* Compare *Text-book*, Vol. I., p. 216.

armature ampere-turns per coil which is required to give regulation for one alternator of a fixed type, will doubtless give equally satisfactory results on machines of different capacities but of the same type; but the marked differences in the magnitude of the effects of self-induction and armature reactions in alternators of different typ ssible to fix any ratio that will even ap r all types of machines. For alternat -core drum armatures the ratio of serie per pole to ampere-turns per armature ally unity. In machines with ring, p l armatures the ratio is doubtless cons . In machines with disc armatures it may b at smaller. The various arrangements of the circuits that may be made in composite windings have already been pointed out (Sect. 71). It is quite common to place a variable shunt around the series windings so that the magnetizing effect may be varied, exactly as is done in compound-wound continuous-current dynamos (Vol. I., p. 225).

83. Regulation for Constant Pressure.—*B. Self-excited Alternator.* The defect in self-regulation of an alternator excited from an independent winding on the armature is practically the same as that of a separately excited machine. In a shunt-wound machine the defect is greater, as already stated (Sect. 74). The regulation of alternators which are self-excited in shunt or by a separate exciting coil, can only be satisfactorily effected by means of a variable re ance or hand regulator placed in the exciting circ The regulation might be effected by moving the brus the rectifying commutator, but only at the expense prohibitive sparking and wear. The variable rheostat ay be operated automatically by the same devices that re sometimes used with separately xcited machines. These do not meet ith favor in America, however.

84. Regulation for Constant Current. –Constant-current alternators may be ither separately or self-excited. Their egulation is made entirely inherent by esigning their armature reactions and elf-induction to be so great that the urrent cannot rise above its normal alue. The armature is wound to generate a pressure upon open circuit uch greater than that required for ll load, and hence the current remains ear its full normal value up to, and ven considerably beyond, full load. uch machines are worked on shortrcuit without injury, but if the circuit opened, they are liable to injury on count of the excessive open circuit ressure breaking down the insulation. hese machines are really worked on part of the characteristic which is used to be almost vertical on account the large self-inductance of the arature. Constant-current alternators have only been ed for arc-lighting. The external characteristic of a tanley arc-light alternator is given in Fig. 150.

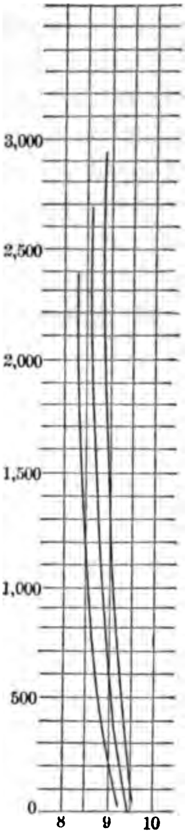

Fig. 150

85. Connecting Alternators for Combined Output. — The conditions required for successfully connecting alternators so that their outputs may be combined, are quite different from those obtaining in the case of continuous-current machines. In order that the output of alternators may be added, it is evident that the pressure waves in upon the circuit must be in exact consor he pressure waves must be of equal pe ronism, and also of corresponding ph ith each other. If this is not the case ill be in opposition during all or a porti t wave.

86. Alternators —The alternators will be assumed in this discussion to be constructed so as to give equal currents at a fixed frequency. The form of the current waves will also be assumed to approximate a sinusoid.

In Fig. 151 *a* let the curves A and A' represent the electric pressure waves of two alternators with their armatures connected in series, the machines being driven independently, but so as to give practically the same frequencies and pressures. The ordinates of curve R are the algebraic sums of the corresponding ordinates of curves A and A', and hence curve R represents the resultant pressure of the two machines. Curve C is assumed to be the curve of current flowing in the circuit. Assuming the two machines to be running synchronously, but to be out of step by an angle 2θ, makes the phase difference between the resultant pressure w and either component wave $\pm\theta$. Finally, the cur lags behind the resultant pressure by an angle ϕ

unt of self-inductance in the circuit. The work put
the circuit by either machine is proportional to the
raic summation of the products of the ordinates of
espective pressure and current waves. The total
done in the circuit is equal to the sum of the prod-
of the ordinates of the current and resultant press-
urves. Therefore, since the pressure wave of the

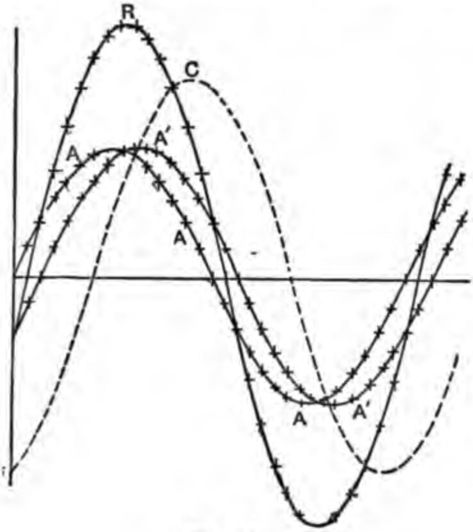

Fig. 151 a

g machine is nearest the current wave, that ma-
furnishes more work to the circuit than does the
g machine. The power loops for the two machines
own by the curves a and a' in Fig. 151 *b*, and the
r delivered to the circuit by the two machines is rep-
ted by the heights of the lines xx and $x'x'$. Were
wo machines rigidly connected together, this condi-

tion would continue indefinitely. In practice, however, the machines are driven by separate belts or attached to separate engines, and the lagging machine, being heavily loaded, tends to fall further behind its more lightly loaded mate, and a still greater percentage of the load is thrown upon it. At the same time, as is shown by Fig. 152*a*, th tal work done in the external circuit, f ssure wave is now of less height than e component curves were more nearly power loops for the condi-

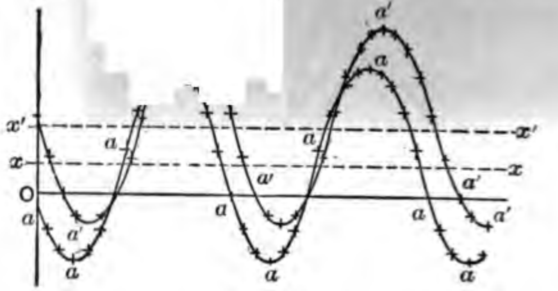

Fig. 151 b

tion of Fig. 152*a* are shown in Fig. 152*b*. The height of the line xx has decreased, and that of $x'x'$ has increased, but the sum of the heights is less than before. The tendency of the lagging machine to fall further behind continues until the pressure waves of the two machines are exactly opposed (Fig. 153). The machines are then in stable equilibrium, but are giving no energy to the external circuit. If the machines were started with their pressure waves in exact tep, they would do equal work, but their equilibrium 'ould be unstable, and any disturbance of their r is would cause them to fall into opposition. It is refore not possible to operate alternators in series

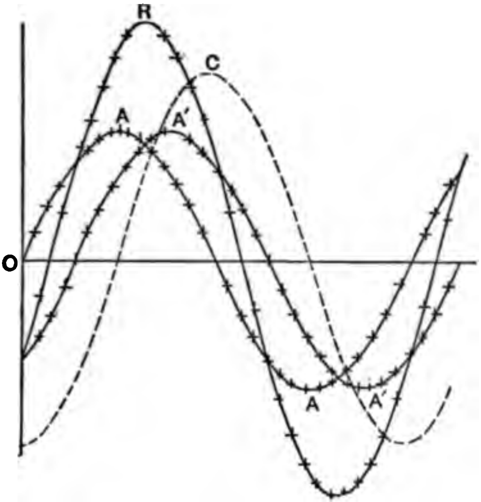

Fig. 152 a

n inductive circuit unless they are rigidly united by echanical coupling.* This result also follows when

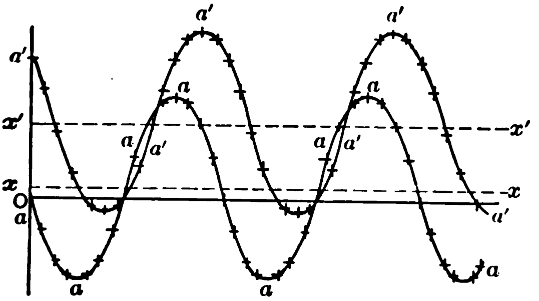

Fig. 152 b

Compare Hopkinson, Some Points in Electric Lighting, *Proc. Inst.* ., 1883, and Theory of Alternating Currents, *Jour. Inst. E. E.*, Vol.

the normal pressures of the machines are different; in which case the pressure impressed on the circuit when equilibrium is attained is the difference of the machine pressures. If there were no inductance or capacity in the circuit on which the machines were working, the resultant pressure and current would have the same phase, and the machines would be in equilibrium, but the equilibrium would be unstable, for after any disturbance of the opera[illegible]chines they would have

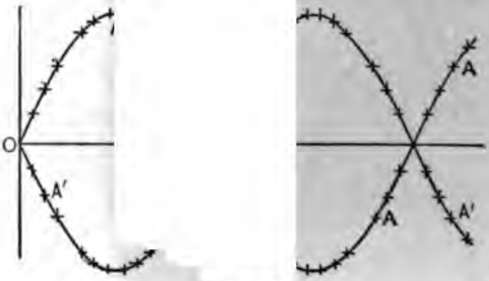

Fig. 153

no tendency to return to their former operating state. No such case is to be met with in any event, because the armature windings of the machines introduce self-induction and current lag into the circuit, even when the external circuit is non-inductive.*

87. Alternators in Parallel.—When the machines have reached opposition of phases, as explained above,

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13, 1884, p. 496; Hopkinson's *Dynamo Machinery*, p. 148; Picou's *Machines Dynamo-Électrique*, p. 279; Kapp's *Dynamos, Alternators, and Transformers*, p. 420; Thompson's *Dynamo-Electric Machinery*, 4th ed., p. 689.

[*] The condition of operation of two alternators connected in ser on an inductive circuit is perhaps more plainly indicated by the follow

a change in the arrangement of the circuit puts them at once in parallel and in step for working in the circuit; for, it will be seen by reference to Figs. 153 and 154 that when machines A and A′ are in opposition, the

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figure (Fig. 1), than by the curves which were used in the preceding demonstration, and which follow those originally presented by Dr. John Hopkinson, in 1883 (*Proc. Inst. C. E.*, 1883; *Jour. Inst. E. E.*, 1884).

Pressure of leading machine = OA.
Pressure of lagging machine = OA'.
Resultant pressure in circuit = OR.
Current in circuit = OC.
Power given to circuit by first machine = $Oa \times OC$.
Power given to circuit by second machine = $Oa' \times OC$.
Total power given to circuit = $Or \times OC = (Oa + Oa') \times OC$.

It is evident from the construction that as the angle θ increases, the ength of OR decreases, and also that Oa decreases; but Oa' increases for

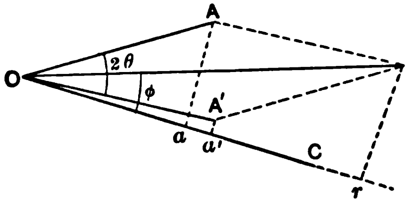

FIG. 1

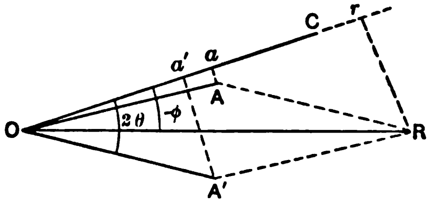

FIG. 2

a time and then decreases at a less rate than Oa, so that the machines tend to get farther apart in phase. When $\theta = 90^\circ - \phi$ the length of Oa vanishes, the first machine gives no power to the circuit, and all the power is furnished by the second machine. When θ approaches more nearly 90°, or the machines are approaching opposition, one machine may actually points *R* and *S* must at every instant be of opposite sign, and that, therefore, the machines will deliver current through the circuit *m*, *m*, *m*.[*] The operation of alternators in parallel was first achieved by Wilde in 1868,[†] but this work was overlooked during the peri of development of the continuous-current dynamo. I 1884 Dr. John Hopkinson showed by mathematic analysis, in the paper already referred to, the imprac cability of working alternators in series and the prac

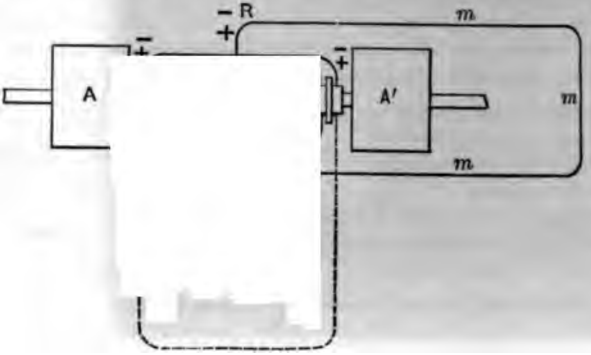

Fig. 154

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run as a motor. Of course, when *OR* decreases, if the resistance of th circuit is unaltered, the current, *OC*, also decreases, but the relative ou puts and phases of the machines are not altered thereby.

In case there is a capacity in the circuit which is sufficient to cause th current to lead the resultant pressure by an angle ϕ, the condition is repre sented by Fig. 2. In this case, *Oa* is greater than *Oa'*, or the leadin machine furnishes the greatest amount of power, ***and the machines tend t come together and run in series.*** This is not a practical condition, how ever, since a capacity in a commercial alternator circuit sufficient to give the current a lead is practically unknown.

[*] Compare references given above, and Fleming's ***Alternate Current* *nsformer***, Vol. II., p. 356.

ee *Philosophical Magazine*, Vol. 37, 4th series, 1869, p. 54.

ticability of working them in parallel. This was done without a knowledge of Wilde's earlier experiments, and it led to some experiments which were carried out by Hopkinson and Adams upon De Meritens magneto machines.* These experiments fully bore out Hopkinson's deductions, but their practical bearing was not fully appreciated until a few years later, when the transformer system of alternating-current distribution was developed.†

88. Synchronizers and Synchronizing. — Mechanical imperfections in engine governors and machine pulleys cause slight differences in the speeds of machines intended to run at equal velocities. Consequently it is desirable to arrange some device for determining the moment a machine is in synchronism with one with which it is to be thrown in parallel. When the alternators are to be connected in parallel, the terminals of each may be connected directly to appropriate 'bus or main conductors through convenient indicating instruments, switches, and safety devices. Before switching a new machine upon the 'bus conductors it must be brought to normal speed, and to the pressure of the other machines. Then at a moment when it is in synchronism and in step with the pressure wave of the 'bus bars, it may be switched into circuit without causing a disturbance among the other alternators.

Any device for indicating the synchronous relation is called a **Synchronizer** or **Phase Indicator.** Its simplest form for low-pressure machines consists of one or incandescent lamps in series, which are connected Fig. 155. One terminal of the alternator is conr directly to a 'bus conductor, while the other is conr through the lamps to the other 'bus. When the ure waves are not in opposition, the lamps w illuminated, and at the moment of opposition th mination will die out. If the frequencies of the nator and the circuit differ materially, the flasl illumination or "beats" are quite rapid. As th

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* Adams, *Jour. Inst. E. E.*, Vol. 13, 1884, p. 515.

† Compare Hopkinson, *Jour. Inst. E. E.*, 1884, and *Dynamo Machinery*, p. 174; Thompson's *Dynamo-Electric Machinery*, 4th ed., p. 696, etc.

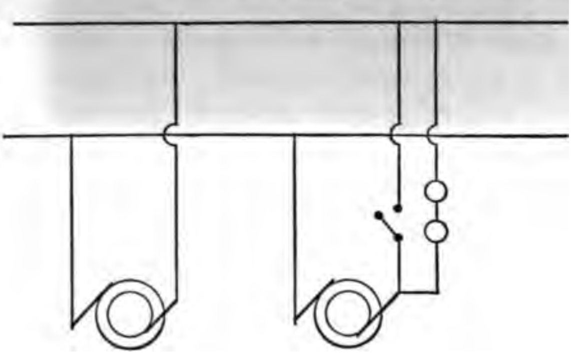

Fig. 155

quencies approach synchronism, the beats lengthe exactly as do the beats of two tones which are app ing unison. The alternator should be connected 'bus bars at an instant of no illumination dui period when the beats are fairly long. This ind that the pressure of the alternator is in synchr with that of the circuit and that it is in propei or phase. Continued illumination or darkness c lamps under these circumstances can only occur the machines produce the same pressure and run at absolutely the same frequencies, which is not a practical occurrence unless the machines are rigidly connected together.

Since alternators are commonly built for high pressures, it is usual to use a transformer with the synchronizer lamps. The primary circuit of this transformer may be composed of either one or two windings. When it is composed of one winding, one terminal of the alternator is connected to a 'bus bar through it, the other terminal of the alternator being connected directly to the other 'bus (Fig. 156). In this case the lamp on the secondary circuit acts exactly as the synchronizer lamps already described. When the primary is composed of two circuits, one is connected between the 'bus conductors and the other between the terminals of the alternator (Fig. 157). In this case the proper phase relation for switching the alternator into

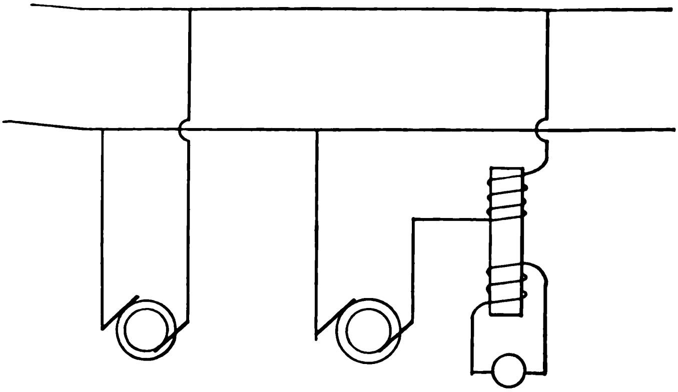

Fig. 156

circuit may be indicated either by darkness or b
mination of the lamps, depending upon the conn
of the synchronizer primaries. This arrangem
advantageous, since it allows the use of a doubl
switch at the dynamo, while the previously des

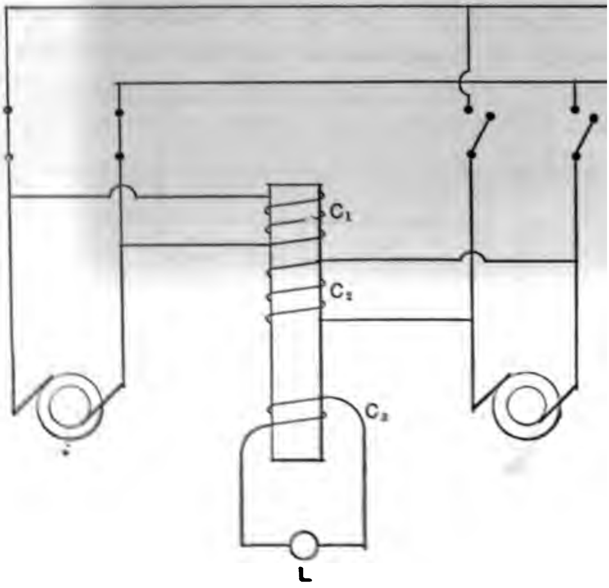

Fig. 157

arrangements require the use of single-pole swi
The latter arrangement may be modified by usin
separate transformers, the primary of one being
nected to the alternator and that of the other t
circuit. The secondaries are connected togeth
series with one or two lamps (Figs. 158 and 159
the secondaries are connected directly in series,

. 158, darkness of the lamps indicates the instant connecting the alternator to the 'bus conductors. the secondaries are cross-connected, as in Fig. 159, ximum illumination of the lamps indicates the mo- nt when the machines are in proper step. In this intry the general practice has been to connect the

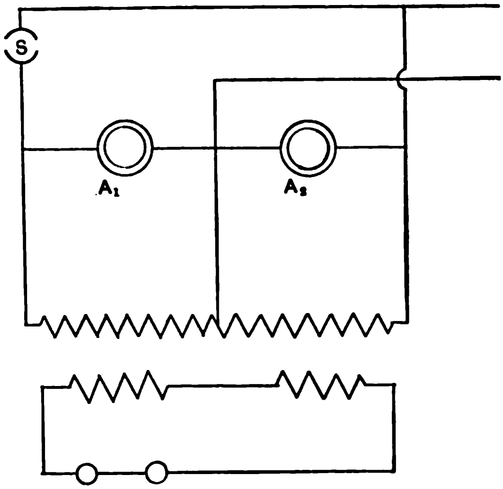

Fig. 158

hronizer so that darkness indicates when the ma- ies are in step. This has the evident advantage it darkness is a condition which is more readily dis- iguished than the condition of maximum brightness. his practice, however, does not seem to be always illowed in England and Europe.*

In any of these methods, the lamps may be replaced by a sensitive high resistance alternating-current amperemeter or galvanometer which is dead-beat.

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* Compare Fleming's *Alternate Current Transformer*, Vol. II., pp. 163 362; Kapp's *Alternating Currents of Electricity*, p. 129; Gerard's *ns sur l'Électricité*, 3d ed., Vol. 1, p. 557.

An ingenious device for use as a synchronizer has lately been developed by the General Electric Company. This consists of two electromagnets made with iron wire cores. The windings of these are connected respectively t and the circuit. Each magnet has of it an iron diaphragm which emits as a pitch due to the

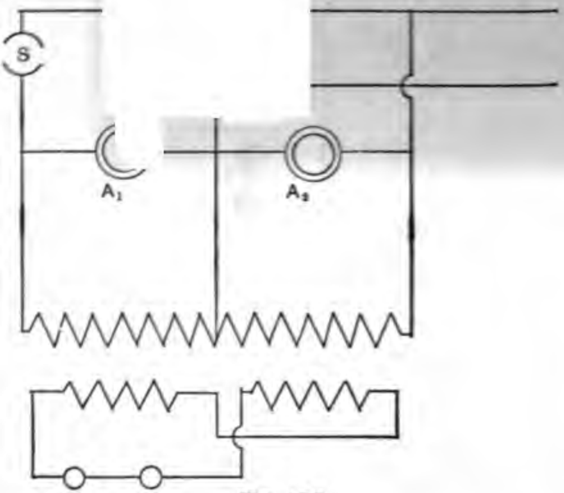

Fig. 159

frequency of the current flowing in the winding. In front of the magnets are placed resonators which magnify the sound emitted by the diaphragms. When the two tones are not in exact harmony, interference uses beats, and the synchronizer emits an inter- tent sound. If the speed of the machines is ught nearer and nearer to synchronism, the beats become less rapid. At exact synchronism, the beats die out and a clear tone results. The alternator may or may not be in step when the clear tone is given, but it may be safely thrown into circuit, for the interaction of the current waves will bring it into proper phase relations. This synchronizer was designed for use with synchronous motors. It cannot give satisfaction with alternators that are furnishing constant pressure current for incandescent lighting, since placing an alternator in the circuit while it is out of step is likely to momentarily disturb the pressure.

It is entirely possible to do without a synchronizer when connecting alternators in parallel, provided they are driven at approximately equal speeds. In this case if the alternator is brought to the proper pressure and is then connected to the 'bus bars, the machine reactions will bring it into step. This plan was practised to some extent in one or two earlier American plants, but it is vicious in its working. Throwing machines onto the 'bus bars under these conditions usually causes a disturbance in the pressure, and it also doubtless strains the armature of the alternator on account of the sudden torque impressed upon it to bring it into step.

89. Usual Practice with Reference to Parallel Operation. — Parallel working of alternators has not been usual in this country heretofore, though it is quite a common practice in Europe. Several of the earlier American plants, installed by the Westinghouse Electric Company, were arranged for parallel working; but the plan was quickly abandoned on account of the machines dividing their load unevenly and thus causing trouble. This

difficulty was quite similar in many respects to that encountered in the early endeavors to operate compound continuous-current dynamos in parallel. It was usual in these early alternating plants, as it is now, to belt the alternators to independent engines. On account of the unsatisfactory results met in the early attempts at parallel working, it has come to be the almost universal practice in this country to operate alternators on separate circuits.
s some complications in
the station s
gements, on account of
the necessity
connections of machines
and circuits
they may be intercon-
nected in an
atisfactory switching ar-
rangements
isfactorily accomplished.
A common ar
itch boards for this pur-
pose is shown
where double-pole throw-
over switches are con
e
such a manner that any
feeder may be connected to any alternator in the system. In some of the later arrangements for large stations, plugs and cords are arranged to be used in combination with the throw-over switches (Fig. 161). By these arrangements the feeders may be almost instantly transferred from one alternator to another, so that any readjustment of the alternator loads may be made without more disturbance than a mere wink of the lamps. It is evident, however, that, with this arrangement, no feeder can be designed to supply a district demanding a greater output than that of the station
ver unit. Figure 160 shows an arrangement for two
nators and four feeders, and Fig. 161 an arrange-
for three alternators and three feeders.

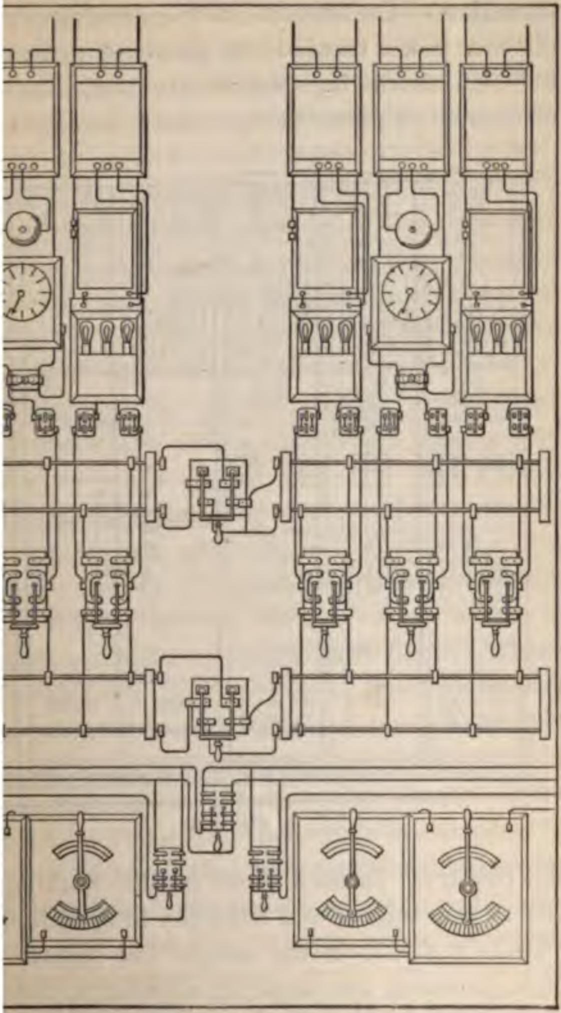

Fig. 160

90. Elements which affect the Success of I
Operation.—On account of the development o
distance power transmission plants of great mag
in which alternating currents are used, parallel w
of alternators seems likely to soon become comi

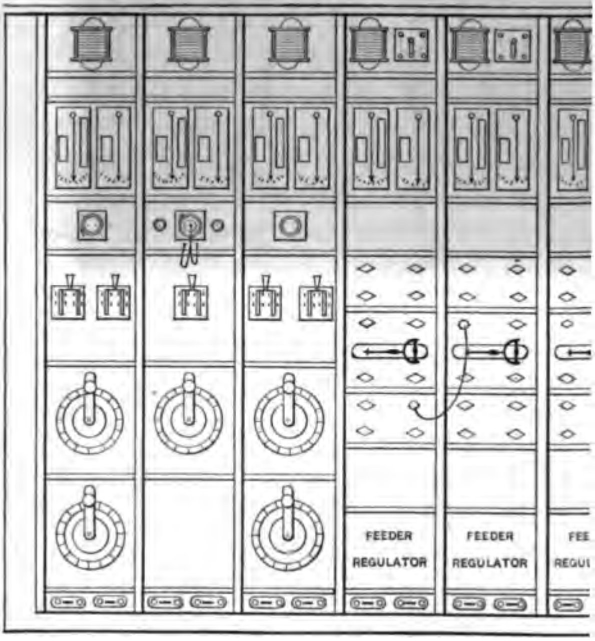

Fig. 161

this country. In such plants parallel working is
cive to convenience and reliability in operation.
permits a saving in the cost of line insulation
high pressures are used, by allowing a concei
of conductors. Parallel working of alternators
advantageous in large electric light stations, w

may be made conducive to convenience, reliability, and economy of operation. The subject is therefore one of considerable interest to us. There is a considerable disagreement amongst builders of alternators, and others, as to why some types of alternators apparently operate well in parallel, while other types do not. There is also an apparent disagreement as to what constitutes successful parallel operation. This is a pure question of practice, and we must therefore rely upon results that have been and are being attained in central station work. Mr. W. M. Mordey seems to have been the first to clearly point the way to truth in the case,* as he was the first to point out categorically some of the relations between the continuous-current dynamo and motor.†

The three elements causing the greatest friction in the discussion of this subject are the effects of frequency, of the form of the pressure curve, and of self-induction. And it is well in such a discussion to consider with particular attention these points: (1) the effect of frequency; (2) the effect of armature inductance; (3) the effect of the form of the current and pressure curves; (4) the effect of regulation by varying the excitation; (5) uniformity of angular velocity. None of these points have been so thoroughly investigated by experiment as to be decided with entire conclusiveness, and the mathematical investigations have in many cases been based upon erroneous premises, and are therefore

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* Alternate Current Working, *Jour. Inst. E. E.*, Vol. 18, p. 583.

† *Philosophical Magazine*, Vol. 21, 5th series, 1886, p. 20, and *The ondon Electrician*, Vol. 16, p. 193.

incorrect; but the experimental investigations hav
sufficiently extended to give a satisfactory clue
true conclusions.

Successful parallel working of alternators req
(1) that they synchronize readily when driven at
which differ by a few per cent; (2) that they sh
stantly fall into step if thrown into parallel when th
practically synchronized, but are out of step; (
they shall continue to operate in parallel, dividi
loads proportionally under the varying conditions
vice and attention; and (4) that the wattless or sy
nizing current passing between the machines, but n
the external circuit, shall be small. The latter co
requires that the apparent watts passing into the
nal circuit shall be appreciably equal to the sum
apparent watts delivered by the individual ma
If the machines do not have a strong inherent te
to remain in step, they are likely to "seesa
"hunt"; that is, one machine takes the lead, the
behind, and after a time again takes the lead, rep
this operation continually. When one of the ma
is leading or lagging with respect to its ma
develops during alternate portions of the per
higher and a lower pressure than its mates. It is
fore alternately electrically driving and being dri
its mates. This results in a considerable flow o
less current which interferes with regulation, ov
the machines beyond the demands of the exter
cuit, and causes irregular and unsatisfactory w
It is not sufficient proof of the adaptability of al
ors for parallel working to show that when two ma

are belted to separate engines, one will drive the other as a motor if the steam is shut off one of the engines. We might equally say the proof that shunt-wound continuous-current dynamos will work satisfactorily in parallel is made when it is shown that one will continue to run (as a motor) when the steam is shut off its engine.

91. The Effect of Frequency on Parallel Operation. — The parallel operation of alternators was practised in the earlier plants installed by the Westinghouse Electric Company in this country. In this case the frequency was about 133. The machines worked together quite well when carrying full load, but when their loads changed they would not properly divide the load, which led to hunting, and consequent injury to the service and damage to the machines. At the best, parallel working increased the attention required at the alternators to a great degree.

Thomson-Houston alternators giving a frequency of 125 are worked in parallel with apparent satisfaction in London, England, St. Brieux, France, and elsewhere.

The classical experiments of Dr. John Hopkinson in paralleling alternators were performed with De Meritens permanent-magnet alternators, giving a frequency of about 120. Of the results obtained in these experiments, Dr. Hopkinson says: "The two machines for Tino were driven from the same countershaft by link bands, at a speed of 850 to 900 revolutions per minute; the pulleys on the countershaft were sensibly equal in diameter, but those on the machines differed by rather more than a millimeter, one being 300, the other 299 millimeters in diameter; thus the machines had not, when unconnected, exactly the same speed. The pulleys have since been equalized. The bands were of course put on as slack as practicable, but no special device for adjusting the tightness of the bands was used. The experiment succeeded perfectly at the very first attempt. The two machines, being at rest, were coupled in series, with a pilot incandescent lamp across the terminals; the two bands were then simultaneously thrown on; for some seconds the machines almost pulled up the engine. As the speed began to increase, the lamp lit up intermittently, but in a few seconds more the machines dropped into step together, and the pilot lamp lit up to full brightness and became perfectly steady, and remained so. An arc lamp was then introduced, and a perfectly steady current of over 200 amperes drawn off without disturbing the harmony. The arc lamp being removed, a Siemens electrodynamometer was introduced between the machines, and it was found that the current passing was only 18 amperes; whereas, if the machines had been in phase to send the current in the same direction, it would have been more than ten times as great. On throwing off the two bands simultaneously, the machines continued to run by their own momentum, with retarded velocity. It was observed that the current, instead of diminishing from diminished electromotive force, steadily increased to about 50 amperes, owing to the diminished electrical control between the machines, and then dropped to zero as the machines stopped." *

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* Theory of Alternating Currents, *Jour. Inst. E. E.*, Vol. 13, and Hopkinson's *Dynamo Machinery and Allied Subjects*, p. 175.

The Mordey alternator with a disc armature is reported to operate well in parallel at a frequency of 100. Mr. Mordey says: "With regard to parallel working, I can only say that we find nothing in practice to lead us to suppose that reducing the rate (frequency) would improve the working. We have no difficulty in parallel working at 100 periods per second, and therefore cannot improve in this respect." The Mordey alternator is working in parallel in a number of English and European plants with apparent satisfaction. In experimenting with these machines,* Mr. Mordey made the following tests:

"(1) The alternators were run up to full speed, and each excited to give 2000 volts. When in phase, they were switched parallel without any external load, and without any impedance coils or resistance between them. They ran in parallel perfectly.

"(2) A considerable inductionless load was then put on, varied, and taken off. They ran equally well under all circumstances.

"(3) They were uncoupled, and then, the load being connected to the mains, they were suddenly and simultaneously switched parallel and on to the mains with perfect success.

"(4) One alternator was excited to give 1000 volts, the other giving 2000 volts. They were then switched parallel, and went into step perfectly, giving a terminal P.D. of about 1500 volts. No impedance or resistance was used in this or any other case. A load was then put on without affecting their behavior.

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* Alternate Current Working, *Jour. Inst. E. E.*, Vol. 18.

"(5) With one machine at 1000 volts, and the othe at 2000 volts, they were switched parallel when out phase, and instantly went into step. A large curre appeared to pass between them for a fraction of a second, but not nearly long enough to enable it to be measured or to do any harm.

"(6) Th [illegible] running parallel while one was discon [illegible] engine by its belt being shifted fror [illegible] ose pulley. It continued to run as a [illegible] sly. A load of lamps was at the same [illegible] it.

"(7) The [illegible] were then uncoupled, and excited up to 2000 [illegible] . They were then switched parallel when out of phase, and without any external load, and went into step instantly.

"(8) Whilst running as in (7), steam was suddenly and entirely shut off one engine. The alternators kept in step perfectly, one acting as a motor, and driving the large engine and all the heavy countershafting and belts. It was impossible to tell, except by the top of the belt becoming tight instead of the bottom, which machine was the motor.

"To find the power exerted by the alternator acting as a motor in (8), a direct-current motor was put in it place, and the power required to drive the engine an shafting was found to be 20 horse power."

The capacity of the alternators here experimente with was about 50 horse power.

Siemens alternators, connected directly to their e gines, are operated in parallel at Bristol, England.*

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* *London Electrical Review*, Vol. 34, p. 274.

Ferranti alternators with disc armatures, giving a frequency of 83, are worked together in England and Europe. At the Deptford station, in London, Ferranti alternators of two sizes, 625 horse power and 1250 horse power, are worked parallel, though their normal pressure is different, and the smaller machines are therefore connected up to the circuit through a transformer.*

Elwell-Parker alternators, giving a frequency of 80, are reported to work together with some satisfaction. These machines are somewhat like the American type turned inside out; that is, they have the equivalent of a drum armature, which surrounds the revolving field magnets.†

In certain European plants Kapp alternators have operated in parallel. These machines have ring armatures, and give a frequency of 70.

Stanley two-phase alternators with frequencies of 60 and 125 are running in parallel with perfect success in this country.

The Gordon alternators, which were among the earliest to be used in commercial service,‡ were shown to be capable of operating in parallel. Of this, however, Mr. Gordon said: "We know that experiments have been made by coupling a number of small alternate-current machines together, and at the South Foreland (Hopkinson's experiment) they were successful, but that was because they were working on arc lamps. Many of us have tried them, and they will, on trial, work together, no doubt; but they do not work together till they have run for three or four minutes; they will in that time jump, and that jumping will take months of life out of the 40,000 lamps. That alone is rather a serious difficulty in coupling machines together, and I think we may take it in practice — I am not speaking about the laboratory, or expe not couple machines." The frequen es was from 40 to 50.

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* Fleming's *Alternate Current Transformer*, Vol. II., p. 359.

† Fleming's *Alternate Current Transformer*, Vol. II., p. 222; Mordey, *Jour. Inst. E. E.*, Vol. 18, p. 588.

‡ See Gordon's *Electric Lighting*.

Alternator tures of the Ganz type, giving a fre e frequently worked in parallel in E In the plant at Rome it has been fou rate Ganz alternators of different sizes together.*

Steinmetz has operated alternators of the General Electric Company in parallel, under the following conditions: †

Two 60 K.W. alternators with toothed armature cores, giving a frequency of 125, were experimented upon. These were first excited so as to give a pressure of 1000 volts. The machines were then switched into parallel without making any effort to first get them into step. They quickly dropped into step, and ran synchronously with an interchange of wattless current of only four amperes. Since the normal full load of these machines was 52 amperes at 1150 volts, this is a remarkably good result. Experiments were then made to determine the momentary rush of current at the instant when the machines were thrown together, under various conditions of phase. To determine the phase relations, a synchronizer was used. The machines were first brought to equality of pressure (1000 volts) and synchronism, and then approximately into step. They were then switched together. The momentary rush of current was from .5 to 6 amperes greater than the regular wattless synchronizing current, and depended in magnitude upon the care taken to bring the machines into exact step before they were thrown together. The machines were then thrown together, when their phases were 180° from step, so that the machines would act in series on short-circuit instead of in parallel. When the switch was closed, a large instantaneous current passed through the machines for a fraction of a second, and the machines came at once into step.

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* Fleming, *Alternate Current Transformer*, Vol. II., p. 134; Hedges, *ntinental Electric Lighting Stations*, p. 14.

† Parallel Running of Alternators, *Electrical World*, Vol. 23, p. 285.

Mr. Steinmetz then made experiments upon the action of the machines when thrown in parallel with their voltages different. The results are given in the following table and the accompanying figure (Fig. 162):

A Machine Pressure.	*B*. Machine Pressure.	*A* – *B*.	Resultant Pressure.	Synchronizing Current.	Phasing Current.
1000	1000	0	996	4.0	2.0
1100	900	200	1000	6.5	.5
1200	800	400	1000	13.0	3.0
1300	700	600	1000	18.0	4.0
1400	600	800	1026	24.0	6.0
1500	500	1000	1010	28.0	6.0
1600	400	1200	1040	39.0	3.0
1700	300	1400	1046	44.0	3.0
1800	200	1600	1060	50.0	6.0
1900	100	1800	1066	56.5	5.5
2000	0	2000	1075 ± 70	62.0	10.0

The meaning of the first four columns is evident from the headings, the fifth gives the synchronizing current necessary to hold the machines in step, and the sixth the additional current that flows between the machines for an instant when they are first thrown together and are out of step. When the difference in the pressures of the two machines became 1900 volts, the resultant pressure began to be unsteady. When the difference was 2000 volts, the resultant pressure varied 70 volts on either side of 1075. The irregularity of the curve of phasing current may be due to the differences in the relative phases of the machines at the instants when they were thrown together.

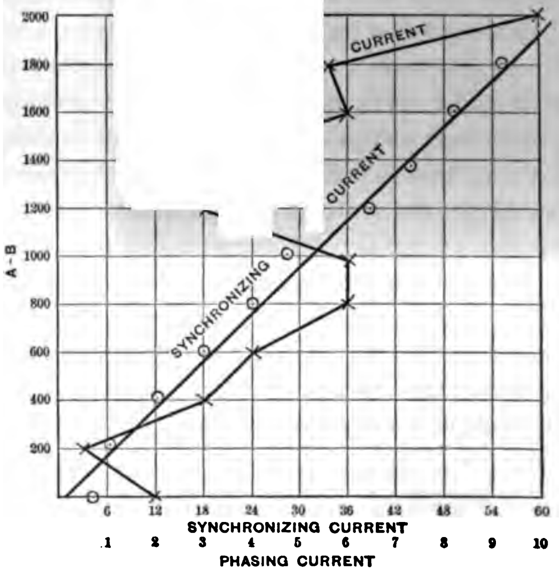

Fig. 162

In discussing these experiments Mr. Steinmetz says: ' We may discard all the usual theoretical statements n parallel working relating to the effect of frequency, elf-induction, etc., as wholly disproved by experience. . . With regard to frequency, I investigated the arallel working of alternators at a frequency as high s 125 cycles, and at a frequency as low as 25 cycles er second, and found no difference whatever; and at he high frequency, as well as at very low frequency, nachines properly designed for these frequencies work perfectly in synchronism."

From all the evidence thus presented, it may reasonably be concluded that frequency, within the limits of common practice, is not an element affecting the sucess of parallel working of alternators. This is in ull accord with the deductions of Mordey and Steinnetz.*

92. The Effect of Armature Inductance on Parallel peration. — Successful parallel operation of alternators epends upon their holding each other in synchronism nd step, even when the prime movers do not naturally ynchronize. *The effort of the machines to do this is the undamental cause for the flow of a wattless synchronizng current.* The total synchronizing current flowing between a machine and the station 'bus bars may be resolved into two components, one in phase with the pressure which causes the synchronizing current to flow, and the other lagging 90° behind the pressure. The former is usually quite small. Thus suppose in Fig. 163 that *b* is the curve of pressure of a machine, and *a* the curve for the station 'bus bars. As *a* and *b* are slightly out of opposition, there will be a resultant pressure *q*, tending to set up a cross or series current. This considered as made up of a component with *q*, and of another component *u*, i ith *q*, the former being the active, and ttless component. The component *s* ha t upon both *a* and *b* during a complete t can have no effect in tending to draw nto phase with the 'bus bar pressure; but the component *u*, which is dependent upon the self-inductance of the series circuit, must, from its position, cause a motor action on the lagging machine (*b*), and a corresponding generator action on the leading machines connected to the 'bus bars; and it thus tends to draw the machines into synchronism (Sect. 86), for it will be noticed by reference to the figure that *u* is in phase with *m* and in opposition to *n*, and that *m* and *n* are respectively the components of *a* and *b*, which are in opposition, and therefore working on the parallel circuit. If the machine, *b*, led the 'bus bar curve, then the synchronizing current would retard instead of assisting it, as the figure plainly shows. The effect of the synchronizing current in dragging the machines into step depends

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* Compare Mordey, Alternate Current Working, *Jour. Inst. E. E.*, ol. 18, p. 591; Snell, The Distribution of Power by Alternate-Current lotors, *Jour. Inst. E. E.*, Vol. 22, p. 280; Mordey, Testing and Workng Alternators, *Jour. Inst. E. E.*, Vol. 22, p. 116; Mordey, On Parallel Vorking with special reference to Long Lines, *Jour. Inst. E. E.*, Vol. 23; orbes, Electrical Transmission of Power from Niagara Falls, *Jour. Inst. E. E.*, Vol. 22, p. 484; and the discussions on these papers; also Steinetz, Parallel Running of Alternators, *Electrical World*, Vol. 23, p. 285; . E. L. Brown, *Jour. Inst. E. E.*, Vol. 22, p. 600.

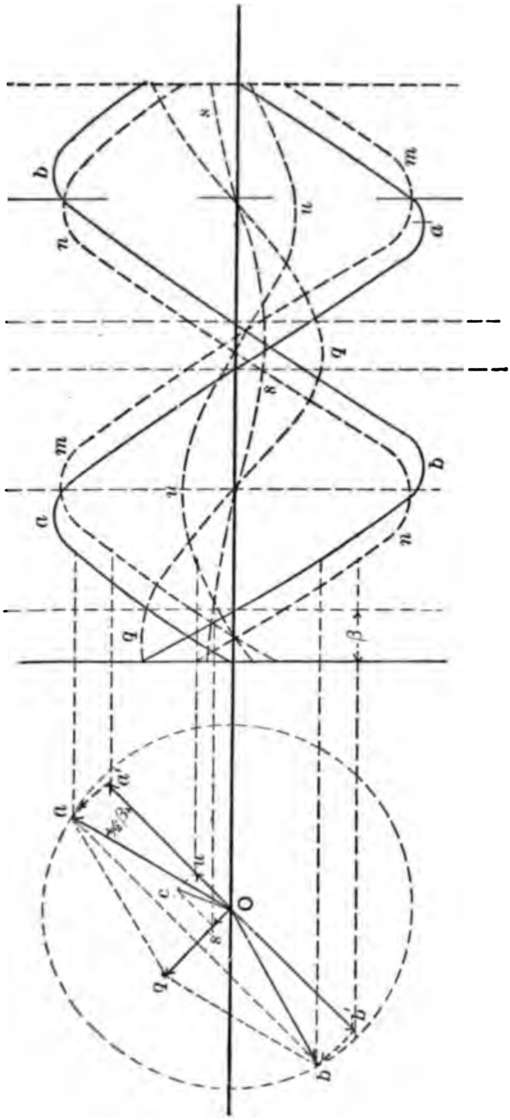

Fig. 163

upon its magnitude and relative phase, while its magnitude depends: (1) upon the algebraic sum, or, what is the same thing, the arithmetical difference between the instantaneous pressure at the 'bus bars and the instantaneous pressure developed by the machine; (2) upon the reciprocal of the impedance of the machine armature. In o istantaneous synchronizing current flo y one machine which is connected t

$$+ e_b$$

$$4\pi^2 f^2 L_a^2$$

where e_a and ntaneous pressures of the 'bus bars and armature, which are always of opposite sign and equal when the machines are in exact synchronism and step, and R_a and L_a are respectively the resistance and the inductance of the armature circuit including the leads from the 'bus bars. It is here assumed that the effective pressure at the 'bus bars and that developed by the machine are equal, which is an essential condition for the synchronizing current to be practically wattless, and is the condition in which the machines are run in practice. Now suppose the pressure curve of the machine under consideration to lag behind the phase of the pressure curve at the 'bus bars by an angle β. Let the effective values o these pressures be E, and the corresponding maximum pressure be e_m. At any moment the instantaneou pressures are e_a and e_b, and

$$e_a = e_m \sin a = \sqrt{2}\, E \sin a,$$

$$e_b = e_m \sin (a + 180^\circ - \beta) = \sqrt{2}\, E \sin (a + 180^\circ - \beta).$$

he instantaneous pressure causing a synchronizing rrent to flow is the algebraic sum of these, or

$$e_a + e_b = \sqrt{2}\,E\,[\sin a + \sin (a + 180^\circ - \beta)].$$

is evident from the figure (Fig. 163) that this is a aximum when e_a and e_b are equal and of similar signs, which case $a = \frac{1}{2}\beta$. Then the maximum value of e pressure causing a synchronizing current is found y substituting this value of a in the expression for $+ e_b$, and $(e_a + e_b)_m = 2\sqrt{2}\,E \sin \frac{1}{2}\beta$.

The effective value of the pressure is then $2\,E \sin \frac{1}{2}\beta$, nd the synchronizing current is

$$C_s = \frac{2\,E \sin \frac{1}{2}\beta}{\sqrt{R_a^2 + 4\pi^2 f^2 L_a^2}}.$$

For smooth and successful working in parallel, C_s must ecome sufficiently great, in case the machines tend to et out of step, to pull the machines together before β ecomes of appreciable magnitude. Hence it is necesary that the denominator in the expression for C_s be as mall as possible. In other words, armature impedance ust be as small as possible. Figure 163 shows plainly at the pressure (Oq) causing C_s is behind the phase of e machine pressure by an angle $90^\circ - \frac{1}{2}\beta$. The synronizing current ($C_s = Oc$) lags behind the pressure by angle ϕ_s (qOc), the tangent of which is $\frac{2\pi f L_a}{R_a}$. onsequently the synchronizing current is out of phase ith the machine pressure by an angle of $90^\circ + (\phi_s - \frac{1}{2}\beta)$. current (wattless) which has a phase difference of 0° or 0° compared with the pressure of a machine has e strongest effect in bringing the machine into step.

This effect is to retard or accelerate the refractory machine depending on whether the phase difference is 0° or 180°, which in turn depends upon whether the machine is leading or trailing. The action in case of a leading machine would be represented by the left-hand half of Fig. 163 if it were reversed.

The wattle [illegible] C_μ) of the synchronizing current just f [illegible]

[illegible] in ϕ_a,

and therefore,

$$C_\mu = \text{[illegible]} \cdot \frac{2\pi f L_a}{\sqrt{R_a^2 + 4\pi^2 f^2 L_a^2}},$$

or, $$C_\mu = 2 \text{[illegible]} \frac{2\pi f L_a}{\text{[illegible]} + 4\pi^2 f^2 L_a^2};$$

and, with a fixed value of R_a, this will have a maximum value when

$$R_a = 2\pi f L_a \text{ or when } \tan\phi_a = 1 \text{ and } \phi_a = 45°.$$

The maximum possible value of C_μ is therefore

$$C_\mu \max = \frac{2E \sin\frac{1}{2}\beta}{2R_a};$$

and the corresponding value of C_s, the total synchronizing current, is

$$C_s = \frac{2E \sin\frac{1}{2}\beta}{\sqrt{2}R_a}.$$

The limits in the value of R_a are fixed by considerations of economy in construction and of efficiency, and the frequency is fixed by conditions of operation; L_a is therefore the only independent variable in the preceding equations. In order to have the most sensiti mutual control, the self-inductance of the armat circuit, which at its least value is always many tin larger than $\frac{R_e}{2\pi f}$, must be as small as possible. If it were possible to reduce the reactance to the value of the resistance, the jerking of a refractory alternator into phase would probably be too severe for good working, but in commercial machines such trouble is not likely to exist on account of the unavoidable magnitude of the self-inductance, which cannot be reduced beyond certain limits. The correctness of the formulas thus deduced has not been studied experimentally, but it might be readily investigated with the aid of Bedell's ingenious phase indicator.[*] [†]

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[*] Bedell, *Trans. Amer. Inst. E. E.*, Vol. 11, p. 502.

[†] Diagrams 1 and 2 given herewith, which are similar in plan to those given in the footnote on page 326, show the conditions of synchronizing very well. Figure 1 applies to an alternator which lags behind its proper phase, and Fig. 2 to one which leads.

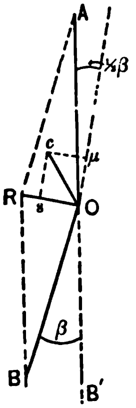

FIG. 1

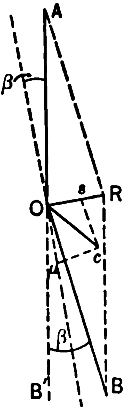

FIG. 2

OA represents the 'bus bar pressure in phase and magnitude.

OB represents the machine pressure in phase and magnitude.

OR represents the resultant, or synchronizing, pressure in phase and magnitude.

The list of examples of parallel working (Se tains machines having armatures of very diffe ances and inductances. The Westinghouse, Houston, and Elwell-Parker machines have s armature cores and fairly low armature induc resistances. The armature inductance of t and Ferranti machines is probably somewh though entirely comparable with these val Kapp machines, and the General Electric with which Steinmetz experimented, have mu armature inductances, and the armatures of t inductor machines probably have inductance mediate values. All these machines have b to run in parallel with similar machines with faction, while Mordey, Steinmetz, and Sta

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Oc represents the synchronizing current in phase and m

Ou represents its wattless, or phasing, component.

β is the angle by which the machine differs from its pr parallel working, *OB′*.

ϕ_s is the angle *cOs* by which the synchronizing curren the resultant pressure.

The figures show plainly that as β increases *OR* increa same time *Oc* and *Ou* increase. The product $OB \times O$ proportional to the synchronizing torque exerted on the negative in Fig. 1 and positive in Fig. 2, so that the torqu *the machine and accelerates it in one case, and it is exerted* *and retards it in the other case.* For any given value of β, maximum when $Ou \times OB$ is a maximum, which occurs wh or $\phi_s = 45^\circ$. In a 1000 volt 50 K.W. machine giving a fr and having a resistance in the armature circuit of .5 ohm that $L = .0013$ to give a maximum synchronizing effect, times smaller than the smallest value which is commerciall any type of alternator.

An inspection of the figures shows that if the synchronizi the resultant pressure there would be no tendency for the ma into parallel with the 'bus bars.

xperimentally shown that Mordey disc armature mahines, the General Electric ironclad armature machines, nd Stanley toothed armature inductor two-phase mahines will run parallel with machines of their own type ith excellent results. On the other hand, Gordon has aid that his machines did not come into step well page 345), and Hopkinson's experiments show that e Meritens machines take an excessive synchronizng current. The armatures of both the Gordon and e Meritens alternators have an excessive resistance nd self-inductance as compared with good machines f the present day,* and their actions fully agree with he indications of the formulas already given.

We are therefore entirely justified in drawing the onclusion that in machines intended for parallel workg both armature resistance and inductance should be mall, but that within the range of resistances and nductances to be found in modern commercial machines f economical design the armature inductance is too nall to have a detrimental effect upon parallel workg though it is much too large to give a maximum synronizing torque, which in fact may be an advantage, it saves the machines from being torn to pieces. me types of modern alternators of less economical sign may have too much armature self-inductance to erate perfectly in parallel.

These deductions are strengthened by the experimen- results gained by Mordey and Steinmetz. In dissing his experiments already summarized (page 343)

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* Gordon's *Electric Lighting*, pp. 156, 162; *London Electrician*, Vol. p. 51.

Mordey says: "It may be pointed out that these tests were made under the most exacting and onerous conditions that could possibly be imposed, and particularly I would point out that on account of the very great momentum of the revolving masses nothing but the strongest and most instantaneous motor action could have kept the machines in place. There never was a single case when they got out of step, even momentarily or when subjected to sudden and violent variations of load. When it is considered that in order to secure this result, it was imperative that the control of all that mass should be exerted in a fraction of $\frac{1}{200}$ of a second (the periodicity being 100), it will be recognized that there was no time to be lost, and that the use of any self-induction or resistance, or of anything else that could in any way choke, retard, check, or interfere with the strength and instantaneity of the action, was above all things to be avoided.

"I should mention that the machines apparently synchronized equally well at speeds varying very considerably.

"As to the self-induction of the machine itself, that is quite negligible. Its characteristic (Fig. 164) is nearly straight, about half the drop in the curve being due to resistance and half to self-induction." *

Steinmetz says in regard to his experiments: "The self-induction of machines within the limits met in well-designed alternators has nothing to do with the success of synchronous running, and I had three phasers of very low armature self-induction running in parallel

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* *London Electrician*, Vol. 23, p. 66.

h each other and ironclad (imbedded armatures) sin-phasers of high armature self-induction." *

Γhe comparatively high self-inductance of ironclad ernator armatures has proved advantageous in an-

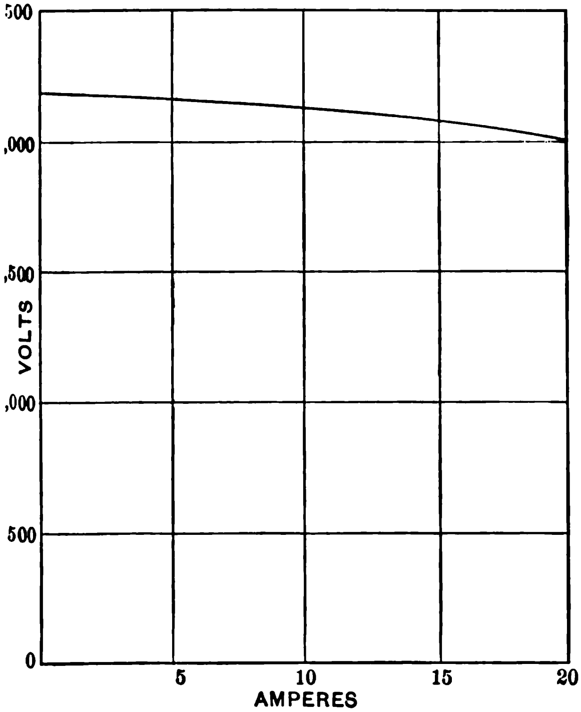

Fig. 164

er direction, that of saving the machine from the naging effects of sharp short-circuits. A sharp rt-circuit upon a machine with a disc armature is ly to cause an enormous momentary rush of current

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* *Electrical World*, Vol. 23, p. 285.

which may damage or destroy the armature, while the higher self-inductance of the ironclad armature chokes down the current to a considerable extent. This quality of the ironclad alternator is an advantage in power transmission plants or small electric light plants, where little stress is laid upon exact regulation, and the station and lines [illegible] for. In large and well-operated elect[illegible], the injurious effect of self-inductance [illegible]ation of the pressure is of preponderat[illegible] and since it is of the

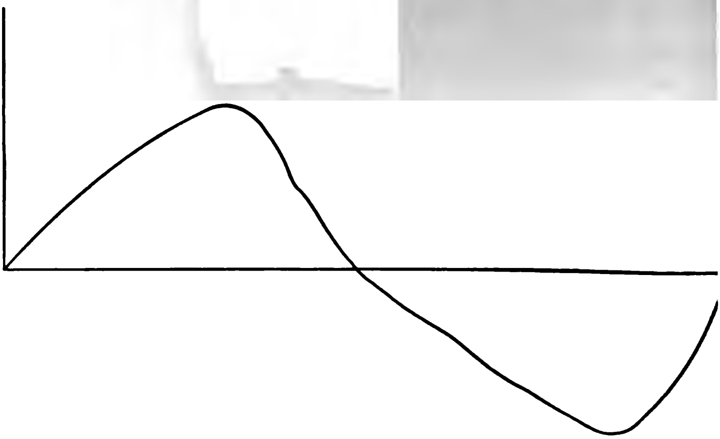

Fig. 165

utmost moment that alternators used in large electric light stations shall have the least possible defect in regulation, the alternator with large self-inductance is not so well adapted to this service (Sects. 67 and 74).

93. Effect of the Form of the Pressure Curve and of Armature Reactions on Parallel Operation. — Little experimental evidence is available bearing upon this question. The able designer, C. E. L. Brown, states that he has operated his own alternators with smooth iron armature cores in parallel with Ganz alternators which have the usual pole-type armatures.* The former machines give a pressure curve which approximates a sinusoid, while the curve given by the latter is quite irregular and peaked (Fig. 165). Mr. Steinmetz has operated machines with smooth iron armature cores in parallel with others having toothed cores.† It is not related how far the pressure curves of these machines differed, but probably some differences existed. These statements seem to show that machines with different pressure curves can be run together satisfactorily, but they undoubtedly require a considerably increased exchange of current as compared with the synchronizing current of machines which give curves which are exactly

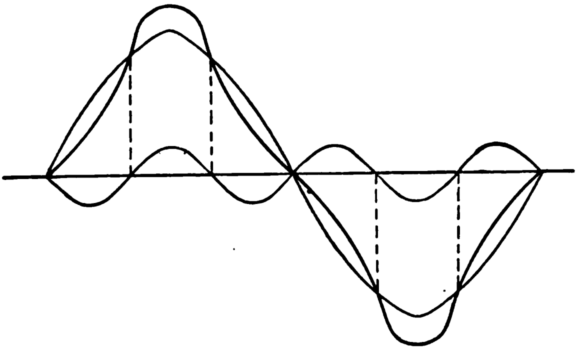

Fig. 166

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* *Jour. Inst. E. E.*, Vol. 22, p. 600.
† *Electrical World*, Vol. 23, p. 285.

alike. For, since the unlike curves cannot coincide even when the machines are in exact synchronism, a current of more or less irregular form must be exchanged by the machines, and is therefore superposed upon the true synchronizing current. This superposed current may have a very different frequency from that of the machines (Fig. :cessarily wattless.

94. The I Operation of Regulation by Varying -Suppose that a Grove's and a Daniel structed that their inter-

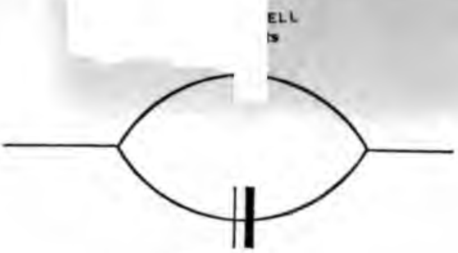

Fig. 167

nal resistances are equal. Their pressures may be assumed to be approximately 1.9 and 1.1 volts. If these cells are connected in parallel (Fig. 167) to a circuit, the pressure impressed upon the circuit is the mean of the pressure developed by the cells, or 1.5 volts. This is brought about as follows: The Daniell cell serves as a shunt across the terminals of the Grove cell, so that current from the Grove cell has two paths, one through the external circuit and one backwards through the Daniell cell. Sufficient current will flow rough the latter to equalize the pressure at the ter-inals by means of the fall of pressure over the internal sistances of the cells. If the external circuit is open, current will flow through the cells which is sufficient cause a loss of pressure in each cell of .4 of a volt. ıe pressure at the terminals of the Grove cell is then ɔ – .4 = 1.5, and at the terminals of the Daniell cell 1 + .4 = 1.5. Suppose the external circuit is closed d demands enough current to cause a pressure loss in e Grove cell of .2 of a volt. Then sufficient addi-nal current will flow through the circuit of the cells make the terminal pressure 1.7 – .3 = 1.4 and 1.1 .3 = 1.4. From an extension of this reasoning it is en that a current flows backwards through the Daniell ll under all conditions of the circuit until sufficient rrent is demanded by the external circuit to make ıe pressure loss in the Grove cell .8 of a volt. Then ıe Daniell cell is exactly balanced, and will furnish *alf of any additional current demanded by the external rcuit.*

The same condition of affairs exists when continuous-urrent dynamos operate in parallel. If their pressures re not exactly equal, the 'bus bar pressure will be a nean value. The machine of lower pressure will sup-ly sufficiently less current so that an equalization of ressure comes about through changes in the loss of ressure due to the resistances of the armature circuits nd the effects of armature reactions. If the low-essure machine falls behind too much, the current rough its armature will be reversed (Vol. I., p. 231). ıe inequality of load can be readily perceived in the case of continuous-current dynamos by observing the amperemeters.

In the case of alternators, similar conditions exist. For illustration we will assume the machines to give sine pressure curves which are in synchronism and step. If the machines give equal pressures, they will feed the exte y if they are of the same capacity, or hey are of unequal capacities. If on is connected to constant-pressure 'bus essure which is too low, it will not f of current to the circuit. The pressure is equalized through the influence of tne ssure ses caused by the impedances of the various armature circuits. If the pressure of the machine becomes much lower than that of the 'bus bars, a reverse current will flow through the machine, and it will run as a motor. This is not shown by the amperemeter, as in the case of continuous-current machines. It may be observed, however, by altering the excitation. If the machine is doing its work properly, decreasing the excitation will decrease its apparent output to a minimum, after which further decrease in the excitation causes an increase in the apparent output. The machine is then receiving more energy from the 'bus bars than it returns. In order, then, that machines working in parallel may be relied upon to contribute energy to the external circuit, it is necessary that they be so sufficiently excited that an increase in the excitation will cause an increase in the parent output. This, of course, is exactly what is ie with continuous-current dynamos, using the indiitions of the amperemeter as a guide, but in the case alternators, the amperemeters, alone, fail to show hat the machines are really doing. This is partly on ccount of their inability to indicate the direction of he transfer of energy in an alternating circuit, and artly on account of the masking effect of a wattless urrent; but if direct-reading wattmeters are used in lace of amperemeters in the circuits of the alternators, he indications of these instruments may be used as a guide in handling the machines when in parallel, exactly as the indications of amperemeters are used as a guide for handling continuous-current dynamos which are connected in parallel.

When two alternators of unequal pressure are connected together, there is quite a curious result. The erminal pressure is a mean between the machine ressures and *the machines run together without a dangerous exchange of current.** The instantaneous value of the current exchanged by the two machines at any moment, on the supposition that the machines re in step and that the pressure curves are sinusoids,

$$\frac{\sqrt{2}\,E'\sin a - \sqrt{2}\,E''\sin a}{\sqrt{R^2+4\,\pi^2f^2L^2}} = \frac{\sqrt{2}(E'-E'')\sin a}{\sqrt{R^2+4\,\pi^2f^2L^2}};$$

nd its effective value is

$$\frac{E'-E''}{\sqrt{R^2+4\,\pi^2f^2L^2}},$$

here E' and E'' are the effective pressures developed by the two machines, and *R* and *L* are the resistance and inductance of the two armature circuits taken in series. Take two 50 K.W. alternators designed to give 50 amperes at 1000 volts with a frequency of 125, assuming the armature resistance and inductance of each to be respectively .45 ohm and .015 henry. Then if the two [illegible]wn in parallel with one excited to [illegible] and the other without excitation, t[illegible]nged amounts to about 42 amperes, [illegible]han the full load of the machines; w[illegible]ted machine be connected to 'bus bars [illegible] the current sent through its armature on [illegible] 35 amperes. The current that would be exchanged under similar conditions by continuous-current machines would be over 1100 amperes, which would immediately ruin the machines. The difference in the two results is due to the inductive effects of the alternating current. If the two alternators have a considerable armature reaction, the difference is likely to be still more marked, since armature reactions tend to weaken an alternator's field when running as a generator, and strengthen it when running as a motor (Sect. 70).

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* Mordey's and Steinmetz's Experiments, Section 91.

95. The Effect of Irregular Angular Velocity on Parallel Operation.—The angular velocity of the steam engines, which are ordinarily used for driving dynamos, is by no means uniform throughout the stroke, though the strokes may be entirely isochronous. Curves showing the instantaneous crank velocities taken from engines of well-known types are often quite irregular. If ternators are driven from separate engines, this irreglarity of angular velocity may be the cause of more or ess difficulty in parallel working, and some machines nay work quite satisfactorily in parallel when driven rom the same countershaft or from separate turbines which give a uniform angular velocity), when they will iot do so if driven by separate engines.

96. Final Conclusions on Parallel Operation.— 1. There s no difficulty in operating alternators of good commerial design in parallel.

2. Within the limits of practice frequency does not naterially affect parallel working.

3. This being accepted, smooth parallel working reuires that the armature impedance be reduced to a ninimum (Sect. 92). Armature resistance must deend upon considerations of economy.

4. Machines with excessively large armature inductnce will run in parallel, but are likely to "hunt."

5. Machines with pressure curves of quite different orms will run together in parallel, but with a continous interchange of current, the magnitude of which epends upon the value of the armature impedances nd the relative form of the pressure curves.

6. The dynamo amperemeters which are ordinarily sed should be replaced, or reinforced, when machines e run in parallel, by direct reading wattmeters. The citation of the machines may then be adjusted so that e wattmeters show a proper division of the load. In at case amperemeters in the dynamo circuits should ow an approximately similar division of the current.

7. If the excitation of the machines is properly adsted the sum of the readings of the machine watt-

meters will equal the reading of a main wattmeter in the main circuit of the 'bus bars, and the sum of the readings of the machine amperemeters will be a little greater than the reading of the main amperemeter (the difference being twice the value of synchronizing current exchanged
s).

8. The effe
makes such an overwhelming part
e of even the best commercial machin
riously excessive current is not likely t
a machine even when switched onto t
tirely unexcited. (This is true to a ma
machines with ironclad armatures.)

9. Parallel working is likely to be more successful when the alternators are driven from the same engine or counter shaft, or when the prime movers are turbines.

These conclusions are based upon the operation of alternators which are driven by self-regulating prime movers; that is, the driving power transmitted to the alternators is automatically adjusted to the demands of the machines, and they are driven at a uniform speed regardless of their load. This is in accord with the usual American practice in operating dynamos. In England it is apparently common to work alternators from engines that are not self-regulating and which furnish a fixed amount of power when running synchronously unless the position of the throttle valve is tered.*

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* Kapp's *Dynamos, Alternators, and Transformers*, p. 399; Snell's *tric Motive Power*, p. 241.

If composite-wound alternators are to be operated in parallel, it is necessary to use an equalizing connection, exactly as in compound-wound continuous-current machines, to equalize the effects of the compounding (Vol. I., p. 237).

VIII.

S, ETC.

97. Genera —The general definitions of efficiencie eady been explained with regard to continuous-curre machines, are applicable to alternators (Vol. I., p. 244). The principal causes of loss in alternators are the same as those in continuous-current machines; but on account of increased frequencies, the effects of foucault currents and hysteresis are intensified. On this account particular care is required in selecting and annealing the iron for the armature core, and in insulating the armature discs from each other. Advantage should also be taken of every opportunity for ventilation. The magnetic density in alternator armature cores is made considerably less than in continuous-current armatures, as already explained (Sect. 61). The following table of satisfactory densities is given by Snell.* The inductions, which are given in lines of force per square centimeter and per square inch, may be assumed as fair guides for proper judgment in designing.

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* *Electric Motive Power*, p. 175.

TABLE OF ARMATURE INDUCTIONS SUITABLE FOR VARIOUS FREQUENCIES.

Frequencies.	B (per sq. cm.).	B (per sq. in.).
50	5000–6200	32,000–40,000
60	4650–5000	30,000–32,000
70	4350–4650	28,000–30,000
80	4000–4350	26,000–28,000
90	3700–4000	24,000–26,000
100	3500–3700	22,500–24,000
110	3250–3500	21,000–22,500
120	3000–3250	19,500–21,000
130	2800–3000	18,000–19,500

As already stated, the values of armature inductions en in this table serve excellently as a guide in de-ning, but they are frequently exceeded in practice. s no uncommon thing to find inductions as great as o to 7000 in alternator armatures, giving a frequency 125 or more. Snell's table is apparently modelled r a table presented by Kolben in an article upon yphase induction motors, and which gives the range nductions to be used in such machines. Kolben's e is given in Section 177.

8. **Methods of Testing Alternators.** — The experi-ital determination of the efficiency of alternators be made by methods quite analogous to those used continuous-current machines (Vol. I., p. 255). How- -, instead of voltmeters and amperemeters for meas-g electrical energy, wattmeters must be used (Fig. , unless the load is entirely non-reactive. In case

a transmission dynamometer is used to measure the power absorbed by an alternator, the commercial efficiency is equal to the electrical output, determined from the readings of a wattmeter, divided by the power shown by the dynamometer readings, if the machine is self-excited. If the machine is separately excited, the
energy sup measured by amperemeter
and voltmet er, must be added to the
power readi ne friction may be deter-
mined from r readings when the ma-
chine is ru nexcited and the external
circuit ope s due to hysteresis and
foucault cur ture core and conductors
may be approximately determined from the dynamome ter readings when the machine is operated with its field separately excited and the external circuit open, and th total loss may be approximately separated into its com ponent parts by proceeding in an entirely analogous wa to that given for continuous-current machines (Vol. I p. 253, *First Method*). The C^2R losses in fields an armature may be readily computed. If the machine i self-excited by a rectified current, the field current will be less than that calculated from the pressure and resistance. In that case it is advisable to use an alternating-current amperemeter, such as an electrodynamometer, to determine the effective current. From this, the C^2R loss may be computed if the field resistance is known. Or, the field loss may be directly determined by a wattmeter in the field circuit.

The following methods are especially adapted to shop esting and determining the efficiency of an alternator.

odification of Hopkinson's Method. It is desir- avoid the use of a power dynamometer, and with view a modification of Hop- s method of testing (Vol. 6) may be made (Fig. 169). ual alternators are rigidly together in proper step llel working. Their arma- e electrically connected to- with a wattmeter and an lynamometer in the circuit. lds being properly excited parate exciter, so that one e will act as a generator other as motor, the system driven by supplying suffi- ower to make up the ma- osses. Assuming the arm- nd stray losses of the two es to be equal, and repre- the wattmeter reading by W, the power sup- y P, and the resistance of the connections by R_1; e efficiency of the generator is

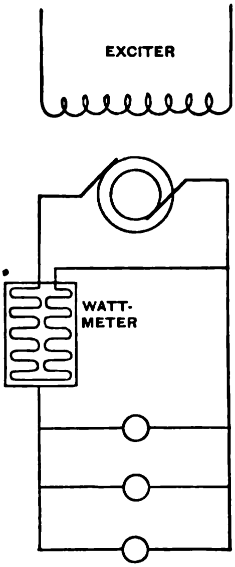

Fig. 168

$$\eta = \frac{W}{W + \frac{1}{2}(P - C^2R_1) + c^2R_f},$$

c^2R_f is the field loss of the generator. If the es are self-exciting, the power in the field circuits e measured and proper allowance made. The supplied to make up the losses may be measured by a transmission dynamometer, or a "rated" continuous-current motor may be used to supply the power. In the latter case, if the efficiency of the motor is known, the power may be determined by measuring the watts supplied to the motor by amperemeter and voltmeter. Or, the losses of the motor may be determined by the stray power method, and being properly deducted from the readings of power absorbed by it when driving the alternators, the value of P is obtained with [illegible]ufficient approximation. The three-machine method [illegible]ght be directly applied, but difficulties due to syn[illegible]onizing are likely to appear.

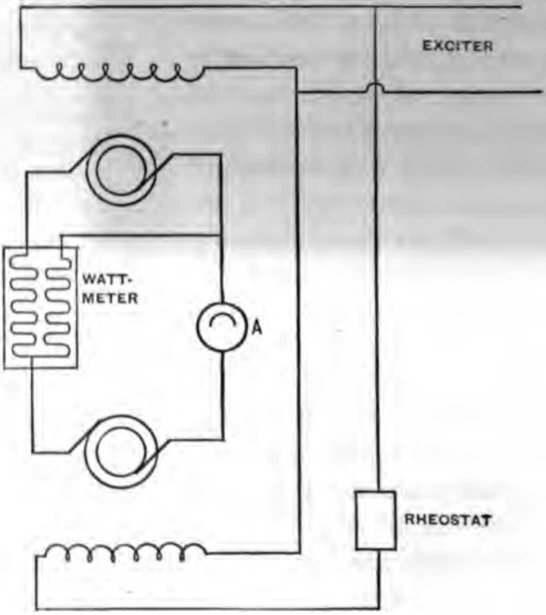

Fig. 169

2. *By Rated Motor.* Where approximate determinaons of the various losses of conversion and of the ommercial efficiency are sufficient, the alternator tested ay be driven directly from a "rated" continuous-current motor. By means of amperemeter and voltmeter he power supplied to the motor may be determined ith the alternator operated under such various conditions as may be necessary to determine the losses. By oading the alternator and simultaneously measuring its output and the power absorbed by the motor, the efficiency of the alternator may be determined with sufficient accuracy for ordinary purposes. In each case the power supplied to the alternator is equal to the power absorbed by the motor multiplied by the efficiency of the motor given in per cent. The motor may be rated by determining its efficiency by the stray power method. If the motor efficiency for various loads is determined by some exact method (Vol. I., p. 255), the power transmitted to the alternator may be determined with considerable exactness. When it is desired to carry the accuracy of this method of testing beyond a fairly good approximation, the efficiency at various loads of the rated motor should be plotted, so that the efficiency at any load may be readily read off.

3. *Mordey's Method.* A neat arrangement for testing a single alternator by a method akin to Hopkinson's method has been devised by Mr. Mordey.* It is to be remembered that the Mordey alternator has a stationary rmature, the individual coils of which may be conected in any desired combination with perfect facility.

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* **Testing and Working Alternators,** *Jour. Inst. E. E.*, Vol. 22, p. 116.

Such an armature may be divided into two parts which are connected in such a way as to oppose each other (Fig. 170). If one part gives a somewhat higher pressure than the other, the first part will operate as a generator and the second as a motor if the alternator is driven in the usual manner. By properly adjusting the difference l ures of the two parts, th current flo ine may be caused to hav

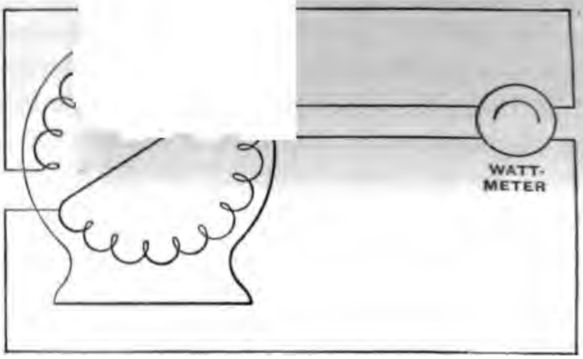

Fig. 170 a

any desired value. The efficiency of the machine gained by measuring the power absorbed by the m chine operating as a self-contained motor-generator a measuring its output by a wattmeter; the pressure c of the wattmeter being connected across the termina of the motor and generator coils, and the series c being connected directly in the circuit. The pow thus measured by the wattmeter is evidently about on half of the total energy of the machine; consequent e corrected efficiency is

$$\eta = \frac{2\,W}{2\,W + P + c^2 R_f},$$

where P is the power supplied to the machine. The difference in the pressure developed in the parts of the machine which is necessary to cause the desired current to circulate may be caused by an unsymmetrical division of the armature (Fig. 170*a*), or by supplying a little additional pressure to the generator side by means of a transformer. The latter may be supplied from another alternator operating in synchronism with the machine under test, or it may be supplied directly from the test machine (Fig. 170*b*). The transformer is likely, however, to introduce uncertain elements of loss.

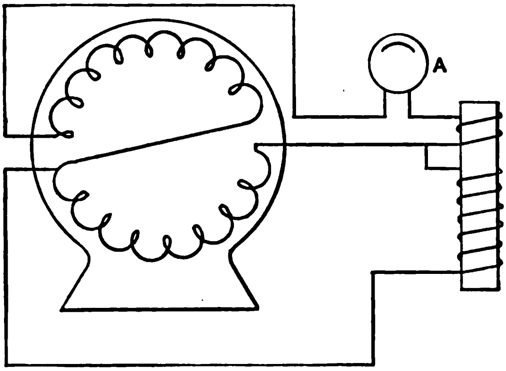

Fig. 170 b

4. *Ayrton's Method.* As pointed out by Professor Ayrton,* this arrangement may be modified so as to apply to alternators with rotating armatures. In this case opposite halves of the fields are magnetized in oppo-

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* *Jour. Inst. E. E.*, Vol. 22, p. 136.

site directions, and the armature is short-circuited through an amperemeter (Fig. 171). By adjusting the relative excitation of the halves of the fields a current of any desired value may be caused to circulate in the armature. If the excitation is practically normal, the measured losses of the machine when any current is flowing will be practically equal to those when the machine is operating normally on the same current, provided we may assume that the losses in an alternator are appreciably equal when driven as a generator and as a motor. This assumption seems entirely reasonable.

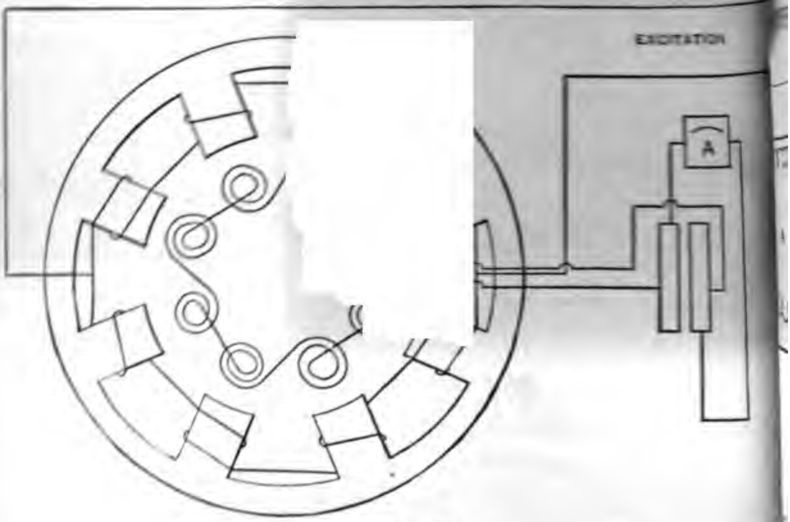

Fig. 171

e arrangement here described is not applicable to hines with armatures with the halves wound in llel, since under the test conditions an amperemeter connected between the terminals of such an armature would not indicate the current circulating in the armature coils.

5. *Motor-Generator Method.* — Mordey has also suggested the following purely electrical method of testing alternators which have stationary armatures.* The machine being properly excited, one-half of the armature is connected as a generator to an external load, R (Fig. 172). The other half of the armature is connected to another alternator and driven as a synchronous motor. The total losses under these conditions are evidently equal to the power absorbed by the motor half of the armature plus the exciting energy and minus the output of the generator half of the armature; that is, $W_m + C^2R_f - W_g$, where W_m and W_g are respectively the power absorbed by the motor and the output of the generator. Since the output, W_g, is the

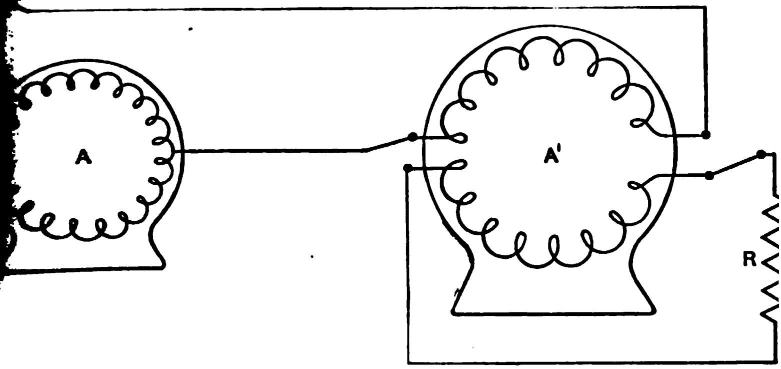

Fig. 172

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* *Jour. Inst. E. E.*, Vol. 22, p. 122.

output of only half the armature, but the losses thus determined are those of the whole machine, the losses will be the same when the machine is run as a generator with an output of 2 W_g, provided generator losses and motor losses are the same. The efficiency of the machir is therefore

$$\eta = \frac{}{2 W_g +} \quad \frac{}{W_g} = \frac{2 W_g}{W_s + W_m + C^2R_f}$$

6. *By Driv* *ous Motor—Applicable to Machines with* *Armatures or Fields.*—

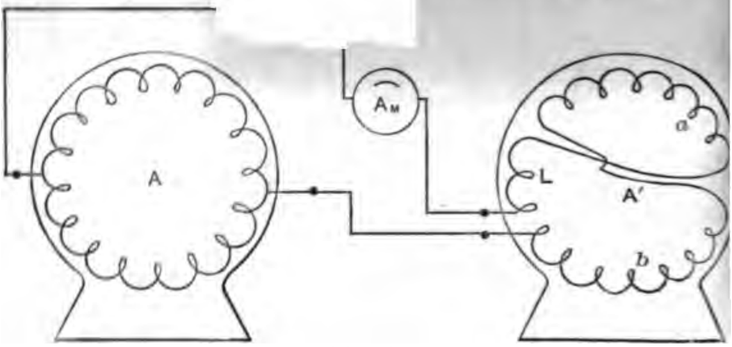

Fig. 173

Another somewhat similar plan is to divide the stationary armature into three divisions which are connected in series. Two of these are made equal and are connected in opposition. The third consists of only a small portion of the armature. The machine is electrically connected to another alternator and operated as a synchronous motor under the influence of the small armature divis- (Fig. 173). By a proper choice of the number of composing the small division any desired current may be sent through the machine to be tested. The energy given to the machine represents the losses in the machine when operated as a generator and producing the same current with the same excitation. This method is also applicable to determining the losses in machines with revolving armatures the coils of which are all connected in series. In this case the fields are

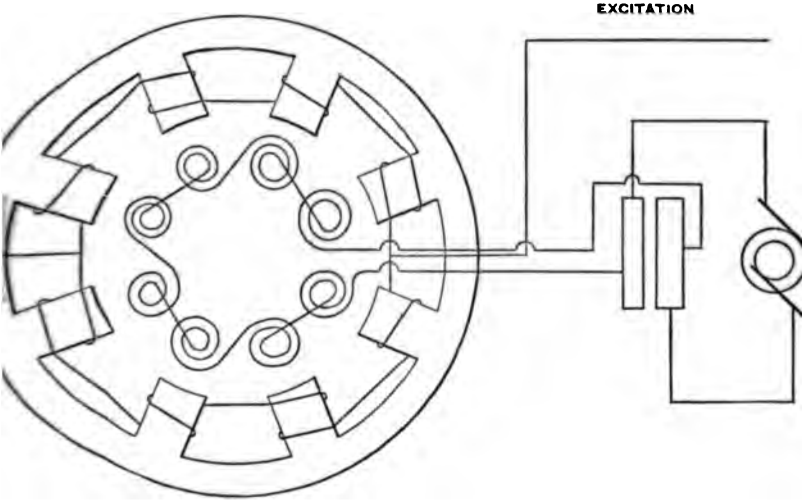

Fig. 174

excited with the poles on one-half reversed (Fig. 174), one-half of the field being slightly stronger than the other. If the armature is connected to that of another alternator, it will run as a motor. The instantaneous counter electric pressure of the machine under test depends upon the relative strength of the two halves of the field, and by adjusting this, with due reference to the impressed pressure which should be of a relatively small value, the current flowing in the armature circuit may be given any desired value. Under these conditions the losses in the test machine are equal to the power absorbed.

99. Shop Tests.—When, in either of the cases mentioned heretofore, the operation of an alternator as a motor is predicated, it is assumed either that the test machine is brought to synchronism with the alternating source, or that the primary generator is started from a state of rest in which case the test machine will start and run with it.

These methods of testing are not only convenient in determining the losses and the efficiency of an alternator, but the tests, according to several of the methods, are made with the consumption of comparatively little power. This makes the methods satisfactory for use in shop tests for determining the reliability in operation and the heating limits of machines. Mordey has suggested that the efficiency of an alternator with stationary armature may be determined from a test of one armature coil, but this is only approximate and cannot serve as a satisfactory shop test which requires a test of the *complete* machine.

100. Wattmeters on High Pressure Circuits.—In using wattmeters where high pressures are met, considerable difficulty is found in arranging a satisfactory non-inductive resistance for the pressure coil. This difficulty may be readily overcome by a plan suggested by Dr. J. A. Fleming,* and which was successfully used in testing high-pressure alternators at the Chicago

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* *Jour. Inst. E. E.*, Vol. 22, p. 99; *London Electrician*, Vol. 30, p. 455.

's Fair. Instead of connecting the pressure coil
wattmeter across the terminals of the test circuit,
imary of a transformer
onnected, and the press-
il of the wattmeter is
cted to the secondary
transformer (Fig. 175).
onstant of the watt-
is then dependent upon
tio of transformation
transformer, which may
dily measured. This
d gives entirely reliable
, since the phases of
rimary and secondary
res of a very lightly
transformer are almost
y 180° apart. If the
eter constant be deter-
without the trans-
r, its constant when in
ith the transformer must be multiplied by the
f transformation.

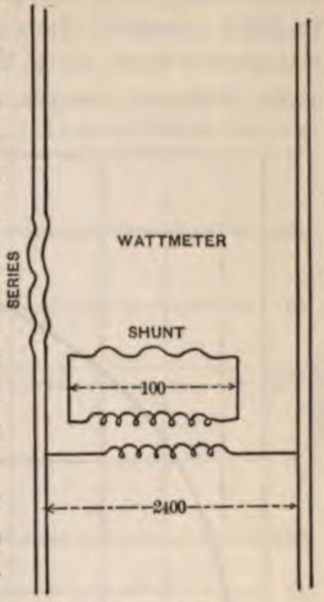


Fig. 175

Variation of Efficiency, Weight, and Cost, with
t. — In general, it is safe to say that the efficiency
alternator should be very nearly the same as that
ontinuous-current machine of the same size and
or the same duty. The electrical losses in the
pes of dynamos should differ but little, and the
sion losses can be held to an appreciable equality
ng proper magnetic densities in alternators, and by

proper construction. The curve showing the relation of commercial efficiency to output, given in Fig. 127 of Vol. I. (reproduced in Fig. 176), may therefore be taken to fairly represent alternator efficiencies. For capacities greater than 100 K.W., the efficiency increases at a very slow rate towards a limit of about 95 per cent.

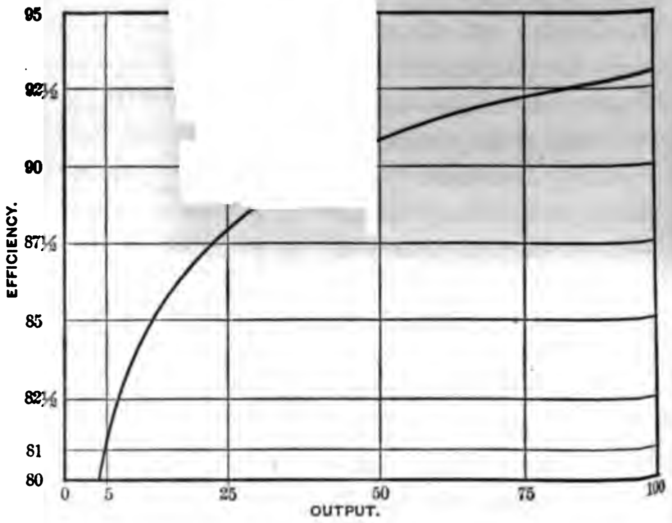


Fig. 176

The great Westinghouse alternators of 1000 horse power capacity, which were tested at the World's Fair at Chicago, showed a commercial efficiency of upwards of 96 per cent. The reported efficiency of various alternators of a capacity of about 200 K.W., varies between 93 per cent and 95 per cent. The efficiencies of continuous-current machines of great capacity are shown by test o be about the same.

The economic curve for alternators is quite similar in rm to those given for continuous-current machines ol. I., p. 268). Figure 177 gives the experimentally termined curve for an alternator of several hundred lowatts output.

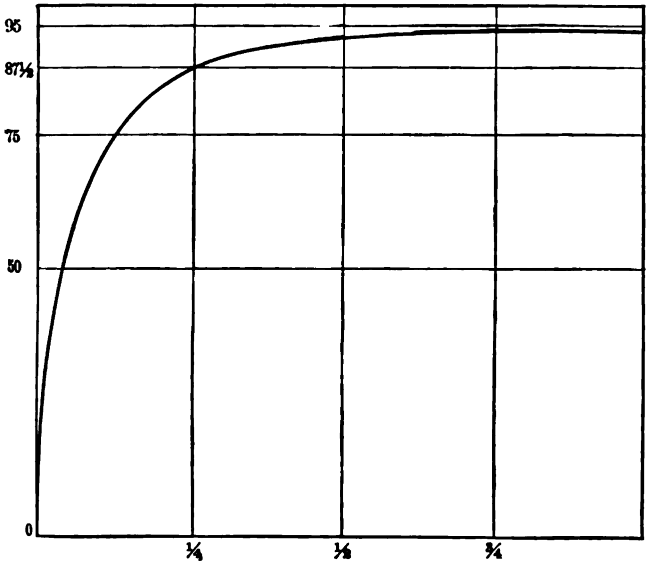


Fig. 177

The relation between weight and output, which has lready been discussed in reference to continuous-urrent dynamos (Vol. I., p. 262), holds equally for ternators, as does also the relation between output id cost. Well-designed continuous-current machines a greater capacity than 10 K.W. weigh between 75 id 200 pounds per kilowatt, depending upon the type machine, the material of which the magnetic circuit is composed, the capacity of the machine, and the use for which it is designed. Practically the same may be said of well-designed *single-phase* alternators. Th almost universal use of toothed cores in alternators o the latest design has decreased their weight as com pared with continuous-current machines having smoot cores. But [illegible]ed cores in the armature

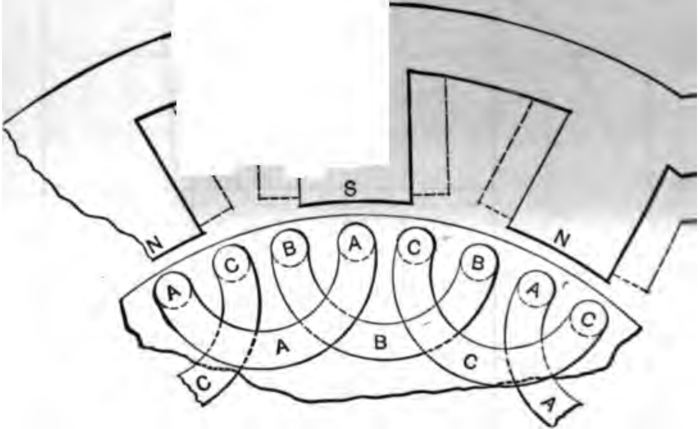


Fig. 178 a

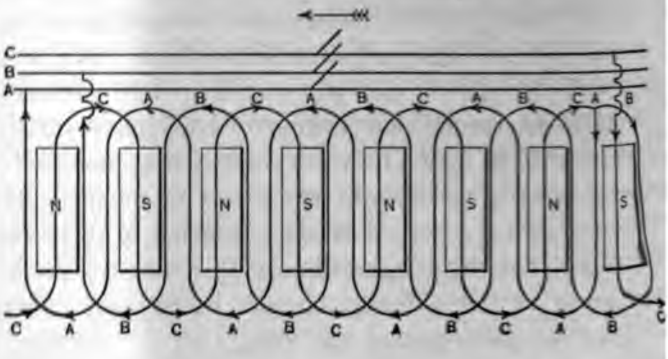
of continuous-current machines makes an equal provement in them.

If the entire surface of alternator armatures could effectively covered with wire, as in continuous-curre machines, the high allowable periphery speeds and oth conditions already discussed (compare Sect. 6) woul give them a largely increased output per unit of weight It has already been shown that this cannot be done for machines with armatures of one circuit. It is possible, however, to wind the armatures with two circuits, which together entirely fill the surface, each circuit occupying one-half the winding space, as in single-circuit machines, the coils being arranged alternately. The pressure waves of two circuits thus arranged have a phase difference of 90° (Fig. 179). It is also possible to economically fill the winding space with three sets of independent and overlapping coils, which are connected in three independent circuits (Fig. 178). In this case the pressure waves of the different circuits follow each other with a phase difference of 120° (Fig. 180).

Alternators which produce a single pressure wave are called **Single-phase Machines,** or **Single-phasers.** Those that produce more than one pressure wave are called **Poly-** or **Multi-phase Machines,** or **Poly-** or **Multi-phasers.** These are divided, according to the number of circuits, into **Two-phasers, Three-phasers,** etc.

The conditions at present existing in the construction of polyphase machines enable them to be built at a considerable reduction of cost and weight per unit of output as compared with single-phasers and continuous-current machines. There is, however, a marked tendency toward a reduction in the speeds and frequencies of such machines, and towards increase in the solidity of their construction. The result of this is to increase the comparative cost of the machines per unit of output until they are approximately on a level with the best continuous-current dynamos.

102. Armature Reactions of Poly-phasers. — The armature reactions in poly-phase generators is materially different from that in single-phase generators. Thus, referring to Fig. 89, it is remembered that when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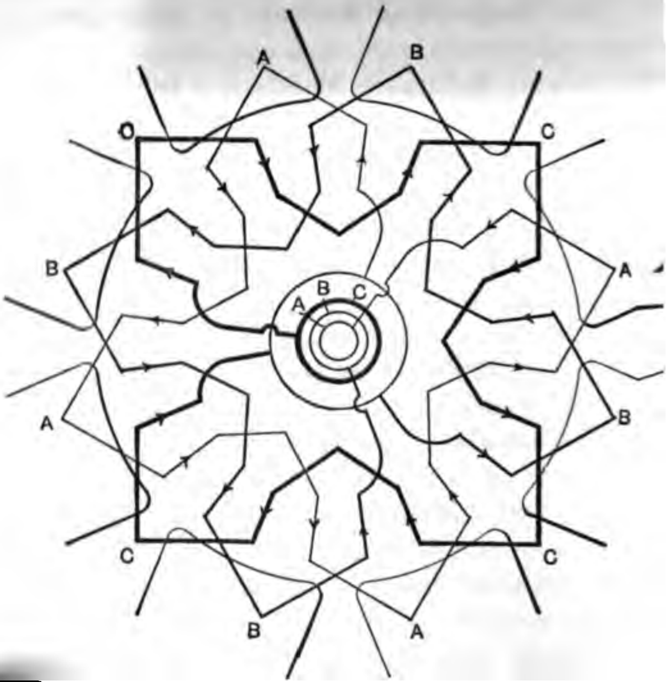
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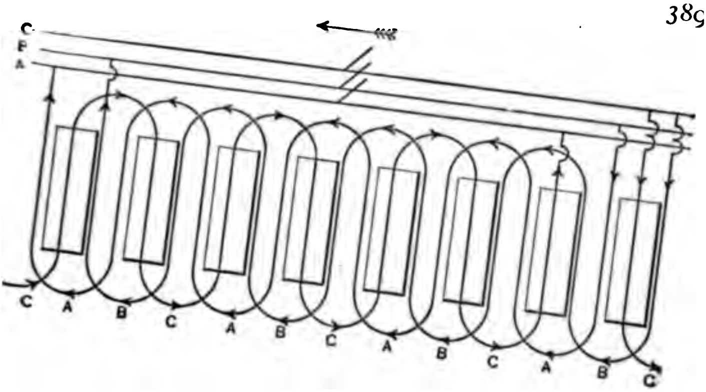
Fig. 178 b



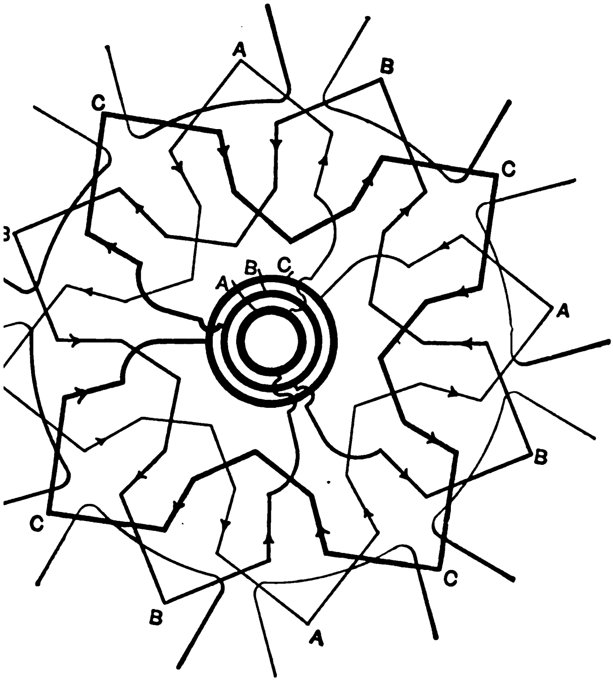


Fig. 178 c

the current of a single-phaser is in phase with the pressure, magnetism is crowded into the trailing pole tips at each time of maximum current, and resumes its initial position when the current falls to zero. In the case of poly-phase machines, in which the wire, in effect, covers the entire surface of the armature, there is a sheet of current at all times under the faces of the pole pieces, and as this sheet of current sets up a nearly constant magnetizing force, the skewing of the magnetic field, due to armature cross-turns, practically constant. The effect of back-turns in single-phase machine having a ring armature is rep sented in Fig. 181 *a*, two coils, *a*, *a*, being shown. T ordinates of the curve *d* represent the magnetizi effect of the armature coils which aids or opposes t field magnetization when the centres of the coils ar in different positions corresponding to the ordinates nd a current of constant strength is passed through m. This magnetizing effect or activity must evi-

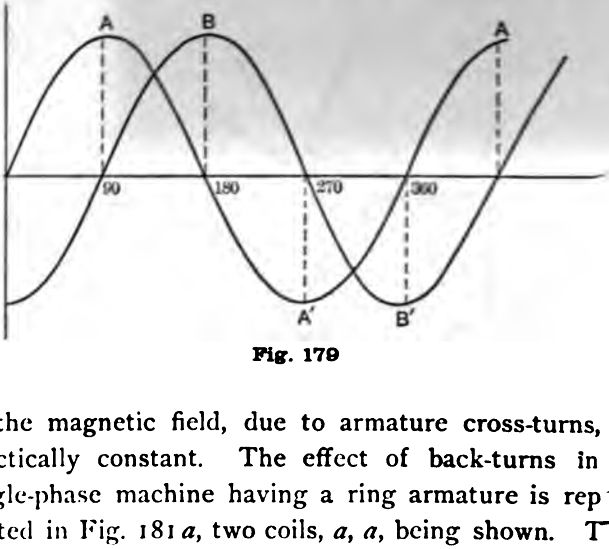


Fig. 179

lently be zero when the centres of the coils are lirectly under the poles. *E* is the pressure curve of he machine, and *c* the current curve in phase with the pressure. By combining the current curve with the curve of activity, the actual effect of the alternating current in the armature coils may be obtained, as is shown by curve *k*. It is seen that the sum of the posi-

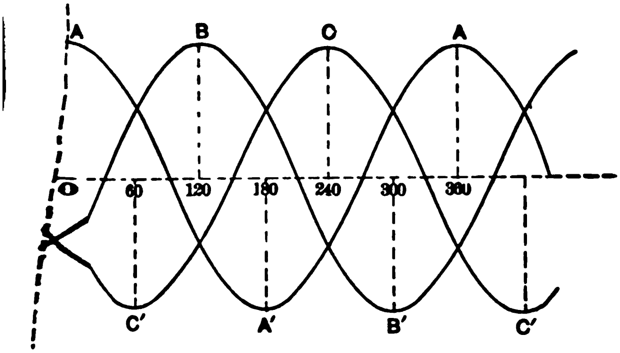


Fig. 180

e and negative effects is zero, but that they have a riodical disturbing effect upon the field.

In the same manner it is shown in Fig. 181*b*, that the current lags behind or leads the pressure, one oop of the curve *k′* is greater, and the fields are periodically either weakened or strengthened.

In the case of a two-phase alternator, a second curve of reaction exactly similar to *k* and one-fourth the pitch, r 90°, from *k*, may be drawn like the heavy lines in ig. 181*c* to represent the action of the second set of oils, and the magnetizing effect of the two phases is dded together and the skewing effect becomes nearly uniform when the armature current and pressure ar phase. Similarly, when the current lags or leads, tl is a constant weakening or strengthening of the fi as shown in Fig. 181 *c*, where the ordinates of the k'' represent this effect. In case the current leads,

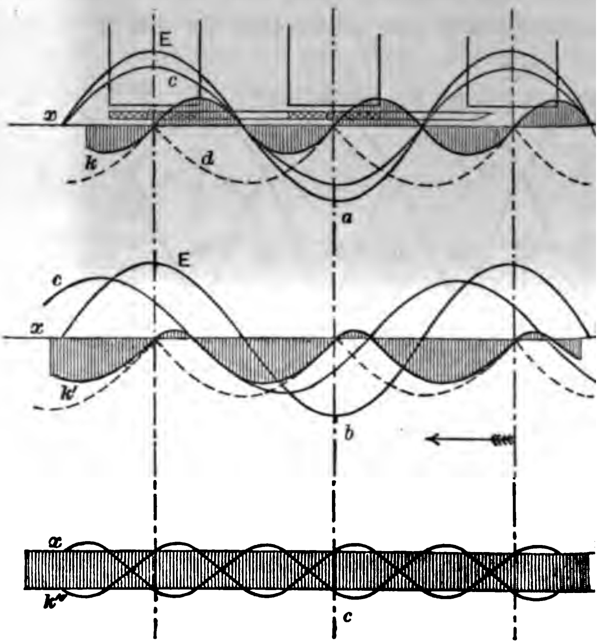


Fig. 181

fields will be strengthened. By the same proce may be shown that the armature reactions* in any phase machine is practically constant and is grea its effect than in a single-phase machine. In machines having large armature reactions this may limit the output of poly-phasers, but in those with small armature reactions poly-phase armatures have much greater capacity than single-phasers.

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* Ossian Chrytræus, Armature Reactions, *Electrical World*, V p. 450.

102 *a*. Connecting up Poly-phase Armatures. — The connecting up of two-phase armatures is very simple. If the armature is wound with independent concentrated coils, each coil may be connected to its individual collector rings, in which case four rings are required and the two circuits are entirely independent; or, one terminal of each coil may be connected to a common collector ring and the other terminals to independent rings, in which case but three rings are required and the circuits have a common point. If the armature has a continuous-current or closed-circuit distributed winding, such as is treated in Section 5, it may be made into a two-phaser by connecting collector rings to the windings at the ends of two diameters which are 90° apart, if the machine is bipolar; if the machine is multipolar, the connections must be made as described in Section 191. Four rings are necessary in this case, as the use of a common ring would cause the permanent short-circuiting of one-quarter of the armature. A continuous-current armature converted into a polyphaser (of any number of phases) in this manner has a capacity when used as a poly-phase alternator which is equal to its continuous-current capacity, though it has already been shown that its capacity as a single-phaser is only seven-tenths as great as its continuous-current capacity (Sect. 5).

The manner of connecting three-phase armatures is not so immediately evident, but is perfectly simple. It is illustrated in Figs. 178 *b* and 178 *c*. Three collecting rings are universally used, and if the armature is wound with three independent coils, these may be connected to the rings in either of two ways: (1) one end of each of the a common point, and the other ends nt rings; or, (2) the coil terminals r d together two and two, forming a s d connections be carried to the collectc se points. The latter arrangement terminate at both ends in a collector ri and, since here are six coil ends and three rings, each ring is connected with two coil terminals. These two arrangements are illustrated respectively in Figs. 178 *b* and 178 *c*, each of which shows the winding diagrammatically as developed and as projected. The following considerations make it perfectly easy to connect the coils in the proper order: Fig. 180 shows that when the current in one coil is at its maximum point, the currents in the other two are equal to each other and opposite to the direction of the first; then, considering the instant at which the conductors of one coil (such as *C* in the figure) are directly under the poles, if we connect its positive end to the common or **Neutral Point,** the negative ends of the other two coils must be connected to the same point. Each of the fre terminals may then be connected to one of the collecto rings, and the connection is completed according to th first or **Star** arrangement (Fig. 178 *b*). To make the sec d or **Mesh** arrangement, the coils must be connecte so that, at the instant considered, the current flows by two paths through the armature from the negative to the positive terminal of the first coil (coil *C* of the figure). Consequently the negative terminals of the first and second coils go to one collector ring, the positive terminals of the first and third coils to another ring, and the free terminals of the second and third coils to the third ring (Fig. 178 *c*).

If the armature has a closed-circuit or distributed winding, the connection is very simple, as the rings are connected to points in the windings 120° apart. If the machine is multipolar, it must be remembered that one cycle, or 360 electrical degrees, is comprised within the space of twice the polar pitch. This is more fully treated in Section 191.

In a three-phase machine, if the armature is mesh-connected, the pressure between any two collector rings is equal to the pressure developed in one coil, while the current leaving a brush is the vector sum of the current in two coils; and, if the armature is star connected, the pressure between the rings is equal to the vector sum of the pressure developed in two coils, while the current leaving a brush is equal to the current in a coil. The vector sum in either case is $\sqrt{3}$ times the arithmetical sum, so that the capacity of a machine is independent of the way in which its armature is connected, but, for a given pressure and output, the windings will differ (though the weight of copper will be the same) for the two arrangements. This is treated more fully in Chap. XIII.

CHAPTER IX.

[illegible] UCTION.

103. The [illegible] nsformer. — The remarkable developm [illegible] f alternating currents for transmitting [illegible] electric power, is mainly due to the f [illegible] my with which they may be transformed from one [illegible] essure to another. The transmission of electric power between two points may be made by alternating currents at high pressure for the sake of economy in the cost of conductors, and the pressure may be reduced at the receiving point by means of induction coils to any value which is deemed entirely safe and convenient for distribution.

The induction coils that are used for this purpose are called **Transformers** or **Converters,** because they are used to transform or convert the electrical energy from one state to another. Formerly transformers were sometimes called **Secondary Generators** on account of the apparent regeneration of the energy of the alternating currents.

The action of transformers is due to the inductive effect which a varying current in one circuit exerts upon an adjacent circuit. Since this effect is a mutually interacting one, it is called **Mutual Induction.** Before proceeding to the study of the commercial trans-

ners, it is essential to examine the relations which
t between two adjacent circuits, one or both of
ch carry an alternating current.

04. **Mutual-Inductance.**—Suppose two adjacent coils
ounded by air and in which a current is flowing:
total number of lines of force passing through
er one of the coils is the number of lines set up
the current in that coil plus the number of lines
up in the other coil which are embraced by the
. If the current is changed in either of the coils,
electrical pressure is set up in the coil under con-
ration, which may be called the first coil, which is
al to $E' = \frac{n'dN'}{10^8 dt}$, where n' is the number of turns
N' the magnetic flux in the first coil. The num-
of lines passing through the coil due to its own
ent is $\frac{10^8 L'C'}{n'}$. L' and C' are respectively the
inductance and the current in the first coil (Chap.
). The number of lines of force due to the first coil
ch pass through the second coil evidently depends
n the relative positions of the coils, but it cannot
greater than the total number of lines set up by
current in the first coil. For any two fixed coils
medium of fixed permeability, this number of lines
roportional to the current. The electric pressure
loped in the second coil due to the change in the
ent flowing in the first is $E'' = \frac{n''dN''}{10^8 dt}$, where n'' and
are the turns and magnetism in the second coil. The
tion $E' = \frac{n'dN'}{10^8 dt}$ may be written $E' = \frac{L'dC'}{dt}$, when

the permeability is constant. The equation $E'' = \frac{n''dN''}{10^8 dt}$ may be similarly written $E'' = \frac{MdC'}{dt}$, where $10^8 M$ is the number of turns in the second coil multiplied by the number of lines of force passing through the second coil which are due to the first coil when one ampere is flowing in it; and M is called, by analogy, the **Mutual-Inductance** or the **Coefficient of Mutual Induction** of the coils. The value of N'' is evidently equal to $N'k$, where k is a constant depending on the reluctance of the path of the lines of force which interlink the two coils. If the coils are long solenoids, $N' = \frac{4\pi n'CA}{10 l}$, where A is the cross-section and l the length of the coil; and if the solenoids are wound one over the other so that their dimensions are practically equal, the value of k is unity because N'' evidently becomes equal to N', and M becomes equal to $\frac{N'n''}{10^8 C}$. If the action of the coils be now reversed, that is, if a current flows in the second coil, we have $N'_1 = \frac{4\pi n''CA}{10 l}$, and $M_1 = \frac{N'_1 n'}{10^8 C}$. But $\frac{N'}{N'_1} = \frac{n'}{n''}$, and hence $M = M_1$. In this case, also, $L' = \frac{N'n'}{10^8 C}$ and $L'' = \frac{N'_1 n''}{10^8 C}$. Consequently $L'L'' = M^2$, or $M = \sqrt{L'L''}$.

If the solenoids be now separated by drawing one out of the other, the value of M continually decreases as the separation continues. The self-inductances of the coils remain constant, so that as the coils separate, the mutual-inductance becomes less than $\sqrt{L'L''}$. As the separation of the coils becomes greater, the value of M decreases towards a minimum of zero. It reaches this value when the coils are at an indefinitely great distance apart or the axis of one is placed symmetrically but at right angles with reference to the axis of the other. The maximum possible value of the mutual-inductance of the two coils is therefore a mean proportional between the values of their self-inductances, and the minimum value is zero. The maximum value can only be attained when all the lines of force due to one coil pass through all the turns of the other, and the value of M may therefore be written $M \overset{=}{<} \sqrt{L'L''}$, from which it is at once seen that the mutual-inductance of the two coils must always be small if the self-inductance of one or both of the coils is very small, while the mutual-inductance may be large if both the self-inductances are large. It is shown on the preceding page that the value of M may be generally written $M = n''N'k$. In this expression N' is proportional to n', whence it is shown that $M \propto n'n''k$.

The product of the number of lines of force which interlink two coils by the number of turns in the individual coils will not change by changing the point of reference from one coil to the other, and, consequently, the mutual-inductance of two coils is always the same when measured from either coil, provided the reluctance of the magnetic circuit is unchanged. When the magnetic circuit is composed wholly or partly of iron, it is necessary to have an equal number of lines of force in each part of the circuit, when the two measurements are made, in order that they may give equal results. When the magnetic circuit is made up partly of iron and partly of non-magnetic materials, and the coils are quite different, as in dynamos, the condition of equal induction is difficult to fulfil, and the value of M may be quite different when measured from the fields and from the armature. This difference is wholly due to the difference in the permeability of the magnetic circuit during the two measurements.

As shown [illegible] tual-inductance is homogeneous witl [illegible] of, the same dimensions as self-induc [illegible] is therefore the henry. The practica [illegible] l-inductance, M, as here developed by [illegible] L, is 10^9 times as large as the absolu [illegible]

If the lines of force [illegible] to one coil which enter another do not all pass completely through the second coil, the definition of the mutual-inductance still holds as already given, but the summation of the number of lines of force passing through each individual turn must be taken (compare Sect. 16). If iron be inserted in the path of the lines which interlink two coils, the mutual-inductance is increased in the ratio of $\frac{P}{P'}$, where P and P' are the reluctances of the path before and after the iron is inserted. The ratio $\frac{P}{P'}$ is dependent on the permeability of the iron in the magnetic circuit, and the value of M must therefore vary with the current in the coils in any case where iron is in the magnetic circuit, while it is independent of the value of the current when magnetic material is absent.

105. The Energy of Mutual Induction. — Assume two adjacent coils with a constant mutual-inductance. In one let a current of C' amperes flow, and in the other current of C'' amperes. The number of lines of force ue to the first coil which pass through the second is $\frac{o^8 MC'}{n''}$, and the change in this number when C' changes s $\frac{10^8 MdC'}{n''}$. If the current C' be varied, the work due o mutual induction which is done in the second coil in hanging the magnetic field against the effect of the urrent C'' is (by Vol. I., p. 69) $dW = MC''dC'$ (compare Sect. 18). If the current in the first coil be hanged from zero to C', the work done on the second coil is $W = \int_0^{c'} MC''dC' = MC'C''$, which is all stored in the magnetic field. If the current C' falls again to zero, this work is restored to the circuit. When M varies with the current, the work is still $MC'C''$, but M in the expression must be assigned its equivalent mean value between the limiting values of the current (compare Sect. 18). As the current in the first coil rises to C', the total work stored in the magnetic field is evidently the sum of the work due to self- and mutual-induction, or $\frac{L'C'^2}{2} + MC'C''$.

If the current varies in both the coils at the same time, the following condition exists at any instant. The total pressure in either coil is the resultant of three elements — the active pressure (cR), the pressure of self-induction $\left(\frac{Ldc}{dt}\right)$, and the pressure of mutual-induction $\left(\frac{Mdc}{dt}\right)$, and

$$c' = \frac{e' - \frac{d(Mc'' + L'c')}{dt}}{r'},$$

$$c'' = \frac{e'' - \dfrac{d(Mc' + L''c'')}{dt}}{r''}$$

where e' and e'' are the instantaneous impressed pressures in the two coils. By transformation we have

$$(e' - c'r')\,dt = M\,dc'' + L'\,dc',$$

$$(e'' - c''r'')\,dt = M\,dc' + L''\,dc''.$$

Multiplying these respectively by c' and c'' and adding, gives

$$(c'e' + c''e'')\,dt - (c'^2r' + c''^2r'')\,dt = L'c'\,dc' + L''c''\,dc'' + M(c'\,dc'' + c''\,dc').$$

The first and second terms of the left-hand member of this equation represent respectively the total work done by the impressed electric pressures and the work expended in the coils in heat during the interval dt. Their difference represents the work done on the magnetic field. The total work done on the magnetic field during any change of the currents, as from zero to C' and C'', is found by integrating the right-hand member of the equation. Thus,

$$L'\int_0^{C'} c'\,dc' + L''\int_0^{C''} c''\,dc'' + M\int_0^{C} (c'\,dc'' + c''\,dc') = \frac{L'C'^2}{2} + \frac{L''C''^2}{2} + MC'C''$$

If the currents now fall to zero again, the work has the same value as above, but the negative sign. In the first case electrical energy is absorbed from the circuit and stored in the magnetic field which is set up, and in the second case the stored work is restored to the circuit as the magnetic field dies away.

106. Transfer of Electricity by the Effect of Mutual luction. — Now suppose that no pressure is initially pressed on the second coil (that is, $e'' = 0$), then ıen the current in the first coil is changed, the con- :ions in the second coil are given from the equations ove; thus

$$c'' = \frac{-d(Mc' + L''c'')}{r''dt}.$$

Whence $$c''r''dt = -Mdc' - L''dc'',$$

d $$\int_0^t c''r''dt = -M\int_0^{C'} dc' - L''\int_0^0 dc''.$$

ıce the last term reduces to zero, the quantity of ctricity which is transferred in the second coil under : inductive influence of the first when its current ınges from zero to C' is

$$\int_0^t c''dt = q'' = -\frac{M}{r''}\int_0^{C'} dc' = -\frac{MC'}{r''} = -\frac{ME'}{r'r''}.$$

the current of the first coil is now brought to its ginal value, we have

$$q_1'' = -\frac{M}{r''}\int_{C'}^0 dc' - \frac{L''}{r''}\int_0^0 dc'' = \frac{MC'}{r''} = \frac{ME'}{r'r''}.$$

e two quantities are equal and of opposite sign, so t the transfer of electricity in the secondary coil ing the rise and fall of the primary current reduces zero, provided the original and final values of the nary current are equal (compare Sect. 19).* If the rent in the first coil is a simple periodic one, a

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Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I., p. 227; Hospi-
r's *Traité sur l'Énergie Électrique*, Vol. I., p. 486.

periodic current of the same frequency is set up in the second coil. Such an arrangement of two coils is a transformer. The first coil is called the **Primary Coil**, and the second is called the **Secondary Coil.** The pressures or currents in the primary and secondary coils are called respectively primary and secondary pressures or curr primary current wave is a
sinusoid an luctance is constant, the
electric pre the secondary is also a
sinusoid, bu se 90° behind the phase
of the prima s is evident from the fact
that the ind s proportional to the rate
of change of on, and the magnetization
is in phase with the primary current (compare Sect. 15). When iron is present in the magnetic circuit, M is no longer constant, and the rate of change of the magnetization is not proportional to the rate of change of the current; consequently the secondary pressure wave is no longer similar to the wave of primary current, but it is always exactly similar in form to the wave of counter electric pressure set up in the primary coil.

107. The Pressure Relations in a Transformer.—If a sinusoidal current is caused to flow in one of two coils, such as have been considered in the preceding paragraph, the relative positions of the pressures in the two coils may be shown graphically as follows. In Fig. 182, let OE_{1a} be the active pressure in the primary coil acting upon the current (OC_1). This current will set up a self-inductive pressure in the primary, $OE_{1s} = 2\pi f L_1 C_1$, and a mutually inductive pressure in the secondary, $OE_{1m} = 2\pi f M C_1$. These pressures are in the same ction and lag 90° behind the current (see Sect. 106). pressure OE_{1m} will set up a current (OC_2) in the

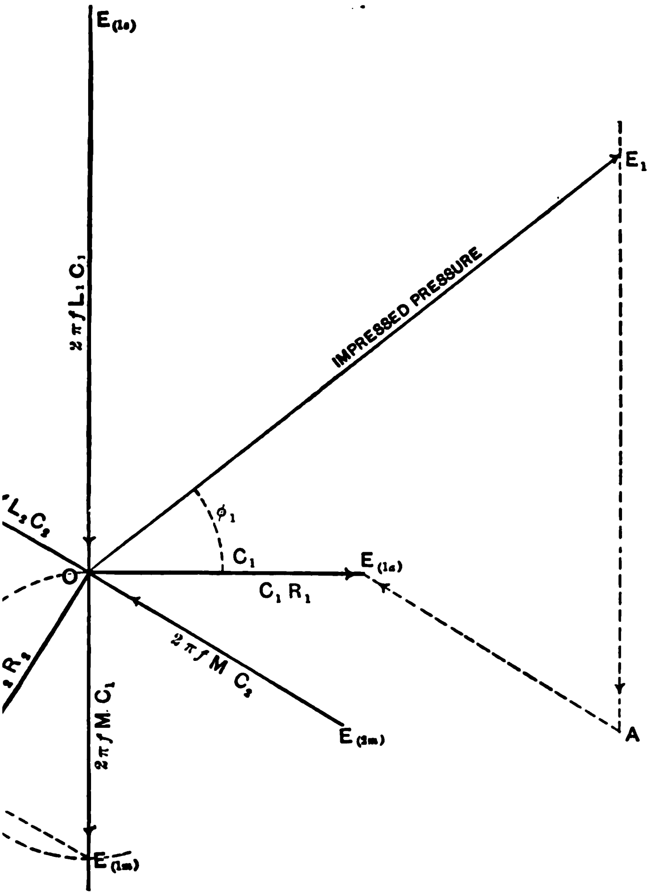

Fig. 182

dary which will cause a pressure in the secondary, $= 2\pi f L_2 C_2$, and a pressure in the primary, OE_{2m}

$= 2\pi fMC_2$, both lagging 90° behind the current. The active pressure in the secondary is the resultant of OE_{1m} and OE_{2s}, or OE_2. The pressure impressed upon the primary (OE_1) must be such that when combined with the self and mutually inductive pressures OE_{1s} and OE_{2m} the resultant will be the active pressure OE_{1a}. The vector ves OE_1 is completed by drawing fro $E_{1a}A$ which is equal and parallel to A the line AE_1 which is equal and pa hen OE_1 is the impressed pressure. I be increased, E_{2s} and E_{2m}, which depend ary current, will be larger, and ϕ_1, the the primary current, will be less, while the secondary current will swing around more nearly into opposition with the primary current. Under these circumstances the active secondary pressure will be smaller if the primary pressure remains constant. If the secondary current be made smaller, the primary pressure remaining constant, the active secondary pressure will be increased, ϕ_1 increased, and the secondary current will swing around towards a phase which is 90° behind the primary current. It is evident that the primary and secondary currents combine to give a resultant magnetizing effect which sets up the magnetization in the magnetic circuit. This property will be used in the chapter on design.

108. Measurement of Mutual-Inductance.—Before leaving this part of the subject, it is well to consider the methods of measuring mutual-inductances. The various practical methods are based on a comparison of the unknown mutual-inductance with a known resistance, capacity, a self-inductance, or another mutual-inductance. The latter may be the mutual-inductance of two tandard coils which are fixed in a position relative to ach other. The mutual-inductance of two such coils ay be determined by calculation if the coils are of e proper shape, or it may be made by careful comarative measurements.

1. *Direct Measurement by Amperemeter and Voltmeter* sing an alternating current). The formula

$$E'' = \frac{MdC'}{dt}$$

Sect. 104) indicates a method of measuring the mutualductance of two coils when a source of sinusoidal ternating current is at hand. When the current is nusoidal and flows continuously through the primary il, the instantaneous pressure induced at any moment the secondary coil is

$$e'' = \frac{Mdc'}{dt} = \frac{Md(c'_m \sin a)}{dt} = \frac{Mc'_m \cos a da}{dt}.$$

e maximum value of the induced pressure is

$$e''_m = \frac{Mc'_m da}{dt} = 2\pi f Mc'_m,$$

ce $\frac{da}{dt} = 2\pi f$ (Sect. 24). The effective value of e induced pressure is therefore $E'' = 2\pi fMC'$, or $= \frac{E''}{2\pi fC'}$. The mutual-inductance of the coils may erefore be measured by passing through one of them sinusoidal current the effective value of which is

measured by an amperemeter, and measuring the e
tive value of the induced pressure (Fig. 183).

2. *Direct Measurement* (using amperemeter and
listic galvanometer). Connect the primary of the
coils, the mutual-inductance of which is to be measu
in series with a battery, an amperemeter, and a
(Fig. 18 the secondary coil conne
ballistic ing the total resistance of
seconda ue R''. Then, when the
in the p ere will be an induced cur

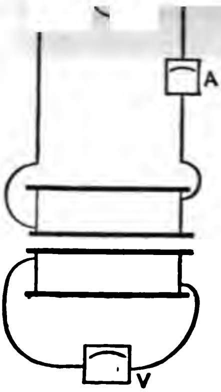

Fig. 183

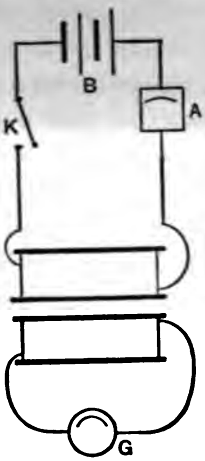

Fig. 184

in the secondary, during the continuance of which
number of coulombs passing will be $Q'' = \frac{MC'}{R''}$ (
106), where C' is the final value of the current i
primary, and M is the value of the mutual induct
which is sought. The value of Q'' is determined
the throw of the ballistic galvanometer. The equ

then contains only one unknown quantity, the value of M, which is therefore determined by the solution

$$M = \frac{Q''R''}{C'} = \frac{R''K\theta}{C'},$$

where θ is the throw of the ballistic galvanometer, and K is its constant. If the known primary current be reversed, the formula becomes

$$M = \frac{Q''R''}{2\,C'} = \frac{R''K\theta}{2\,C'}.$$

3. *Comparison with a Known Capacity* (Carey Foster's Method). By modifying the preceding method, it is possible to make the desired determination without knowing the constant of the ballistic galvanometer. Thus, after the observations have been taken as described, a condenser with the galvanometer in series may be shunted around the resistance r', which is in the primary circuit (Fig. 185). Then when the key is closed, the quantity of electricity which passes through the galvanometer is (Sect. 35 *a*)

$$Q_1 = sC'r'.$$

If the resistance r', shunted by the condenser, is adjusted without altering the total resistance in the circuit, so that the galvanometer deflections in the two positions are equal, or $Q_1 = Q''$, then

$$sC'r' = \frac{MC'}{R''};$$

whence $$M = sr'R''.$$

If the deflections, and therefore the quantities, of el tricity are not equal in the two cases, this becomes

$$M=\frac{sr'R''\theta}{\theta_1},$$

where θ and θ_1 are the respective throws of the galvanometer in the two positions. In order that the adjustment may b he arrangement shown in Fig. 186 is ariable resistances r' and r'', which a y and secondary circuits, respectively, are adjusted until the galvanometer gives no throw upon closing the primary circuit. Then the electric flow due to charging the condenser is exactly equal and opposite to the flow in the secondary. In this case we have

$$sC'r' = \frac{MC'}{R''},$$

or, as before, $$M = sr'R''.$$

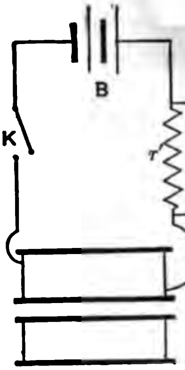

Fig. 185

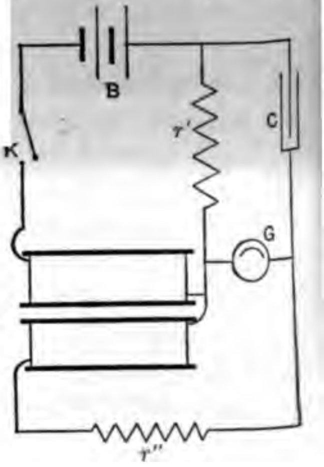

Fig. 186

r that the self-induction of the circuits may urb the observations, the ballistic galvanometer ve a rather sluggish needle, so that it will not preciably during the duration of the discharge * ɔ).

Pirani's Method.) This method has much in with the preceding, but the arrangement of the is quite different 7). Here the pri- d secondary circuits tain variable resist- and r''. These are ed together at the ere they join their ve coils ; at the other are joined through nser. The battery anometer are con- respectively in the and secondary cir- shown in the figure. current is set up in ary, a charging cur- ds to flow through r'' into the condenser which $sC'r'$ coulombs of electricity The average e of electric pressure at the terminals of r'' he period of charging, t, is therefore $\frac{sC'r'r''}{t}$. rage pressure set up by induction in the second-

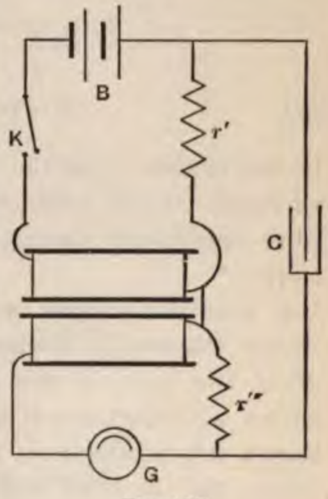

Fig. 187

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ophical *Magazine*, Vol. 23, 3d Series, p. 121 ; Gerard's *Leçons ricité*, 3d ed., Vol. I., p. 327; Gray's *Absolute Meas. in Elect.* Vol. II., p. 303.

ary circuit, which is opposite in direction to the charging current, is $\frac{MC'}{t}$. When the resistances r' and r'' are adjusted so that the galvanometer shows no deflection, the average fall of pressure in r'' during the period of the transient current which is caused by the condenser charging curre average pressure developed in the sec nsequently,

$$\frac{C'}{t},$$

and

In this method, as in ous one, the galvanometer needle must be sufficiently heavy, so that it does not move appreciably during the period of the transient current.*

4. *Comparison with a Known Self-inductance by Bridge* (Maxwell's Method). The mutual-inductance M of two coils in this case is compared with the known self-inductance of one of the coils. The coil of known self-inductance is connected in one arm R of a bridge, and the other coil is connected in the battery circuit (Fig. 188); the connections being so made that the magnetic effects of the two coils are in opposition. The resistances of the other arms of the bridge are represented by R_1, A, and B. The bridge is balanced by trial and approximation for both steady and transient currents, when the fall of pressure in the bridge arm R is equal to that in the arm R_1. From this con

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* *Elektrotechnische Zeitschrift*, 1887, Vol. 8, p. 336; Hospitalier's *Traité de l'Énergie Électrique*, Vol. I., p. 301.

dition the following equations are formed. If c and c_1 are the currents in the arms R and R_1, at any instant the fall of pressure in the arm R is

$$Rc + \frac{Ldc}{dt} + \frac{M(dc + dc_1)}{dt},$$

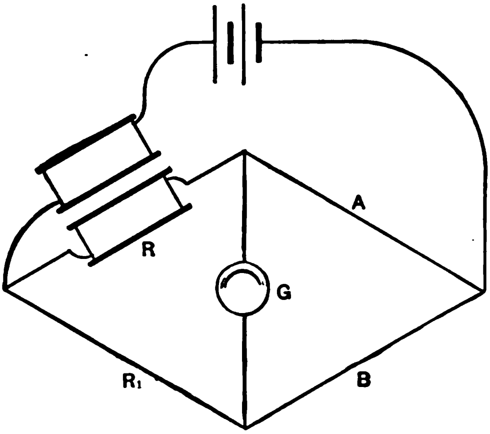

Fig. 188

and the fall of pressure in the arm R_1 is R_1c_1. The condition of balance for both transient and steady current requires that

$$R_1c_1 = Rc + \frac{Ldc}{dt} + \frac{M(dc + dc_1)}{dt} \text{ and } R_1c_1 = Rc.$$

Hence
$$\frac{Ldc}{dt} + \frac{M(dc + dc_1)}{dt} = 0.$$

That is, the current, $c + c_1$, flowing in the battery circuit, and that in the arm R, c, must have such a ratio that the effects of self and mutual induction are equal opposite. Integrating the last equation gives

$$Lc + M(c + c_1) = 0,$$

and combining this with $Rc = R_1c_1$ gives

$$\Big) = -\Big(1 + \frac{A}{B}\Big).$$

In order nvenience of the trial
approxim balancing, a variable resi

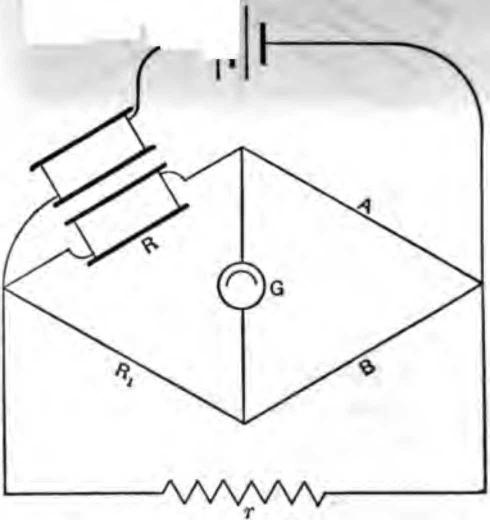

Fig. 189

ance, r, may be connected between the battery termi of the bridge (Fig. 189), and the required relations tween the transient currents in the battery circuit rm R may be gained by adjusting this resistance w ut disturbing the steady balance of the bridge. Then ve have, by a solution similar to the above,

$$\frac{L}{M} = -\left(1 + \frac{R}{R_1} + \frac{(R + A)}{r}\right).$$

These equations show that the value of L must be greater than M, in order that the method may be used. To make the method generally useful, the coil of known self-inductance should be inserted in the shunt circuit with the resistance r. Then, when the balance is made,

$$\frac{L}{M} = -\left(R + \frac{R_1}{r'}\right),$$

where r' is the total resistance of the shunt branch.

In order that this method may be reliable, the inductances of the bridge coils must be entirely negligible or a proper correction must be made. To gain greater sensibility in the method, a secohmmeter may be used with the bridge (Sect. 37).*

4*a*. (Niven's Method.) In this case, the mutual-inductance, M, of two coils is compared with the known self-inductance of another coil. The coil of known self-inductance is connected in one of the bridge arms, R. One of the coils whose mutual-inductance it is desired to measure is connected in the battery circuit, and the other is connected in series with a variable resistance, as a shunt to the galvanometer (Fig. 190). When the bridge is balanced for steady currents, a balance for transient currents may be gained by adjusting the vari able resistance in the shunt circuit. This being don we have

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* Maxwell's *Electricity and Magnetism*, 2d ed., Vol. II., p. 365; Gray's *Absolute Measurements*, Vol. II., p. 465.

$$\frac{L}{M}=\frac{(R+R_1)^2}{Rr},$$

where r is the resistance of the circuit shunting t galvanometer. The galvanometer needle must have

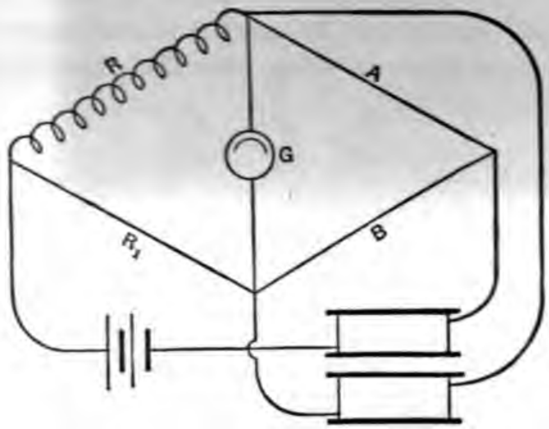

Fig. 190

considerable time of vibration as before, and a secohm meter must be used to give sensitiveness.*

5. *Comparison of Two Mutual-Inductances* (Maxwell Method). The primaries of the two pairs of coils a connected in series with a battery and key, and tl secondaries are connected in series with variable r istances. A galvanometer is connected as a shu tween the secondaries (Fig. 191). The variable resis

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* Gray's *Absolute Measurements*, Vol. II., p. 475.

:es are adjusted until the galvanometer shows no lection upon opening and closing the key. Then

$$\frac{M_1}{M_2}=\frac{R_1}{R_2},$$

re R_1 and R_2 are the total resistances in the sec- ary circuit on either side of the galvanometer.

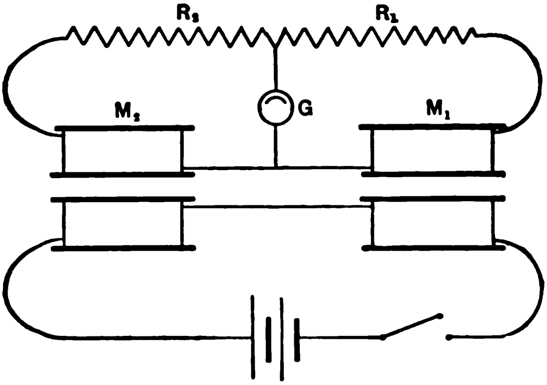

Fig. 191

his method may be modified by connecting the anometer in series with the secondaries, which are lected in opposition. A shunt is then connected veen the lead wires, as in Fig. 192. When the re- ınces on either side of the shunt have been adjusted hat the galvanometer shows no deflection on open- and closing the key, the relation obtaining is

$$\frac{M_2}{M_1}=\frac{R_2+R_s}{R_s},$$

re R_s is the resistance of the shunt connection.

: the shunt connection is placed in the primary circuit

(Fig. 192 *a*) and R_3 is the total resistance of the prima circuit to the right of the shunt, the relation becomes

$$\frac{M_2}{M_1} = \frac{R_p}{R_p + R_3}.$$

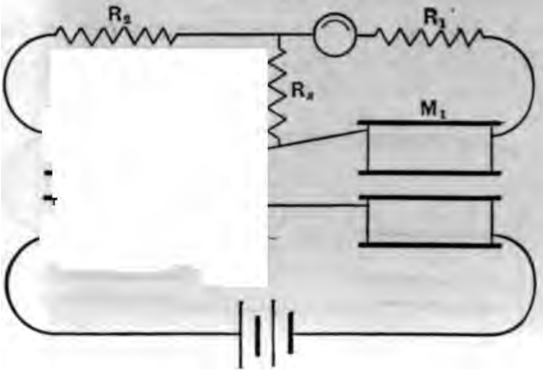

Fig. 192

Finally if a shunt is placed in both primary and s ondary circuits (Fig. 192 *b*), the relation becomes *

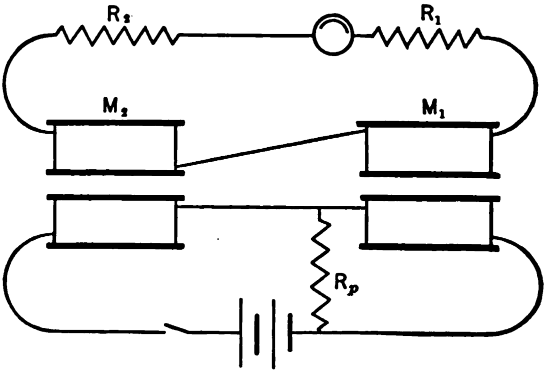

Fig. 192 a

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[*] Maxwell's *Electricity and Magnetism*, 2d ed., Vol. II., p. Gray's *Absolute Measurements*, Vol. II., p. 444.

$$\frac{M_2}{M_1}=\frac{R_p(R_2+R_s)}{R_s(R_p+R_3)}.$$

In this method a secohmmeter may be used, and then e galvanometer terminals will be reversed at each versal of the current. Therefore, a dead beat galvameter may be substituted for the ballistic form, and

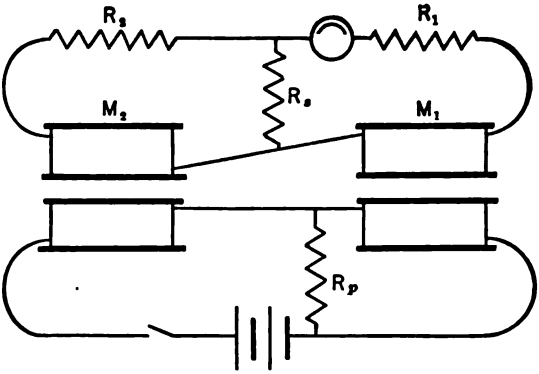

Fig. 192 b

en the desired condition obtains, it will indicate that current is passing.

09. **Coils with Iron Cores.** — When measurements the mutual-inductances of coils with iron cores are le by either of the preceding methods, the value rved will depend upon the magnitude of the cur- used in making the measurements. Since it is not ticable to use currents of much magnitude in the ge methods, they are not adapted to the measure- t of the mutual-inductances of coils with iron cores h are designed for use with large currents. The

last method is a laborious one, and therefore is not well adapted to general use unless modified by employing a variable standard, as described below. The first, second, and third methods are fairly convenient, and may be used with any desired current in the primary coil. They are also fairly reliable. If the value of M is to be determin iron-cored coils using a certain curr necessary to adjust the resistance of cuit so that the required current will ey is closed. The standards of self f mutual-inductance employed in th ethods must evidently be constructed wi es so that the coefficients are independent of the v e of the testing current. A variable standard of mutual-inductance may be made up to serve a purpose similar to that of the Ayrton and Perry self-inductance standard (Sect. 36, 3 *a*). In fact, the Ayrton and Perry self-inductance standard may be used as a variable standard mutual-inductance by using the fixed and movable coils for the mutually interacting pair, in which case the mutual-inductance of the pair may be varied at will by rotating the movable coil. With such a variable standard the last method enumerated above may be somewhat simplified. In this case the adjustments required to gain a balance may be made by changing the variable standard. If the unknown mutual-inductance is beyond the range of the standard, the balance may still be gained by making $\frac{R_1}{R_2}$ (page 99) some satisfactory fixed ratio and then balancing by adjusting the standard.

110. Mutual Induction of Parallel Distributing Circuits. —Where two or more electric light or power circuits carrying alternating currents run parallel to each other, they act inductively upon each other, and in some cases the mutual induction may cause considerable interference with the uniformity of the pressure on the lines. The mutual inductance of any two parallel circuits of indefinitely great length may be easily calculated, provided the distances apart of the different wires composing the circuits are known. The number of lines of force which pass through or link with one circuit, due to one ampere flowing in the other, is numerically equal to 10^9 times the mutual-inductance of the two circuits, and this number of lines of force is equal to the algebraic sum of the number of lines of force embraced by the first circuit which would be set up by the current in the individual conductors of the second circuit taken separately. The method of Section 47 is therefore directly applicable to the calculation of the mutual-inductance of two long and parallel, narrow circuits. The following examples represent the commonest arrangements of circuits on pole lines. Suppose that a, a' and b, b' represent the conductors of two circuits, and that the order of the wires is $a - a' - b - b'$, the distance apart centre to centre of the wires of circuit A is x, of circuit B is y, and of the adjacent wires of the two circuits $(a' - b)$ is z; then, if we consider the currents as concentrated at the centres of the wires, which makes but an insignificant error with the ordinary dimensions of conductors and circuits, and consider the space between two planes perpendicular to the circuits and one centimeter apart, the number of lines of force due to a current of one ampere in a', which pass through the circuit B between the planes, is (Sect. 47)

$$N_{a'} = \int_{z}^{y+z} \frac{2\,da}{a} = 2 \log_e \frac{y+z}{z},$$

and the number of lines of force due to a current of one ampere in a which pass through the circuit B between the planes, is

$$N_a = -\int_{x+z}^{x+y+z} \frac{2\,da}{a} = -2 \log_e \frac{x+y+z}{x+z}.$$

The total number of lines of force set up by the current of one ampere in circuit A, which pass through the circuit B between the planes, is $N_a + N_{a'}$, the number which link through the B circuit in a length of l centimeters is

$$l(N_a + N_{a'}),$$

and the mutual-inductance of the parallel circuits of length l is

$$M = \frac{l(N_a + N_{a'})}{10^9} = \frac{2\,l}{10^9}\left(\log_e \frac{y+z}{z} - \log_e \frac{x+y+z}{x+z}\right)$$

$$= \frac{2\,l}{10^9} \log_e \frac{(x+z)(y+z)}{z(x+y+z)}.$$

If $x = y$, this becomes

$$M = \frac{2\,l}{10^9} \log_e \frac{(x+z)^2}{z(2x+z)} = \frac{4.60\,l}{10^9} \log_{10} \frac{(x+z)^2}{z(2x+z)};$$

and if $x = y = z$, it becomes

$$M = \frac{4.60\,l}{10^9} \log_{10} \frac{4}{3} = \frac{.575\,l}{10^9},$$

where l is the length of the parallel circuits in centimeters.

Exchanging the order of the wires so that circuit A is between the conductors of circuit B, thus $b-a-a'-b'$, changes the formulas. Here the algebraic sum of the number of lines of force set up by the circuit A which link with circuit B, is equal to the total number of lines of force set up by circuit A minus the number passing backwards through $b-a$ and $a'-b'$. Suppose $a-a'$ is equal to x and $b-a$ and $a'-b'$ are each equal to y, then the total number of lines of force set up by one ampere in a length of one centimeter of circuit A, is

$$N_a = 4 \log_e \frac{x}{r}$$

(r being the radius of the conductor), and the number of lines due to circuit A, which pass between the planes through the space $b-a$, is

$$N_{ab} = 2\left(\log_e \frac{y}{r} - \log_e \frac{x+y}{x}\right) = 2 \log_e \frac{xy}{r(x+y)},$$

and $$M = \frac{l}{10^9}(N_a - 2\,N_{ab}) = \frac{4\,l}{10^9}\left(\log_e \frac{x}{r} - \log_e \frac{xy}{r(x+y)}\right)$$

$$= \frac{4\,l}{10^9} \log_e \frac{x+y}{y} = \frac{9.20\,l}{10^9} \log_{10} \frac{x+y}{y}.$$

If $x = y$, $M = \frac{9.20\,l}{10^9} \log_{10} 2 = \frac{2.77\,l}{10^9}$.

If the circuits are not in the same plane, as, for instance, they are arranged thus,

$a - a'$

and the distance $a - a'$ is x, the distance $b - b'$ is y, the distance $a' - b'$ is z, the distance $a' - b$ is w, $a - b$ is v, and $a - b'$ is u; then the formulas are

$$N_a = 2\left(\log_e \frac{u}{r} - \log_e \frac{v}{r}\right) = 2 \log_e \frac{u}{v},$$

$$\text{og}_e \frac{z}{r}\Big) = 2 \log_e \frac{w}{z},$$

and

$$M = \frac{l}{10} \qquad \text{g}_e \frac{uw}{vz} = \frac{4.60\, l}{10^9} \log_{10} \frac{uw}{vz}.$$

If one c beneath the other, $x = y$, $v = z$, and and the formula becomes

$$M = \frac{2\, l}{10^9} \text{l} \quad \frac{x^2 +}{z^2} \qquad \frac{4.60\, l}{10^9} \log_{10} \frac{x^2 + z^2}{z^2}.$$

If $x = y = v = z$,

$$M = \frac{4.60\, l}{10^9} \log_{10} 2 = \frac{1.38\, l}{10^9}.$$

These results plainly show that the mutual-inductance of two circuits is entirely independent of the actual distances apart of the conductors composing the circuits, but depends wholly upon the relative values of the distances. The mutual-inductance of two circuits is a maximum when the circuits are exactly superposed, in which case $M = \sqrt{L'L''} = L$, and decreases as the distance between the circuits is increased in comparison with the distance apart of the conductors of each circuit; consequently, mutual-inductance between circuits on the same pole line may be reduced by decreasing the tance apart of the conductors of each circuit and incr ing the distance apart of the circuits. A better way to 'oid mutual induction in some cases is to transpose e position of the circuits with reference to each other, is done in long distance telephone lines, so that the ductive effects of the circuits on each other are in position in different parts of the line, and neutralize ch other for the line as a whole.

The effect of mutual induction between two circuits to set up an electrical pressure in one when the cur- nt in the other varies. If the current is a sinusoidal ternating one, this pressure is (Sects. 107 and 108) πfMC, and the effect of an alternating current in one rcuit upon another circuit is easily determined if M is nown. When the two circuits are fed from the same ngle-phase alternator, the induction of one upon the ther is in quadrature with the current in the first, nd the relative phase of the pressure induced in the cond depends on the current lag in the first. If this zero, the induced and impressed pressures are in uadrature, while they are in opposition if the lag is 0°. The result is a displacement of the pressure aves and a drop of pressure along the lines. If the rcuits are fed from different alternators, the frequency which is slightly different, the inductive pressure and npressed pressure interfere so as to form pulsations or ats, the frequency of which is equal to the difference the two alternator frequencies, and the amplitude of hich is the sum of the two pressures. This may use a perceptible winking of incandescent lamps con- cted to mutually inductive circuits of nearly the same quency.*

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* C. F. Scott, Polyphase Transmission, *Electrical World*, Vol. 23 *ndon Electrician*, Vol. 32, p. 642.

CHAPTER X.

OPERATION OF [illegible]MER, AND EFFECT OF IR[illegible] LOSSES.

111. Ratio of [illegible] **in a Transformer.**—The formulas of [illegible] that the electric pressure developed in [illegible]oil is at any instant

$$e'' = $$ [illegible]

If the current wave is a sinusoid, this becomes

$$e'' = \frac{d(Mc'_m \sin a)}{dt}.$$

If the conditions require that M be treated as a variable dependent upon the varying permeability of an iron core, this equation is practically unsolvable. For practical purposes, as has already been said, it is sufficient to assume M as having a constant average value which depends upon the iron of the core and the magnetic density used in the transformer. The equation then becomes $e'' = \frac{Mc'_m \cos a da}{dt}$. The maximum value of the electric pressure is therefore $e''_m = 2\pi f Mc'$[illegible] where f is the frequency of the current wave. Th[illegible]ective value of the secondary electric pressure is the[illegible] dently $E'' = 2\pi f MC'$, where C' is the effective pr[illegible]

:y current. If the secondary circuit is open, the owing equations may be written,

$$C' = \frac{E'}{\sqrt{R'^2 + 4\pi^2 f^2 L'^2}},$$

le $E'_s = 2\pi f L' C' = \frac{\sqrt{2}\,\pi f n' N}{10^8}$, where E'_s is the self-ıced primary pressure, E' is the impressed pressure, N is the maximum number of lines of force in the le. If the resistance of the primary be considered ligible, the former equation becomes

$$C' = \frac{E'}{2\pi f L'},$$

$$E' = 2\pi f L' C = E'_s.$$

the primary and secondary coils are so completely erposed that there is no magnetic leakage, the value M becomes $M = \sqrt{L'L''}$; whence

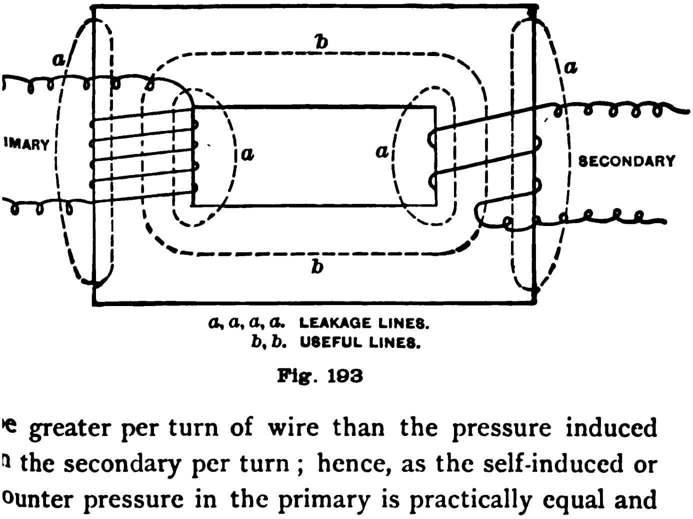
$$\frac{E'}{E''} = \frac{2\pi f L' C'}{2\pi f M C'} = \frac{L'}{\sqrt{L'L''}} \quad \text{or} \quad \frac{E'}{E''} = \frac{\sqrt{L'}}{\sqrt{L''}}.$$

t $\frac{\sqrt{L'}}{\sqrt{L''}} = \frac{n'}{n''}$, since the reluctance in the magnetic :uit of the two coils is assumed to be the same (Sect. , and therefore

$$\frac{E'}{E''} = \frac{n'}{n''}.$$

other words, if the active pressure in the primary y be considered negligible when compared with the ›ressed pressure, and there is no leakage of magnetic :s, the ratio of the impressed pressure in the primary a transformer to the induced pressure in the s ondary *is equal to the ratio of the number of turns of wire in the two coils.* This ratio of pressures is called **the Ratio of Transformation.** The ratio of transformation of well-designed transformers is practically equal to $\frac{n'}{n''}$ when the secondary circuit is open, showing that the assumpt re pressure and magnetic leakage are ercial transformers, when the secondar s entirely allowable. An example wil striking manner. In a certain tran K.W. capacity the resistance of the ically 1 ohm and the inductance is 9.1 henrys. a frequency of 70 and a pressure of 2000 volts, for which the transformer was designed, the value of $4\pi^2f^2L'^2$ is 16,000,000. In another transformer of 11.25 K.W. capacity designed for 2400 volts primary pressure, the value of the primary resistance is 6.45 ohms and the value of $4\pi^2f^2L'^2$ is 10,000,000. In three other transformers designed for 1000 volts pressure and respectively of 7.5, 4.5, and 1.5 K.W. capacity, the primary resistances are 1.16, 2.15, and 8.90 ohms, while, at a frequency of 125, $4\pi^2f^2L'^2$ is equal respectively to 100,000,000, 125,000,000, and 400,000,000; and in a transformer of .5 K.W. capacity the primary resistance is 25 ohms and $4\pi^2f^2L'^2$ is 400,000,000. In each of these cases, which represent common practice in the construction of transformers, the value of R'^2 is entirely negligible when compared with $4\pi^2f^2L'^2$. If R'^2 were not negligible, it would evidently increase the ratio of transformation (that is, for a given impressed primary pressure the secondary press-e would be decreased) on account of the loss of essure due to the current flowing through R'.

112. Magnetic Leakage.—The primary and secondary ils in each of these cases are so sandwiched together at magnetic leakage is certainly negligible when there no current in the secondary coil. A case when leakge is always present is shown in Fig. 193. From the gure it is evident that if there is no current flowing in e secondary, the counter pressure in the primary will



a, a, a, a. LEAKAGE LINES.
b, b. USEFUL LINES.

Fig. 193

e greater per turn of wire than the pressure induced n the secondary per turn; hence, as the self-induced or ounter pressure in the primary is practically equal and pposite to the impressed pressure, the ratio of transrmation will be increased. If a current flows in the econdary, the self-induction due to magnetic leakage n the secondary (lines of force linking with the secndary coil but not linking with the primary coil) will ill further reduce the active secondary pressur e ratio of transformation will be further i

The effect of magnetic leakage in i
of transformation (decreasing the pr
induced in the secondary by decrea
induction passing through it) is sh
ment reported by Professor Ryan.
ment recorded by him, the primary
were wound on opposite sides of a l
(Fig. 194). The number of turns
secondary were respectively 500 an

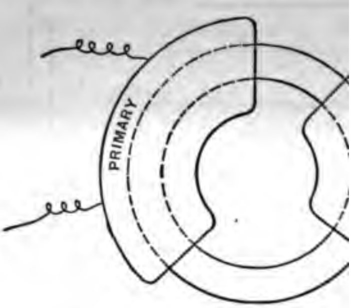


Fig. 194

When a pressure of 75.6 volts was
primary with the secondary open,
16.4 volts was induced in the seco
The whole difference in the two
magnetic leakage, and the magnitud
shows that M was much less than
$\frac{E'}{E''} = \frac{1.44\,n'}{n''}$, and assuming that the
magnetic circuits of the two coils were

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* Some Experiments upon Alternating Current
Inst. E. E., Vol. 7, p. 324.

by Sect. 104). The magnetic leakage was therefore o per cent; that is, the number of lines of force that assed through the primary but not through the sec-ondary coil, was 30 per cent of the total magnetic nduction set up in the magnetic circuit. The effect f magnetic leakage on a transformer is analogous to he effect produced on one without leakage, of insert-ng coils having self-inductance, or **Impedance Coils,** in the primary and secondary circuits outside of the trans-former (Fig. 195). These coils would have such self-inductances as to increase the self-inductances of the primary and secondary circuits in the ratio of $a : 100$, where a is the magnetic leakage in per cent. Since leakage causes a proportional increase in the apparent self-inductance of the primary and secondary circuit, it causes an equivalent lag of the currents in the two circuits.

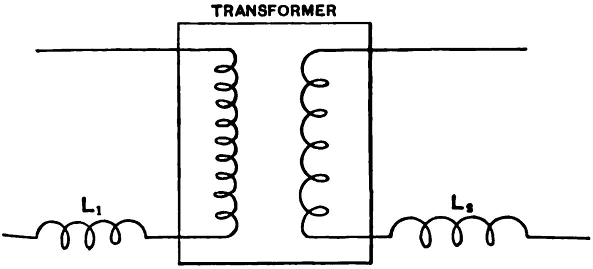

Fig. 195

113. Exciting Current. — In the case of an ideal trans-former without losses, the lag of the primary current, when the secondary circuit is open, is 90° with respect to the impressed pressure; for, $\tan \phi' = \frac{2 \pi f L'}{R'}$ (Sect.

28), and R' is assumed to be zero. Since the induced secondary pressure lags behind the magnetism, which is in phase with the primary current when the secondary circuit is open, by an angle of 90°, the phases of the primary impressed pressure and the secondary induced pressure are exactly 180° apart, or *they are in exact o* [illegible] urrent in the primary circuit of an [illegible] r, when the secondary is open, is all [illegible] magnitude which depends only upon t [illegible], L', of the primary coil.

The loss [illegible] sis and foucault currents in the iron [illegible] ce in the primary coil are by no mea [illegible] commercial transformers, but are of such a [illegible] le as to decrease the lag of the primary current until the power factor of the primary circuit is ordinarily between 50 per cent and 80 per cent, when there is no current in the secondary circuit; but the magnetism in the core remains in phase with the wattless component of the primary current and is 90° behind the phase of the primary pressure, so that *the primary and secondary pressures are still in opposition.*[*] The current which flows in the primary circuit when the secondary circuit is open, may therefore be considered as composed of two components, one of which supplies the energy required to make up the transformer losses, and the other of which serves simply for magnetizing power, and is therefore wattless. This primary current is often called the **Leakage Cur-**

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[*] Fleming, Experimental Researches on Alternate Current Transformers, *Jour. Inst. E. E.*, 1892. Ford, Tests of Modern Transformers, *Bull. Univ. of Wisconsin*, Vol. 1, No. 11.

rent or **Open-Circuit Current,** but a more satisfactory term is **Exciting Current.** The term **Magnetizing Current** is also applied to the exciting current, but we will reserve the term for its wattless component which is truly a magnetizing current.

114. Core Magnetization. — The maximum magnetic induction during a period, in the iron core of an ideal transformer, is dependent upon the maximum value of the current in the primary circuit, the effect of iron losses being omitted by assumption, and is equal to

$$N = \frac{4\sqrt{2}\,\pi n' C_1'}{10\,P} = 1.25\sqrt{2}\frac{n' C_1'}{P} = 1.77\frac{n' C_1'}{P},$$

where C'_1 is the magnetizing current and P is the reluctance of the magnetic circuit at the time that the current has its maximum value.

Closing the secondary circuit so that a current may flow in it under the impulse of the induced secondary pressure, materially changes the conditions heretofore explained. We will first assume that the secondary circuit is without self-inductance, and continue to neglect hysteresis and foucault current losses in the iron core and resistance losses in the windings, in which case the secondary current C'' will be in unison with the induced or secondary pressure, E''. This current has its own magnetizing effect on the magnetic circuit. If c' and c'' be the primary and secondary currents at any instant, the total magnetizing force in the circuit is $\frac{4\pi(n'c' + n''c'')}{10} = N_t P$, where N_t is the instantaneous value of the magnetic induction, and P is the assumed constant value of the reluctance of the magnetic

From this is found

$$c' = \left(\frac{10\,PN_i}{4\,\pi n'}\right) - \left(\frac{n''c''}{n'}\right),$$

and whence $$c' = \frac{10\,PN_m}{4\,\pi n'}\sin a - \frac{n''\sqrt{2}\,C''}{n'}\cos a;$$

but $$\frac{10\,P}{4\,\pi}$$

whence $$a - \frac{n''}{n'}C''\cos a\Big)$$

or $$n' \quad a - n''C''\cos a),$$

and $$n'^2C' \quad \sin^2 a$$

$$- 2\,n'n''C_1'C''\sin a\cos a$$

$$+ n''^2C''^2\cos^2 a)da,$$

where C' is the effective value of the primary current when the secondary current is equal to C'', and C_1', as before, is the wattless primary current when the secondary is open. Performing the integration gives

$$n'^2C'^2 = n'^2C_1'^2 + n''^2C''^2.$$

Remembering that C_1' and C'' have 90° difference of phase, the three terms of this formula may be represented by the three sides of a right-angled triangle (Fig. 196). The current C', which flows in the primary when the secondary is loaded, is in advance of the current C_1' by an angle ψ, the tangent of which shown by the figure to be $\tan\psi = \frac{n''C''}{n'C_1'}$. We hav already seen that C_1' is inversely dependent upon th

nductance of the primary circuit, and therefore—
ı the value of n' is fixed—directly upon the re-
nce of the magnetic circuit, which in commercial
formers is made very small. Tan ψ is therefore
: large when C'' has any considerable magnitude

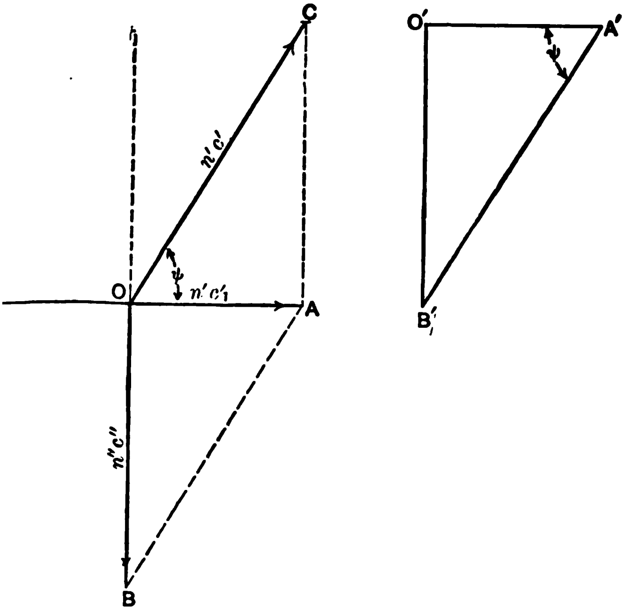

Fig. 196

ψ approaches 90° as C'' increases, so that the pri-
and secondary currents are practically in opposite
s in a well loaded transformer. As a transformer
ıded up, its power factor is therefore rapidly in-
ed.* The effect of the secondary current on the

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ompare Fleming, Experimental Researches on Alternate Current ormers, *Jour. Inst. E. E.*, Vol. 21, p. 594; Ford, Tests of Modern ormers, *Bull. Univ. of Wisconsin*, Vol. 1, No. 11.

primary circuit is to apparently decrease its tance and therefore to decrease its impedanc lag of the primary current.

115. Effect of Copper and Iron Losses on R —Consideration of the effect of C^2R, hyst foucault current losses has thus far been neg it has been shown that the effects of these by no means negligible. It is shown in S that the effect of the primary resistance, R', a fall in the secondary pressure and theref crease the ratio of transformation. The re the secondary winding, R'', evidently acts decrease in the pressure at the terminals of t ary and therefore to increase the apparent rati formation. The magnitude of the apparent the ratio of transformation is dependent upc of the products of the resistances with the c the respective circuits; that is, to the pressu to pass the current through the resistances cuits. The loss of pressure at the secondar in volts, due to this cause, when current C'' fl secondary circuit is

$$V = C''R'' + \frac{n''}{n'}C'R',$$

and since approximately, with core losses neg

$$C'' = \frac{n'}{n''}C',$$

this is
$$V = C''\left[R'' + \left(\frac{n''}{n'}\right)^2 R'\right].$$

The percentage increase of the ratio of tran s $\frac{V}{E''}$, where E'' is the total secondary pressure, the terninal pressure becoming $E'' - V$. This shows that the erminal pressure falls off proportionally as the load on he secondary of the transformer is increased, if the mpressed primary pressure remains constant. The ormula also shows that an ideal transformer (*i.e.* one vithout resistance, core losses, or magnetic leakage) is nherently self-regulating, and will therefore give a contant pressure at the secondary terminals at all loads if ed with a constant primary pressure.

The effect of core losses (hysteresis and foucault urrent losses) is to increase the primary current to a ertain extent and therefore to slightly affect the gulation.

116. Perfect Regulation of an Ideal Transformer.—The atements of the preceding section may also be proved follows: Considering the phases of the primary current and magnetization to be practically 90° apart, then e counter electric pressure of self-induction is in position to the impressed pressure, and the active essure in the primary circuit at the instant when the pressed pressure is a maximum is

$$E'_m - E'_{sm} = c'R',$$

being the counter electric pressure of self-induction. this instant the value of the primary current, c', is

$$c' = \sqrt{2}\, C' \sin \psi = \sqrt{2}\, C' \frac{n''C''}{n'C'}$$

ce (Sect. 114) $\sin \psi = \frac{n''C''}{n'C'}$

The counter electric pressure of self-induction is evidently equal to

$$E'_s = \frac{E''n'}{n''} = C''(R'' + R)\frac{n'}{n''},$$

wnere R is the external resistance in the secondary circuit. Substituting these values for c' and E'_s and dividing by $\sqrt{2}$ gives

$$E' = \frac{n''C''}{n'C'}C'R' + \frac{n'}{n''}C''(R'' + R),$$

whence, by transposition,

$$C''R = \frac{n''}{n'}E' - \left(\frac{n''}{n'}\right)^2 C''R' - C''R''.$$

From these equations it is seen that the pressure at the secondary terminals, $C''R$, becomes $\frac{E'n''}{n'}$ provided $\left(\frac{n''}{n'}\right)^2 R'$ and R'' can be taken as very small in comparison with R, and therefore under these circumstances the secondary pressure is constant provided th impressed pressure be kept constant. *An ideal transformer is therefore an inherently self-regulating instr ment for transforming electric currents at one constan pressure into equivalent currents at another constan pressure, and the faulty regulation found in commerci transformers is wholly due to electrical losses and ma netic leakage.* By transforming the last formula int the equivalent form $C'' = C'\frac{n'}{n''}$, it is seen that *an ide transformer which is fed with a constant current is a inherently self-regulating instrument for the transform tion of that current into an equivalent constant curre* *er pressure.* In this case the primary impressed will vary with the resistance of the secondary

ffect on Regulation of Self-inductance or Capacity dary Circuit. — In the service to which trans-

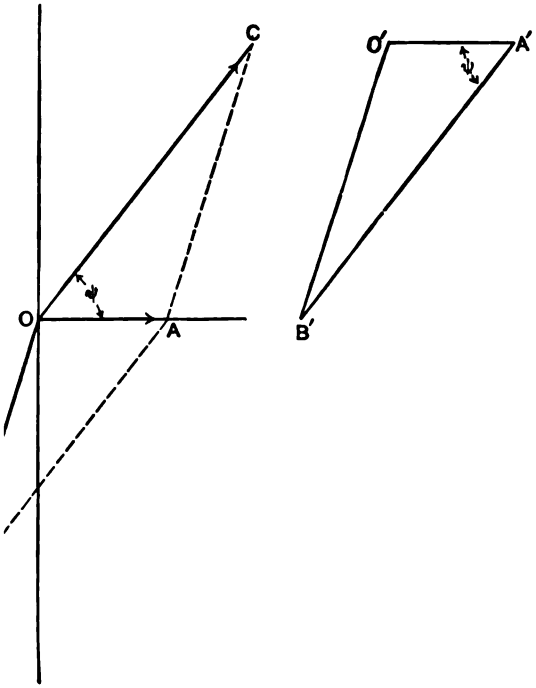


Fig. 197

have heretofore been generally applied, the n of incandescent lamps, the external secondary s practically non-inductive, but when motors or ps are operated on the secondary circuits of mers, they may add a considerable inductance to the circuits. In this case the secondary current is caused to lag behind the secondary pressure. The relation which must exist between the secondary and primary ampere-turns and the resultant magnetizing ampere-turns (Fig. 197) shows that such a lag of the secondary current must ca n the lag of the primary current. The as though additional self-inductance w e circuit of the primary coil, and *an l secondary circuit therefore causes ion on the part of the transformer.* increase in the ratio of transformatio upon the resistance and reactance of the secondary circuit, since $\tan\phi = \frac{2\pi fL}{R}$.

The effect of a capacity in the secondary circuit is exactly opposite to that of an inductance, since it causes the current to lead the pressure. Consequently, *a secondary circuit having capacity tends to aid regulation,* and may, if the capacity is sufficient, even cause a decrease in the ratio of transformation. That is, the pressure in a secondary circuit may be increased by the mere insertion of a condenser.

118. Graphical Method for Determining Current and Pressure Relations. — The effects discussed may all be shown very plainly by a graphical construction based upon the triangle of electrical forces (Sect. 15). In Fig. 198, OC'' on the vertical axis represents the value, on a convenient scale, of the product $n''C''$. If the condary circuit may be considered non-inductive, as the transformer is feeding incandescent lamps, condary pressure wave is in unison with the cur-nd OE'' may be taken to represent the value sition of the pressure. The current component

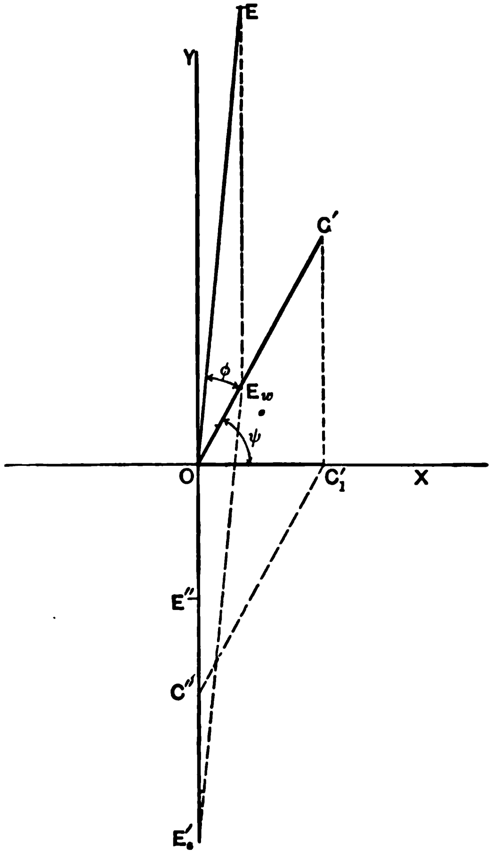


Fig. 198

s effective in producing magnetization must be advance of this. Accepting the conventional : direction for harmonic rotation as left-handed or counter-clockwise, the magnetizing ampere-turns $n'C_1'$ must be laid off on the horizontal line to the right of the vertical and may be represented by OC_1'. The ampere-turns of the primary, when C'' flows in the secondary, are found by completing the parallelogram on OC'' of which OC_1' is the diagonal. This gives the line OC' to represent $n'C'$. In order that the diagram may be readily intelligible the value of $n'C_1'$ is taken as about $\frac{1}{2}$ of $n''C''$, while in commercial transformers it is generally less than $\frac{1}{20}$ of $n''C''$ and is sometimes as small as $\frac{1}{50}$ or $\frac{1}{60}$ of $n''C''$. The angle YOC' in the diagram is therefore much exaggerated in comparison with its value in commercial transformers.

It now remains to find the value and position of the impressed primary pressure. This is the resultant of the counter pressure of self-induction in the primary circuit, E_s', and the active pressure — the pressure which is effective in making up the losses in the magnetic circuit caused by hysteresis and foucault currents and in the conductors of the primary coil caused by its resistance. The second component of the pressure is in unison with the primary current and may be laid off on the line OC', its length being OE_w. The self-inductive primary pressure is in unison with and in the same direction as the induced secondary pressure, and is equal to $E''\frac{n'}{n''}$ if $M = \sqrt{L'L''}$. It is represented in the figure by OE_s'. Completing the parallelogram gives line OE', which represents the direction and magni- of the impressed electric pressure. In the figure ngle $E'OC' = \phi'$, and the angle $C'OC_1' = \psi$. The ases of the pressures and currents are shown ative angular positions of the lines radiating The value of the primary current is taken ɔm the length of the line *OC'*. When the

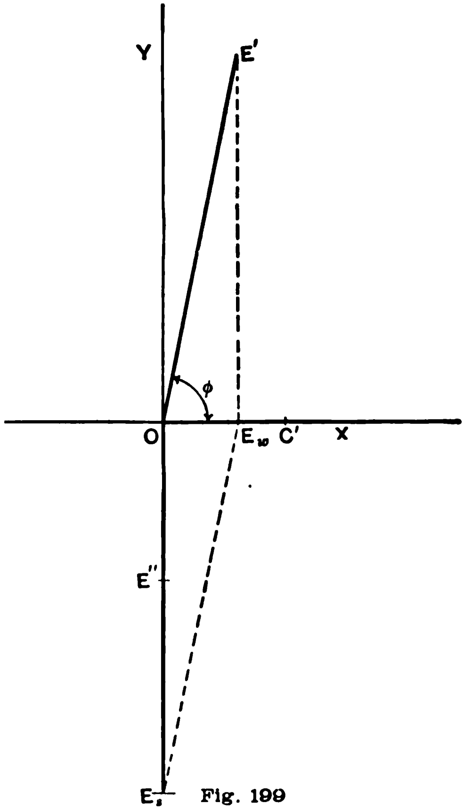


Fig. 199

circuit is open, the construction is similar to ing, but the value of the primary *CR* loss king the length of *OE'* slightly smaller (Fig. : exciting current is taken, as before, directly

from the length of the line OC', and the figures sho
that the ratio of transformation is increased by loadin
the transformer on account of the drop of pressure i

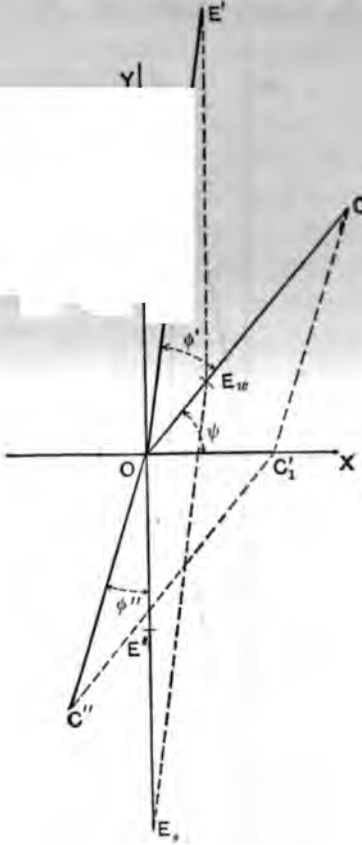


Fig. 200

the windings. The figures plainly show that the d
ation of the secondary pressure from the form of
imary pressure is proportional to the value of C'

ysteresis and foucault current losses may be d to be independent of the secondary current 127).

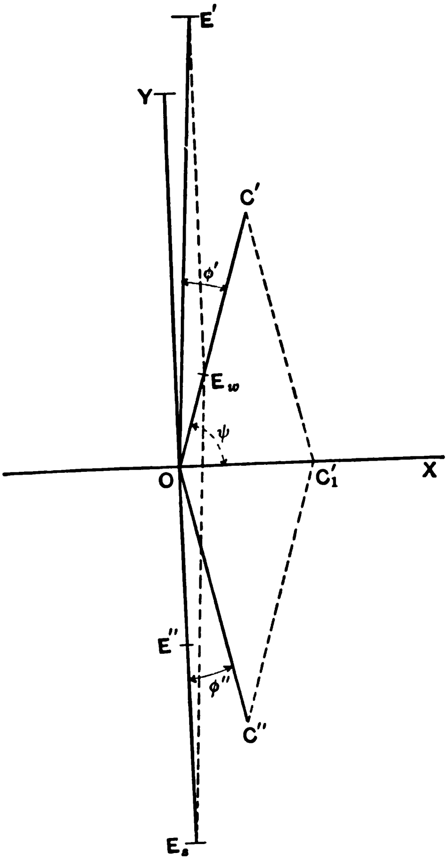


Fig. 201

effect of inductance in the external secondary is shown in Fig. 200. As before, OC'' repre-
he secondary ampere-turns $n''C''$. If it is sup-

posed that the inductance in the secondary circuit be sufficient to cause a lag of ϕ'', then the induced secondary pressure is in advance of the current by an angle ϕ'', and is represented in magnitude and direction by OE''. The position of the magnetizing ampere-turns is 90° in advance of OE'', and is represented by OC_1'. Completing on OC'' and OC_1', gives OC'. The p pressure is then found as before, and Figs. 198 and 200 shows that the sel e secondary circuit increases ϕ'.

A similar c ng the effect of capacity, is given in Fig. 201. This ffers from the preceding only on account of the secondary current leading the pressure.

119. Transformation from Constant Pressure to Constant Current. — The effect of magnetic leakage can also be satisfactorily shown in the same manner (Fig. 202). Remembering that the effect of leakage is the same as that of self-inductance coils placed in the primary and secondary circuits, the construction is exactly the same as in the case of a transformer working on an inductive secondary circuit, with an additional correction applied to the angle of lag between the primary pressure and current to account for the direct effect of the leakage on the primary circuit. The construction shows that, as the leakage is increased so that the secondary angle of lag ϕ'' approaches 90°, the deficiency in the inherent tendency to regulate for constant pressure becomes so great that the secondary terminal pressure actually tends to vary inversely with the current. Such

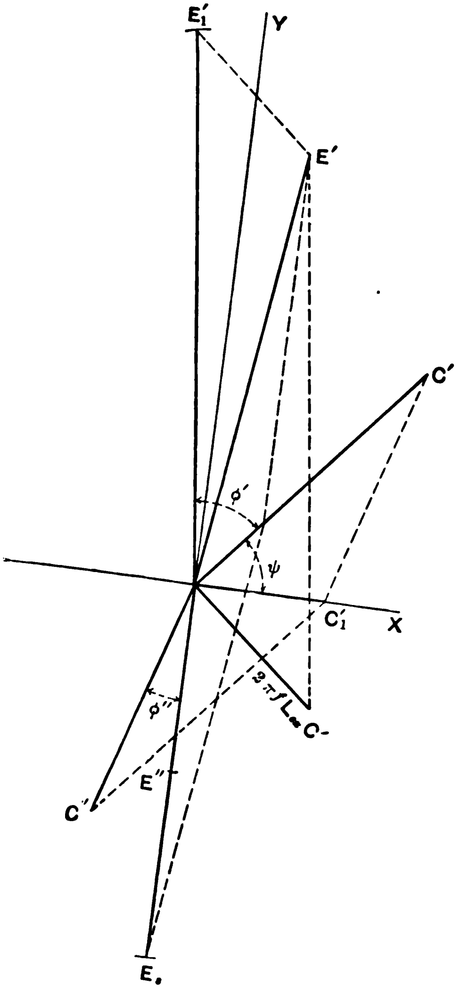


Fig. 202

a transformer would therefore tend to transform a variable current at constant pressure into a constant current at a variable pressure, which would enable it to be used for series arc lighting from a constant-pressure circuit. When the lag angle becomes 90°, the transformer can of course do no work, consequently it is impossible to get very exact regulation in thus transforming from constant pressure to constant current, but it is possible

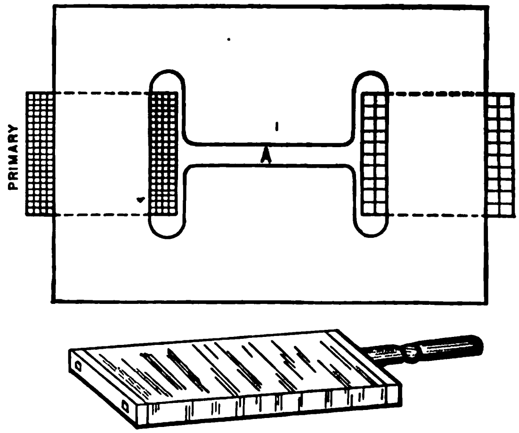


Fig. 203

to arrange the transformer so that the percentage can be varied when necessary by partially closing a shunt magnetic circuit by a slab of iron strips, as was first proposed by Elihu Thomson (Fig. 203). Figure 204 shows the results of a test of a Wood transformer, in which the constant-current regulation is wholly due to magnetic leakage. In the upper half of the figure, one curve shows the efficiency as a function of the current ıe secondary circuit, and the other curve shows the rnal characteristic, or the secondary terminal press-

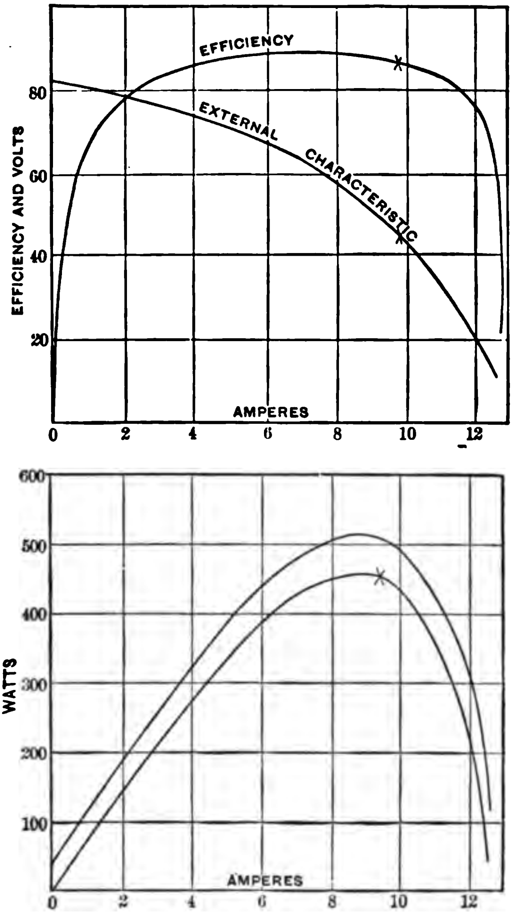


Fig. 204

; a function of the secondary current. The lower f the figure has curves which show the watts in the primary and secondary circuits as a function of secondary current. The crosses on the curves sh the points corresponding to normal load, which is t required to operate one arc lamp. The primary pr ure of this transformer was 1000 volts.

When makes itself evident ir transforn ıdary circuit open, the ceding to constant-pressure regı tion are ratio of transformation no longe n well-built transformers signed f re, magnetic leakage is likely to be of much magnitude, and in fact it can o be brought to a large value by making the space oc pied by the primary and secondary coils very la compared with the cross-section of the iron core, using iron of a low permeability, or by specially arra ing leakage paths.

120. The Effects of Variable Reluctance, Hystere and Foucault Currents on the Form of the Primary C rent Wave.—In the preceding discussions it has b assumed that the reluctance of the magnetic circuit transformers can be taken at an average constant va which is practically equal to that when the curren at its maximum point. The low induction whict used in commercial transformers as ordinarily structed, makes this assumption entirely allowa though it is by no means exact. If the induction pushed above the bend in the curve of magnetizat however, the influence of the lowered permeabilit the iron becomes marked. The curve *OM* in Fig.

e taken to represent the curve of magnetization ransformer core, plotted with ampere-turns as as and volts induced in the primary windings

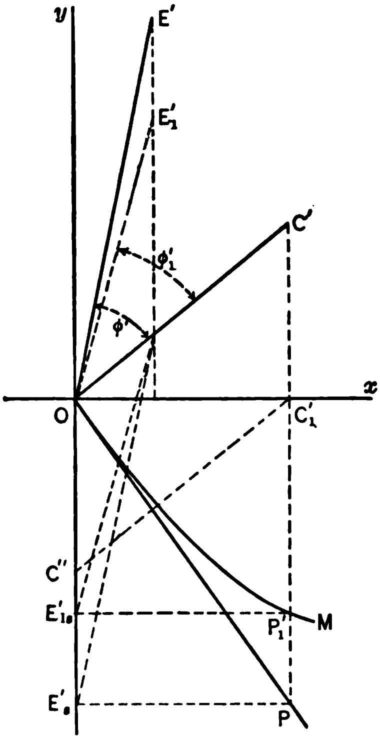


Fig. 205

inates, supposing the effect of hysteresis to be ble. Then when the magnetizing turns equal he induced pressures in the primary and second- cuits are reduced from E_s' and E'', which would hed with a constant reluctance, to E_{1s}' and E_1''.

The construction shows that this decreases the angle o lag between the primary current and pressure, and makes necessary more turns of wire on the primary and secondary coils in order that a given output may be obtained. Since, in this case, the permeability varies through each period with the magnetizing ampere-turns, the variation of ϕ', and the primary c istorted from the form of the prima sure wave.

The effe the core of a transformer is to disto primary current wave to a still more than would magnetic saturation alone, and the m r the maximum magnetic density is carried, the greater the distortion becomes. The ordinates of the primary current wave are at each instant proportional to the difference of the corresponding ordinates of the primary pressure wave and the wave of counter electric pressure. The latter is of course exactly similar to the form of the secondary pressure wave. With the primary pressure wave sinusoidal and an approximately uniform magnetic reluctance, the primary current wave would be sinusoidal. With a variable reluctance, but no hysteresis, the current wave becomes peaked, but remains symmetrical; but when hysteresis is taken into account, the symmetrical form is lost. This is conveniently illustrated by Figs. 206 and 207. In Fig. 206 the heavy line represents the hysteresis cycle for a piece of wrought iron, and the line *b'Ob* may be taken to represent the cyclic curve of magnetization which would be given by the iron if hysteresis were absent. In Fig. 207 the curve *M* represents the curve of magnetic induction in a transformer with its ordinates plotted on the same scale as Fig. 206. When the transformer is worked with its secondary circuit open, the drop of pressure due to the exciting current flowing through the primary winding is negligible, so that the primary impressed pressure at each instant is proportional to the tangent of the curve of magnetism and is 90° in advance of the magnetization. In this case, the curves of pressure and magnetism are assumed to be sinusoidal. The tangent relations between the curves of magnetization and pressure (Sect. 77) must exist as long as $C'R'$ is negligible. As an illustration of the fact that the self-induced pressure is of the same form, and is equal and opposite to the impressed pressure, and that the exciting current varies in such a way as to furnish this induced pressure in a transformer with the secondary open (supposing the C^2R losses negligible), we may consider an inductance coil of negligible resistance with a sinusoidal pressure (E) of, say, 100 volts impressed upon it. Suppose the frequency is $127\frac{1}{2}$, the self-inductance (L_1) is .01 of a henry, and the resistance

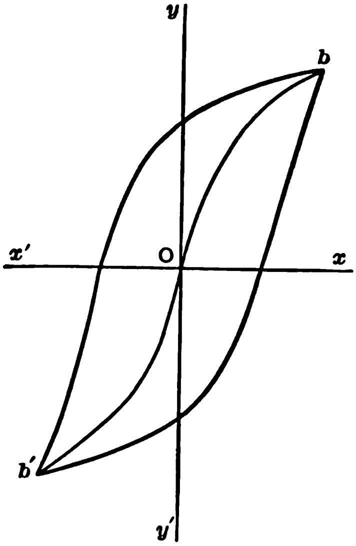


Fig. 206

is negligible; then $2\pi f L_1 = 8$ will be the impedance (see Chap. IV.). The current (C_1) flowing will be $\frac{E}{2\pi f L_1} = 12.5$ amperes with a lag angle of 90°. The inductive pressure (E_1) will be $2\pi f L_1 C_1 = 100$ volts, which is equal and opposite to the impressed pressure. Suppose the L_1 change to $L_2 = .005$; then the cur

25,

and the indu) will be

$= 100,$

as before. It is seen that whatever value the self-inductance may have, the exciting current will take such a magnitude that the counter pressure of self-induction will be equal and opposite to the impressed pressure. As the reluctance of the magnetic circuit is proportional to the self-inductance, when the reluctance changes, the exciting current flowing will change so that, as before, the counter pressure will equal the impressed pressure. Now, neglecting hysteresis, when the magnetism is carried through the cycle $ObOb'O$ (Fig. 206), the current corresponding to each ordinate of the curve M in Fig. 207 must be proportional to the current required to produce the corresponding magnetization in Fig. 206. In this way curve C_e is plotted to represent the wave of exciting current in the transformer if there were no hysteresis in the iron. This curve is symmetrical and of the same phase as curve M, and *it represents the true wattless magnetizing current.*

is easy to plot the curve which shows the form xciting current of the transformer with the effect ysteresis included, since it is only necessary to plot urrent ordinates in Fig. 207 the currents which espond to the various values of the magnetization he hysteresis cycle. This curve is curve *C* in Fig. It may be considered as made up of a wattless netizing component, C_e, and an active component

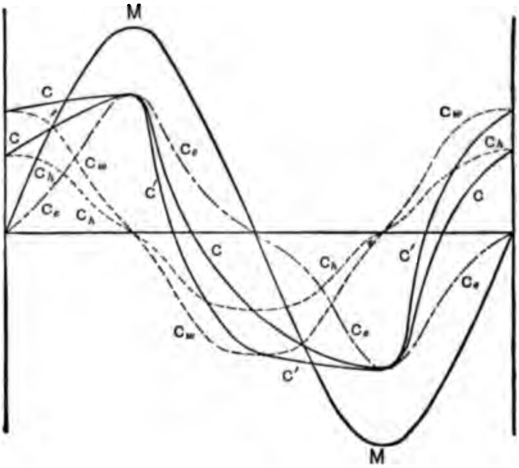


Fig. 207

aused by hysteresis. The ordinates of the active ponent are proportional to the differences of cor- onding abscissas of the curve of magnetization and hysteresis cycle (Fig. 206), and the curve is nearly soidal. The co-ordinates of the points *b* and *b'* ig. 206 are not altered by hysteresis and the effect ysteresis therefore does not alter the position or nitude of the maximum value of the exciting cur-

rent, though it distorts and enlarges the loop of
curve, and advances its zero points slightly so
the equivalent lag angle becomes slightly less than

The effect which foucault currents in the core
upon the exciting current may be shown in a si
manner. The instantaneous value of the portion o
exciting current which is required to make up the l

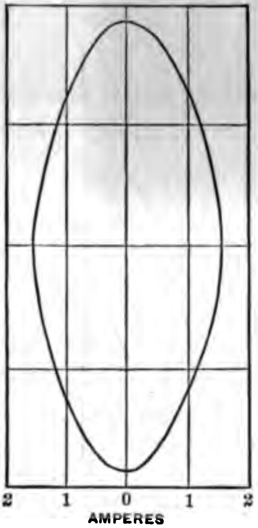


Fig. 208

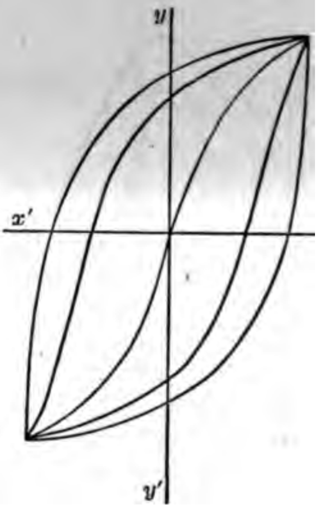


Fig. 209

due to foucault currents at any moment is equal to
corresponding instantaneous value of the foucault
rent loss divided by the instantaneous value of
primary pressure. The foucault current loss ma
represented by the symmetrical cycle shown in
208, where the ordinates and abscissas are respect
proportional to the magnetism and its rate of ch

e area of this cyclic curve is proportional to the total cault current loss. Its effect on the total core loss y be shown by plotting a cycle having abscissas ıal to the arithmetical sum of the corresponding ;cissas of the hysteresis and eddy cycles (Fig. 209). ıally, the total transformer exciting current may be tted from this as shown by *C′* in Fig. 207. It is

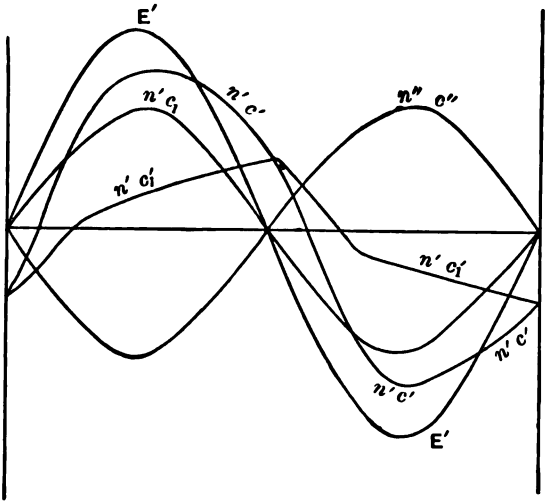


Fig. 210

ıdent that the hysteresis and eddy components of the citing current add directly together, giving the total tive component C_w, which is nearly sinusoidal when e impressed pressure is sinusoidal. If the foucault rrent cycle has a large area, its effect may cause a :kwards displacement of the maximum point in the :iting current.

ıWhen the secondary circuit is closed, the form of the primary current is changed by the effect of the secondary current. In Fig. 210, curve E' represents the primary pressure curve, $n'C'$ represents the exciting current times the primary turns. Now, if by closing the secondary circuit through the non-inductive resistance, a current C'' is caused to flow, and its ampere-turns may be represented by the curve $n''C''$, the effect of the current flowing in the secondary is to cause a corresponding increase in the current flowing in the primary (Sect. 114), and the ampere-turns of this increase may be represented by curve $n'C_1$. The total primary wave is similar to curve $n'C'$, which is the sum of $n'C_1'$ and $n'C_1$, and the primary current may be directly shown from this curve by a simple change of scale. It is thus shown by these figures that the

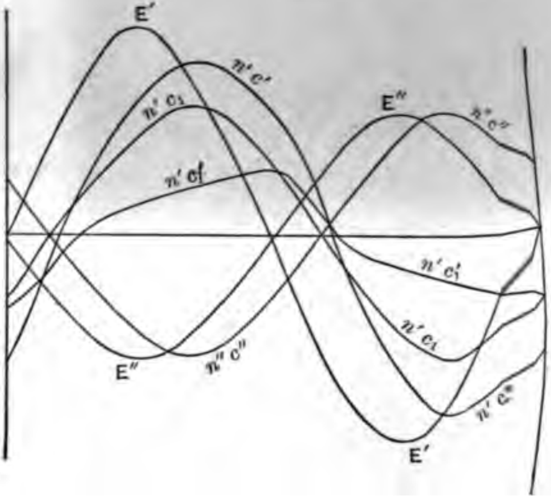
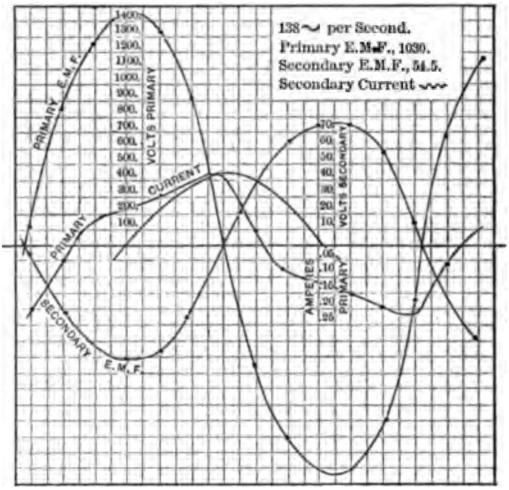


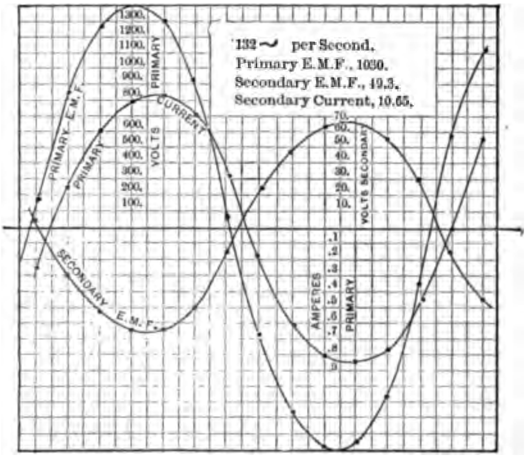
Fig. 211



OPEN CIRCUIT.

Fig. 212

SECONDARY CLOSED THROUGH 10 LAMPS.



FULL LOAD.

Fig. 213

secondary current, when in phase with the secondary pressure, tends to reduce the distortion and lag of the primary current, exactly as has already been proved analytically (Sect. 114). If the secondary circuit were inductive, the effect would be altered so that the lag of the primary current would be larger, as shown in Fig. 211, an [illegible] been proved (Sects. 117 and 118). [illegible] 213 show transformer curves expe [illegible] ed by Professor Ryan,* and which s [illegible] esemblance to the hypothetical curv [illegible] he loss cycles.

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* [illegible] . *E.*, Vol. 7, p. 71.

CHAPTER XI.

EFFICIENCY AND LOSSES IN TRANSFORMERS.

21. Transformer Core Losses and Magnetic Densi- .—The commercial efficiency of a transformer is ratio of the electrical output of the secondary coil he corresponding power absorbed by the primary . It may be written

$$\eta = \frac{W''}{W'} = \frac{W''}{W'' + L},$$

re W' and W'' are the power absorbed and de- red respectively by the primary and secondary coils, l L is the total loss in the transformer. This total ; is made up of the C^2R losses in the primary and ondary coils and the losses due to hysteresis and cault currents in the core. The C^2R loss in the ondary winding is directly proportional to the square he load (secondary output), while the C^2R loss in primary is nearly proportional to the square of the , though it contains a small approximately con- t term due to the exciting current (Fig. 214). The eresis and foucault current losses, which together titute the **Iron Losses** or **Core Losses**, have been shown experimentally to be independent of the load.* The hysteresis loss is directly proportional to the frequency and approximately proportional within the limits of magnetic density used in practice to the 1.55 or 1.6 power of the magnetic density. The foucault current loss is proportional to the square of both the frequency and the magnetic density. Consequently, for fixed values of the iron losses in transformers designed for use with different frequencies, the magnetic density should vary inversely with some power of the frequency between one and a half and two. The table in Section 97 gives satisfactory values of magnetic densities to be used in transformers for various frequencies, though the values there given are commonly exceeded in American transformers.

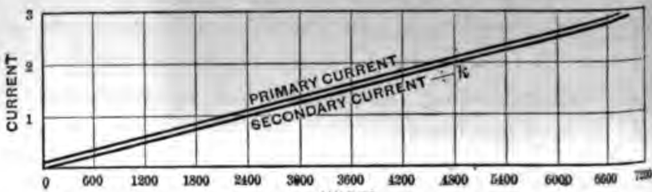

Fig. 214

The actual magnetic densities aimed at in recent transformers of one large maker may be represented, for a frequency of 133, by the following formulas and the first curve shown in Fig. 215. Where the output is below 1500 watts, $B_{\max} = \frac{250{,}000}{\text{Output}} + 3330$, and when the watts

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* Ewing, The Dissipation of Energy through Reversals of Magnetism in the Core of a Transformer, *London Electrician*, Vol. 28, p. 111; Ewing and Klaassen, Magnetic Qualities of Iron, *London Electrician*, Vol. 32, p. 713.

utput is above 1500 watts, $B_{max} = 3500$. These trans-
ormers may be used on circuits having frequencies
rom 60 to 135, in which case magnetic densities are
nversely proportional to the frequencies. In older
ransformers, where poorer iron was used, the densities
imed at by the same maker are shown by the second
urve of Fig. 215.

The magnetic densities in the transformers of another
arge manufacturer lie between 3600 and 2800 at a
requency of 125, in transformers of capacities between
00 and 30,000 watts. Similar magnetic densities are
imed at in the transformers of a third large manu-
acturer, and all successful American manufacturers
eep pretty closely within these limits.

The percentage which the core losses bear to the out-
ut in well-designed transformers varies greatly. The
verage for transformers not smaller than 6 K.W.
apacity and not larger than 20 K.W. capacity, may
e said to range between ¾ and 2 per cent. For smaller
ransformers, this percentage increases. For 3 K.W.
ransformers 2 per cent is a fair value, though 3 per
ent is exceeded in some transformers of this size; and
r 500 watt transformers 5 per cent is not bad.

First class transformers should have core losses not
xceeding the following: 1 K.W., 30 watts; 1½ K.W.,
0 watts; 2 K.W., 50 watts; 2½ K.W., 60 watts;
K.W., 80 watts; 6½ K.W., 100 watts; 17½ K.W.,
50 watts. Intermediate sizes will have proportional
sses. In some transformers these figures are bettered
the higher commercial frequencies, as in the case of
o of 7500 watts capacity, built by different makers,

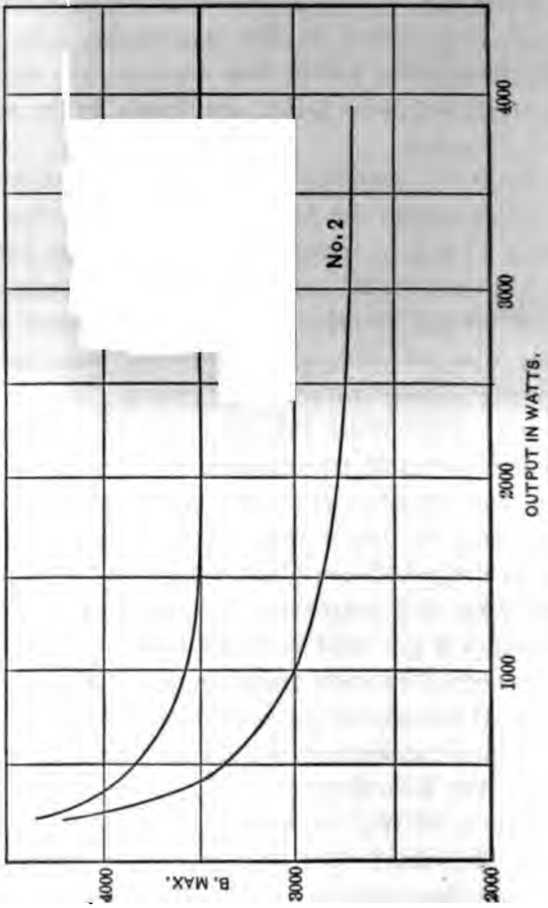

Fig. 215

/hich the core losses varied from 75 to 125 watts :nding on the frequency, the test frequencies being nd 125.*

he following table of the exciting currents of good sformers is taken from the results of numerous ; by Professor Ryan.†

Capacity.	Exciting Current.	Approximate Per Cent of Primary Full Load Current.
250	.040 Ampere	14.
500	.050 "	7.2
1000	.055 "	5.0
2000	.080 "	3.8
6500	.100 "	1.5
17500	.200 "	1.1

ıe data of this table were gained from tests of sformers of various makers designed for a primary sure of 1000 volts at a frequency of 133, but are er high for the better grade of transformers.

2. Copper Losses. — The C^2R loss in transformers rdinarily between 1½ per cent and 3½ per cent. ; is divided with approximate equality between the ıary and secondary windings. Sometimes this loss :rmitted to reach 5 per cent, but in the better transıers it is more often between 2 per cent and 3 per . The primary and secondary coils of good com:ial transformers of later design are so disposed

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Bull. Univ. of Wisconsin, Vol. 1, No. 11; Jackson, *Electrical nal*, Vol. 1, p. 78, and *N. Y. Elect. Engineer*, Vol. 20, p. 183.

Ryan, The Efficiency of Alternating Plants, *N. Y. Elect. Engineer*, 13, p. 12.

that the magnetic leakage is practically negligible, though in earlier transformers this was not true.* Consequently, the change in the ratio of transformation causing a drop in the secondary pressure as the load increases, is practically all caused by the copper losses. If the total C^2R loss is equally divided between the primary and ngs, and magnetic leakage is negligibl relations exist for transformers hav ircuit wholly of iron:

$$\frac{R'}{R''}=k^2,$$

$$\frac{L'}{L''}=k^2,$$

$$M=\sqrt{L'L''}=\frac{L'}{k}=L''k,$$

and, approximately,

$$\frac{C'}{C''}=\frac{1}{k}+\frac{C'_1}{C''},$$

where k is the ratio of transformation. $\frac{C'_1}{C''}$ is usually very small compared with $\frac{1}{k}$.

123. Rise of Temperature and Radiating Surface.—The windings of transformers are usually embedded largely in the iron core, and the whole transformer is enclosed in a water-proof iron case; and their rise of temperature is as much due to the heating of the core by core losses as to the copper losses. If transformers were placed in the open air, the entire external surface could be assumed to be effective in dissipating heat by radiation and convection, but on account of he enclosing case, convection from the surface can-ot take place, and all the heat must be radiated to the all of the case, or conducted thereto through the poor eat conductors which are used to electrically insulate he transformer from its case. The conditions there-re point to the conclusion that for a given liberation f heat per square centimeter of surface, the tempera-ure is likely to be higher in transformers than in ynamo fields. On the other hand, transformer coils ay always be designed to be lathe wound, and there-re may be more effectually insulated than dynamo eld coils; and as space is not so valuable in transfor-ers, more liberal use may be made of mica, varnished nvas, fibre, and wood. It is therefore possible to fely run transformers with the windings at a consid-ably higher temperature than dynamos, and 60° Centi-ade (108° F.) may be set as a safe limit to the rise in mperature. A high temperature limit has a marked sadvantage in causing an undue drop in pressure as e transformer heats up, by increasing the resistance the windings, and while many transformers exceed e temperature limit named, many of the best types oid the difficulty from drop in the pressure by not ceeding 40° rise. As the rise in temperature also reases the electrical resistance of the iron core, it creases the foucault current loss, so that, as sug-sted by Elihu Thomson, it would be advantageous have the core of a transformer operated at a high perature while the windings were kept cool. This not be conveniently arranged in small transfor-rs, but the cooling of the conductors of very large transformers has been effected by making the conductors tubular and passing a cool liquid through them.

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* Ryan, *Trans. Amer. Inst. E. E.*, Vol. 7, p. 11.

It is practically impossible to fix any averages for the external surface of transformers per watt lost in the core and windings, on account of the very varied arrangements of the coils with reference to the core, and the effect of the containing case. For small and medium transformers it is usual to make the design as compact as possible, and no particular trouble from heating is experienced if the losses are not excessive, since the losses ought to be quite small. In large transformers the same plan may be adopted, and some device may be arranged for cooling the conductors, such as circulating a liquid through them or blowing air through ducts in the core. Figure 215 *a* shows the rise of temperature of the conductors as a function of the period of operation of a 200 K.W. air-blast transformer with and without the blast. Point *D* shows the temperature of the discs after the transformer had been operated seven hours with blast on; curve *A* shows the temperature of the windings when operated at full load without blast; curve *B* shows the temperature of the windings when operated at full load with blast of 1040 cubic feet per minute; and curve *C* shows the temperature of the air issuing from the transformer. The core and windings of some transformers are immersed in oil, which fills the case, so as to give a better opportunity for the heat to escape.

124. Current Density in Transformer Conductors.—The current density in transformer windings varies between 1000 and 2000 circular mils per ampere. It is not unusual to make the density somewhat smaller in the secondary than in the primary, and the values in the best transformers frequently fall between 1000 and 1500 circular mils per ampere for the primary coil, and between 1200 and 2000 for the secondary coil. On the other hand, some designers make the density of current greater in the secondary windings, while others make the density about the same in each. As the primary

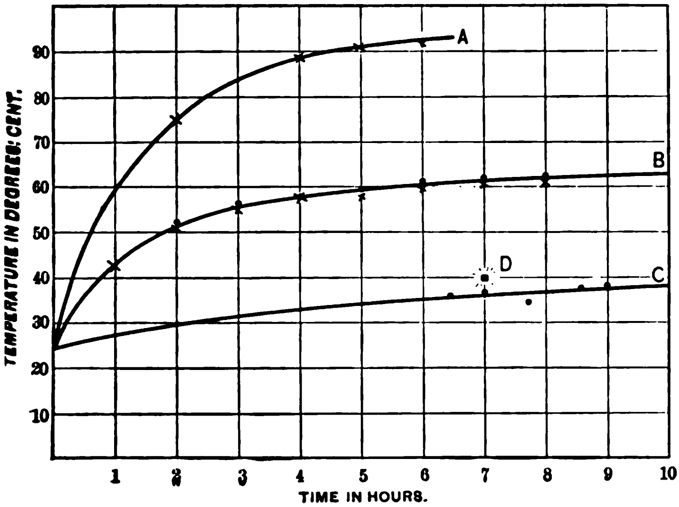

Fig. 215 a

ire of nearly all commercial transformers is much maller than the secondary conductor, insulation takes p much more space in the primary coil, and this coil ccupies more space than the secondary coil unless the rimary current density is considerably greater.

125. Testing Transformers.—Methods for testing the fficiency of transformers and for determining their ore losses have received a large amount of attention.

The more important methods are explained in the following pages.

Transformers are ordinarily worked on loads composed of incandescent lamps, which are practically non-inductive. Consequently there is no difficulty in determining the power in the secondary circuit, since
the indicati mperemeter and voltmeter
are sufficien n watts is given by the
product of t ıce the current and press-
ure agree in omposed of a liquid resist-
ance serves the load is composed of a
non-inductiv which is not appreciably
heated by the current deliv ed by the transformer, the readings of the amperemeter and voltmeter may be checked by comparing their indications with the measured values of the resistance.

The measurement of the power absorbed by the primary is not so simple, since the current and pressure do not agree in phase. The same is true of the measurements in the secondary circuit if the load is reactive.

1. *Ryan's and Merritt's Method.* — One of the earliest thorough tests on commercial transformers was that carried out by Professors Ryan and Merritt in 1889.*

In this series of tests the curves of current and electric pressure were determined by Ryan's method (Sect. 78, 4), and the effective values of these quantities and of the power in the circuit were determined from the curves, by the first method given in Section 81, except in the case of the secondary current, which was directly measured by an amperemeter. The connections are shown in Fig. 216; *T* is the transformer, *MM* are the primary leads to the transformer, *LL* are incandescent lamps connected as load to the secondary, *KK* is a non-inductive resistance connected across the primary leads, *I* is an amperemeter, *A* is a contact maker, *E* is a Ryan electrometer with accompanying devices, and *GG* is a series of switches. The object of these switches is to

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* *Trans. Amer. Inst. E. E.*, Vol. 7, p. 1; *Elect. World*, Vol. 14, p. 419.

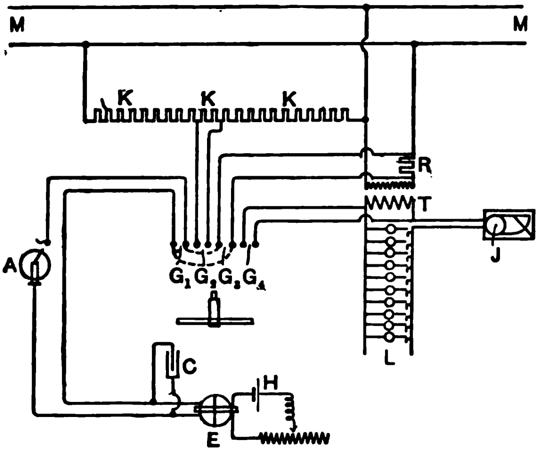

Fig. 216

onnect the contact maker alternately with different parts f the circuit, so that the curves of primary and secondry pressure and primary current may be traced.

The curve of pressure at the terminals of the nonnductive resistance *R* is evidently the same, if taken o a proper scale, as the curve of secondary current. In order to avoid handling the large primary pressure t the contact maker and electrometer, the calibrated resistance *KK* is used. The curve of pressure taken between any two points on this resistance, when a proper scale, is evidently the same as the cur total pressure. For the non-inductive resistanc and *KK*, Ryan and Merritt used incandescent l which they had "rated."

These tests were carried out on a ten-light (500 transformer when operated with the secondary ci

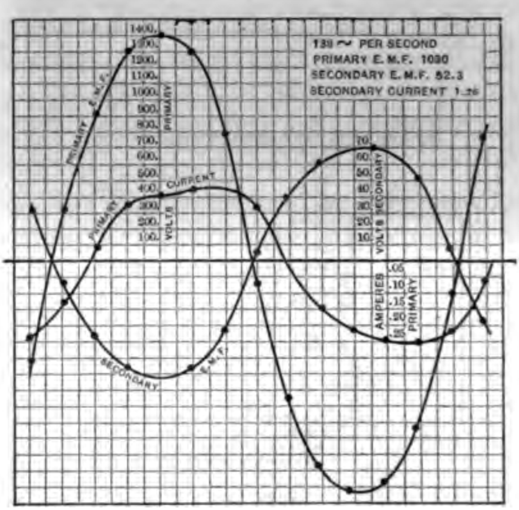

Fig. 217

open, and with the secondary loaded respectively one lamp, with five lamps, and with ten lamps. curves gained under these conditions are given in 212, 217, 218, and 213.

In addition to the determination of the curv current and pressure, the various quantities were directly measured. The effective values deduced the curves agree quite closely with those directly mined, but are of no use in determining the performance of the transformer, unless the phase differences of pressures and currents be determined in some manner. This is practically done by using the curves.

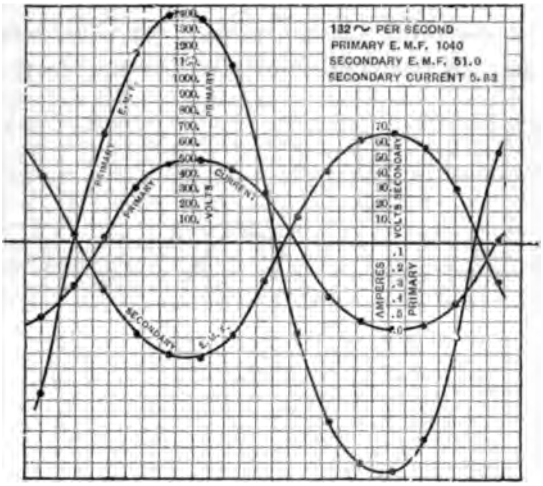

Fig. 218

The results given by these tests reduced to a uniform primary pressure of 1020 volts are as follows:

	No Load.	1/10 Load.	1/2 Load.	Full Load.
Secondary pressure	52.3	52.3	50.1	47.5
Watts absorbed by primary	96.1	159.1	388.6	607.9
Watts delivered by secondary	0.0	64.3	300.9	525.0
Commercial efficiency	0.0	41.1	77.5	86.6
Total loss in watts	96.1	94.8	87.7	82.9
C^2R loss in primary	0.4	0.9	3.3	8.7
C^2R loss in secondary	0.0	0.0	1.3	4.5
Core losses	95.7	93.9	83.1	69.7

half load, .96 per cent; full lo

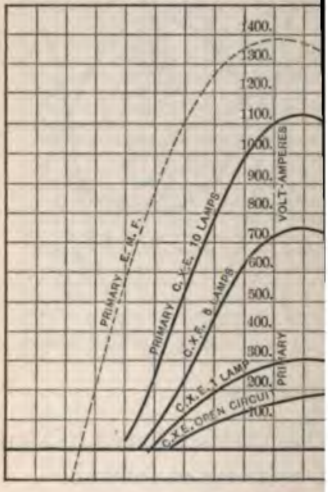

Fig. 219

power factors in the different ca
per cent, 92.2 per cent, 98.1 per

ross-section of the transformer is given in Fig. 220, hich shows the arrangement of the primary and secndary coils. In this transformer the magnetic leakage mounted to 1.2 per cent on open circuit and increased ith the load to as much as 5.4 per cent on full load. t the same time the secondary pressure wave fell back rom exact opposition to the primary pressure wave 180° lag) to nearly 190° lag.

In Fig. 218½ are given the curves of a Stanley transrmer with a capacity of 17500 watts, when the second-

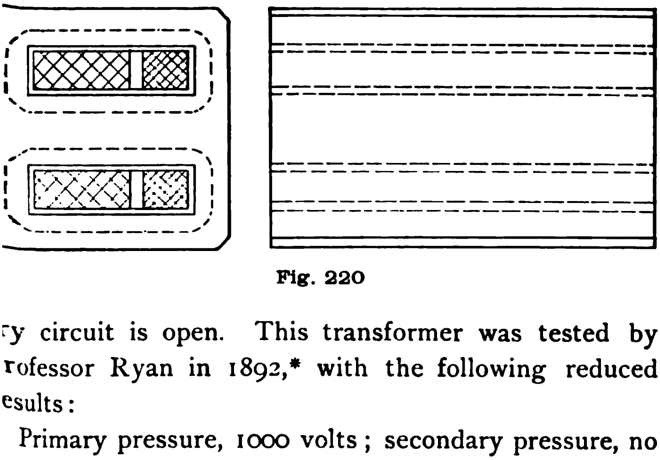

Fig. 220

y circuit is open. This transformer was tested by rofessor Ryan in 1892,* with the following reduced esults:

Primary pressure, 1000 volts; secondary pressure, no oad, 50.80 volts; full load, 49.67 volts; drop, 1.16 volts r 2.32 per cent; efficiency at full load, 96.9 per cent; t half load, 97.1 per cent; one-quarter load, 96.0 per ent; and one-eighth load, 93.0 per cent; exciting urrent, .195 amperes; power absorbed with secondy open (core losses), 137 watts; magnetic leakage practically negligible. The frequency on which t transformer was tested was practically 133. The pr portionately small value of the magnetizing compone

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* *N. Y. Elect. Engineer*, Vol. 14, p. 298.

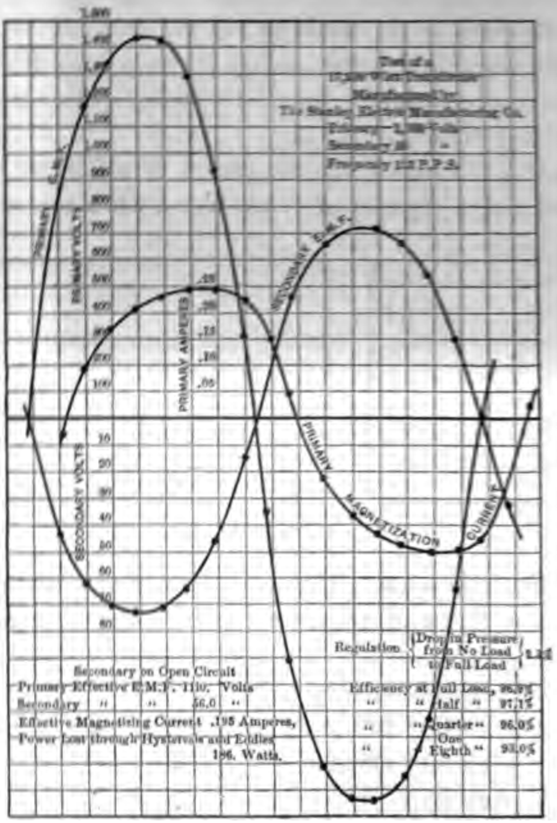

Fig. 218½

of the exciting current in this transformer (the pow factor shown by the results marked on the figure is . decreases the distortion of the curve of exciting curr

id causes its maximum point to be displaced to a posi-
n in advance of the position of zero pressure (Sect. 120).

2. *Hopkinson's Method.* In the preceding method
e losses and efficiencies are determined by taking dif-
rences between measured quantities which are of con-
derable magnitude, each of which may be affected by
nsiderable errors of observation, which errors enter
rectly into the value of the difference. The differ-
ces thus obtained are therefore not reliable.

Dr. John Hopkinson modified the method by using
o similar transformers, connected (Fig. 221) so that
e transformed up the pressure supplied to its thick
ire coil, and the other transformed this pressure down
ain. The respective pressures and currents in the
w-pressure coils of the two transformers when thus
ranged are therefore nearly equal. By measuring
eir differences and determining their phase relations,
e losses in the two transformers are obtained. Assum-
g the two transformers to be similar, one-half the total
ss is due to each. The efficiency is then obtained
m this loss measurement and a measurement of the
rrent and pressure in the secondary circuit of the
cond transformer, which consists of a non-inductive
sistance. To determine the differences and their
ase relations, a contact maker and electrometer were
ed (Sect. 78, 3).

The connections were arranged as shown (Fig. 221)[*]
d a series of curves were obtained for various loads
ich are shown in Fig. 222 (*a*, *b*, *c*, and *d*).

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[*] Hopkinson's *Dynamo Machinery and Allied Subjects*, p. 187; *Elect.
rld*, Vol. 20, p. 40.

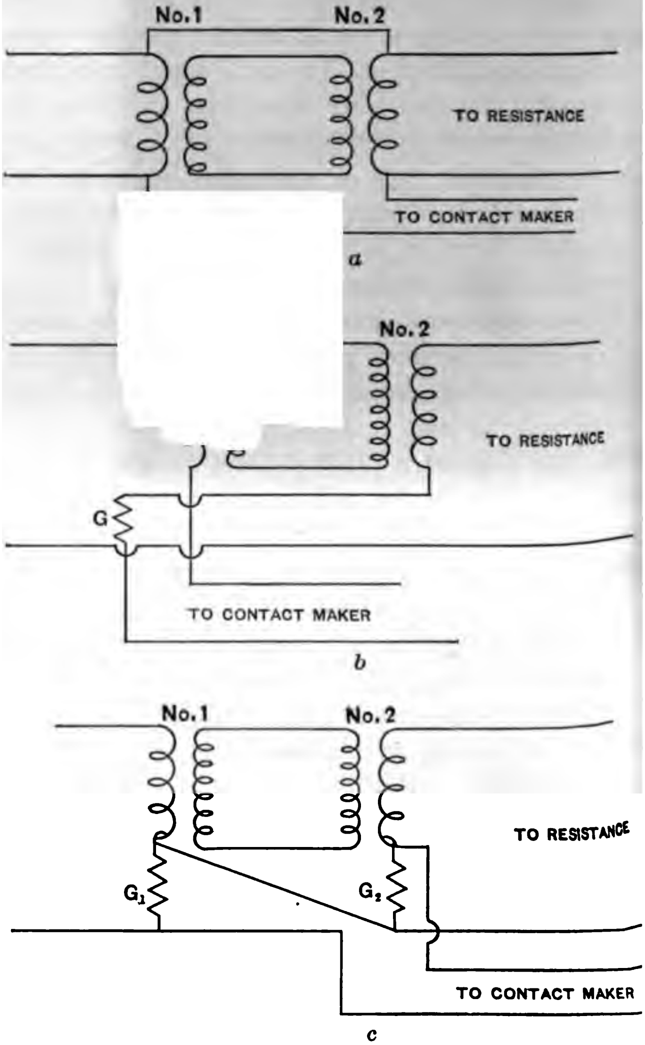
No. 1
No. 2
TO RESISTANCE
TO CONTACT MAKER
a
No. 2
TO RESISTANCE
G
TO CONTACT MAKER
b
No. 1
No. 2
TO RESISTANCE
G_1
G_2
TO CONTACT MAKER
c

Fig. 221

'he results of these tests showed the efficiencies of stinghouse transformers of 6500 watts capacity to be

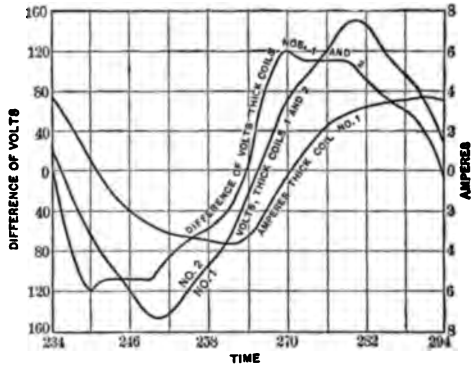

Fig. 222 a

' per cent at full load, 96 per cent at half load, and per cent at quarter load; the drop of pressure veen no load and full load to be between 2.2 and 3

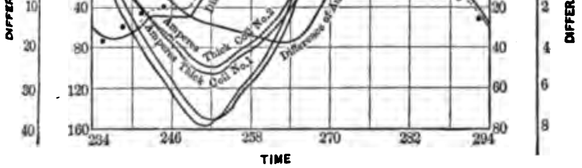

Fig. 222 b

cent; the core losses in each transformer to be t 114 watts; the magnetic leakage in one transformer appeared to be small, but in the other to be co siderable.

3. *Mordey's Method.* Mordey suggested the follo ing method for determining the losses in a transform at any desired load:* The transformer to be tested worked at its normal load until a constant temperatu is reached which is determined by a thermometer.

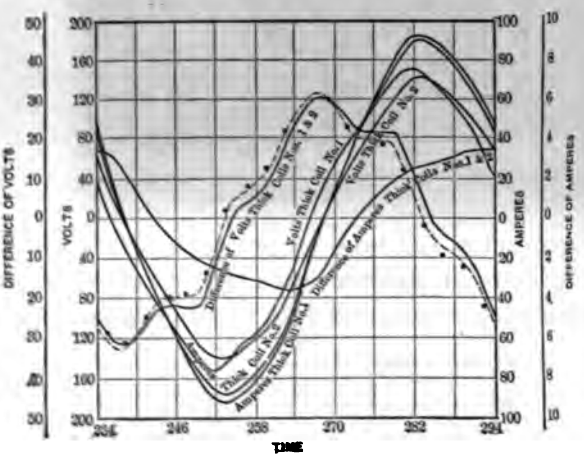

Fig. 222 c

continuous current is then passed through the windin of such a magnitude that the C^2R loss from this cu rent is sufficient to maintain the temperature of t transformer. The continuous current C^2R loss is th equal to the total losses with the alternating curre The continuous current C^2R loss is readily determin by voltmeter and amperemeter, or by wattmeter. T sts by this method are laborious and impractical on :count of the time required.

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* Alternate Current Working, *Jour. Inst. E. E.*, Vol. 18, p. 608.

4. *Calorimetric Method.* This method has been used y many experimenters.* The transformer is taken :om its iron case and sealed up in a tin or copper case 'hich is immersed in a water calorimeter (Fig. 223). 'he loss in the transformer at any desired load on the

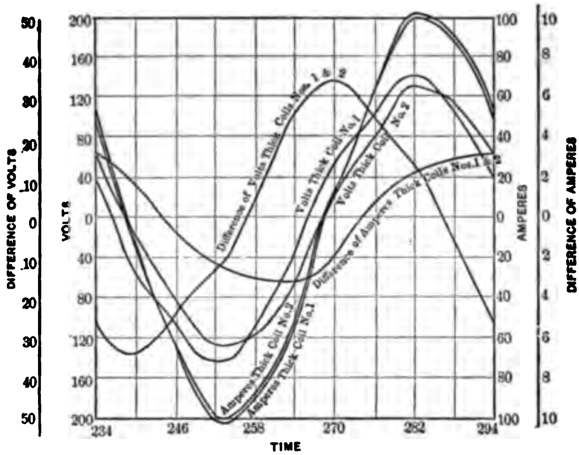

Fig. 222 d

:condary is determined from the rate at which the ater in the calorimeter rises in temperature, provided e heat absorbed by the transformer and calorimeter ater equivalent), and the rate of loss of heat from the es of the calorimeter, have been determined, so that roper correction may be applied to the results. The :ermination of the elements entering into this correction is ikely to introduce considerable errors. Tl are deci eased somewhat by using oil in the calorim

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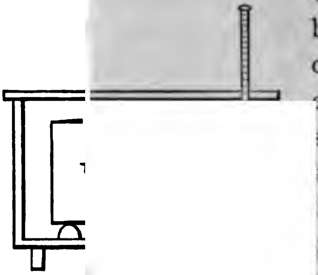
* Duncan, *Electrical World*, Vol. 9, p. 188; Roiti, *La Lumière* *trique*, Vol. 35, p. 528.



when the transformer be directly immersed w out the enclosing case. errors due to the heat sorbed by the transfor and calorimeter may be e nated by arranging the paratus so that a slow stant flow of water is pas through the calorimeter (l 224). Th entering water should have a const temperature. If the flow of water is of const volume, and the transformer has attained a c stant temperature, there will be a constant differe of temperature between the water at entrance :

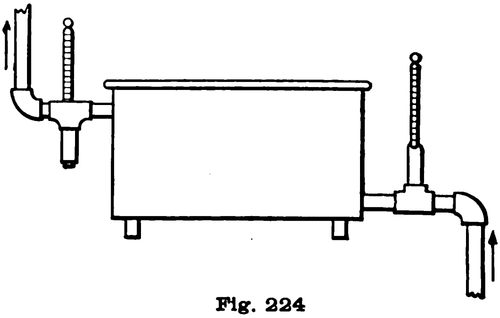


Fig. 224

exit. This may be measured by thermometers. F this difference of temperature and the weight of w flowing per minute through the calorimeter, the en expended in the transformer and given up to the w may be deduced. A correction must be applied on account of the loss of heat due to radiation from the calorimeter. A more satisfactory arrangement of the calorimeter test is to place the hermetically sealed transformer in a vessel with ice packed around it. If the vessel is properly protected from external heating effects, the melting of the ice will practically all be due to heat from the transformer, and the losses in the transformer may be determined from the amount melted in a given time. A correction must be applied for the melting before the transformer has reached a constant temperature.

It is difficult, at best, to get very satisfactory results from the calorimetric methods, on account of the difficulties inherent in the exact determination of temperatures or of heat losses. The best method of work is probably to directly calibrate the calorimeter by inserting in the place of the transformer a known resistance and passing through it such a continuous current as will give the same heating effect as the transformer. Then the energy absorbed by the resistance is equal to the transformer losses.

5. *Split Dynamometer Method.* The split dynamometer may be used for directly determining the power absorbed by a loaded transformer if the magnetic leakage is negligible. In this case an electrodynamometer s connected with one coil in each circuit. Then, practially, $e' = R'c' + \left(\frac{n'}{n''}\right)e''$. But $e'' = c''(R + R'')$, where ', e''; c', c''; n', n''; and R', R'' are respectively the ressures, currents, turns, and resistances in the pri-

mary and secondary coils, and where R is the resistance of the external secondary circuit. Then

$$e'c' = R'c'^2 + \frac{n'}{n''}(R + R'')c'c''.$$

The energy absorbed by the primary circuit is, therefore,

$$W' = \frac{1}{T}\int_0^T e'c'\,dt = \frac{R'}{T}\int_0^T c'^2 dt + \frac{n'}{Tn''}(R + R'')\int_0^T c'c''dt.$$

The integral of the first term of the right-hand member is equal to the square of the primary current, and that of the second term is $C'C''\cos\phi = kD$, where D is the reading of the split dynamometer and k is its constant (Sect. 44, 5). Consequently,

$$W' = C'^2R' + \frac{n'}{n''}(R + R'')kD.$$

As already said, this method is only accurate when magnetic leakage is negligible.

6. *Voltmeter and Ammeter Methods.* The power in the primary and secondary circuits may be measured by any of the methods given in Section 44 for measuring the power in an alternating-current circuit by means of voltmeters and amperemeters. The numerical efficiency will be found from the ratio of the secondary and primary energies, and the transformer losses from their difference.

7. *Wattmeter Method.* Where satisfactory wattmeters are at hand, it is evident that one may be placed in the primary circuit of a loaded transformer and another in the secondary circuit. Then the ratio of their readings is the efficiency of the transformer. Or,

wattmeter may be used in the primary circuit and an mperemeter and voltmeter may be used to determine he output, if the transformer is worked on a non-inducive load. The wattmeter was used by Fleming in a ery extended series of transformer tests, and found to e more satisfactory than either of the methods in which amperemeters and voltmeters are used to determine the energy absorbed by the primary circuit.*

8. *Stray Power Methods.* A very convenient method of measuring the efficiency of transformers is to determine the various losses directly, and thence the efficiency. The iron losses may be determined by measuring with a wattmeter the power absorbed by the transformer when the secondary circuit is open. The copper losses for any load are readily calculated when the secondary and exciting currents and the primary and secondary resistances are known. The exciting current may be measured at the same time that the iron losses are determined, and the resistances may be measured by a bridge. For a given load, the secondary current is a fixed quantity. The efficiency is then, practically,

$$\frac{W''}{W''+I+C''^2R''+\left(\frac{C''}{k}+C_1'\right)^2R'},$$

where I represents the measured iron losses and k the atio of transformation.

8 a. A still more convenient method, which may be eadily used in central stations for testing transformers, to measure the iron losses by a wattmeter, as ex-

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* *Jour. Inst. E. E.*, Vol. 21, p. 623; *Elect. World*, Vol. 20, p. 413.

plained above. The copper losses may then be measured by short-circuiting the secondary through an amperemeter, and adjusting the primary pressure until the full load current, or any desired fraction thereof, passes through the amperemeter. The reading of a wattmeter in the primary circuit is now nearly equal
to the copp e pressure and maximum
magnetic de y small, and the iron losses
are therefor ely negligible. The exact
copper loss mined by measuring and

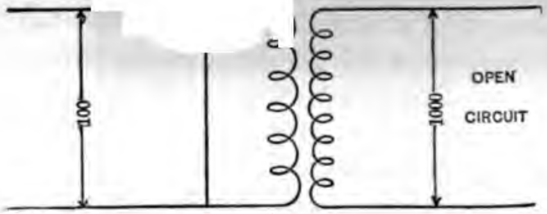

Fig. 225

correcting for the small iron loss. The tests for the iron losses may be most conveniently made by using the low resistance coil of the transformer as the primary coil (Fig. 225).

This method of testing may be used with satisfaction where numerous transformers must be tested, since the losses and efficiency may be determined expeditiously and with the expenditure of little power. When combined with a run of several hours with full load current,
he secondary circuit being made up of impedance coils,
e method proves very economical for shop tests. The

od was adopted by Mr. A. H. Ford for a very com-
series of tests of American transformers.*
. Ayrton and Sumpner have devised a method
so serviceable as the last, but still quite useful,
hich two transformers of the same size and make
pposed to each other. The method of connecting

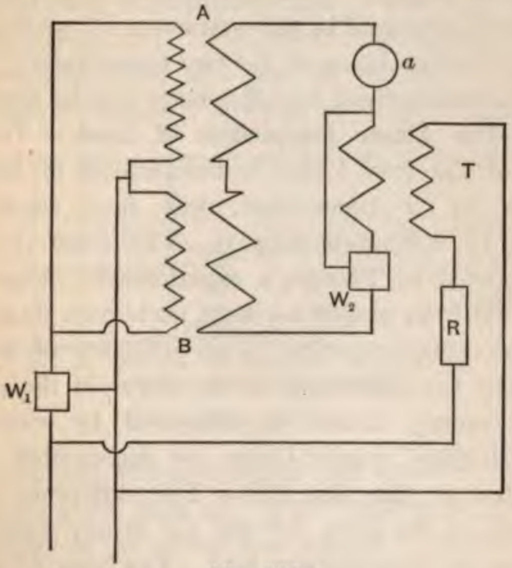

Fig. 225 a

as in Fig. 225 *a*, in which *A* and *B* are the trans-
ers with their primaries connected in relatively
site directions to the leads and their secondaries in
s. The pressures of the secondaries are thus op-
d. A transformer *T* is inserted with its secondary

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ests of American Transformers, *Bull. of Univ. of Wisconsin*, Vol. 1, in circuit with the secondaries of A and B. By varying the resistance R, the pressure of T may be regulated so that any desired current will pass through the secondaries of A and B. Then the output of T measured by the wattmeter W_2 will give approximately the copper losses of A and B plus the loss in leads and instruments. The power supplied to the primaries of A and B by the alternator, measured by the wattmeter W_1, gives approximately the iron losses of the two transformers. From the data thus derived the efficiencies may be obtained.

126. Iron Losses Independent of Load. — That the value of the iron losses is independent of the load carried by a transformer, was first conclusively proved by Ewing (Sect. 127). The same thing has been proved by Fleming's experiments. Figure 226 is plotted from one of his tests made on a transformer of 4000 watts capacity. The ordinates of line AB represent the differences of the power in the primary and secondary circuits as measured by wattmeters. The calculated copper losses are represented by the ordinates of the line CD. The difference of the ordinates of the lines AB and CD at any point is the iron loss for the particular load. The lines AB and CD are approximately parallel, which shows that the iron losses are practically constant, regardless of the load. Therefore the stray power methods of testing transformers give efficiencies which are entirely reliable.

127. Ewing's Experiment showing that Iron Losses are Constant. — Ewing's very neat plan for proving this point was designed to get at the matter directly. Two small transformers were made up exactly alike, the of which consisted of insulated iron wire wound he form of a ring. Over this were uniformly two layers of wire making a primary coil, and r two layers making a secondary coil. In operate primary and secondary coils were respectively ted in series, but the two halves of each coil in ansformer were so connected as to be in magnetic tion (Fig. 227). The core of one transformer was re magnetized and that of the other was not. the C^2R losses at any load were equal in the two,

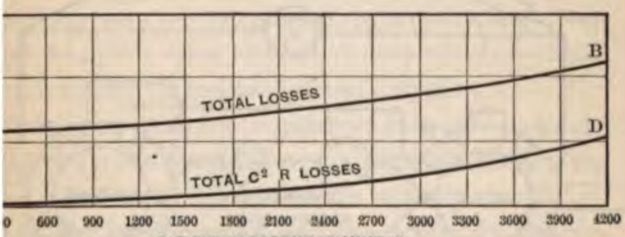

Fig. 226

nsformer with magnetized core heated more when operation than the other transformer, but the rature of the second was brought to equality with the first by passing a continuous current through sulated wire of the core. The energy expended ting the second core by the continuous current us equal to that expended in the first core due losses. The equality of temperature was deter- by means of thermo-electric couples embedded in res, which were connected in series through a ometer (Fig. 227). In this experiment it was that after a balance of temperature was once obtained it was unaltered by any changes in the loads of the transformers, thus showing that the core losses in the magnetized transformer were independent of the load.*

128. Regulation Tests. — The regulation of transformers which are used in incandescent lighting is a matter of much moment, and regulation tests are of almost equal importance to the tests of losses and temperatures. The ordinary method of making regulation tests is to place a voltmeter across the primary circuit and another across the secondary circuit of the transformer to be tested. At no load, the reduced readings

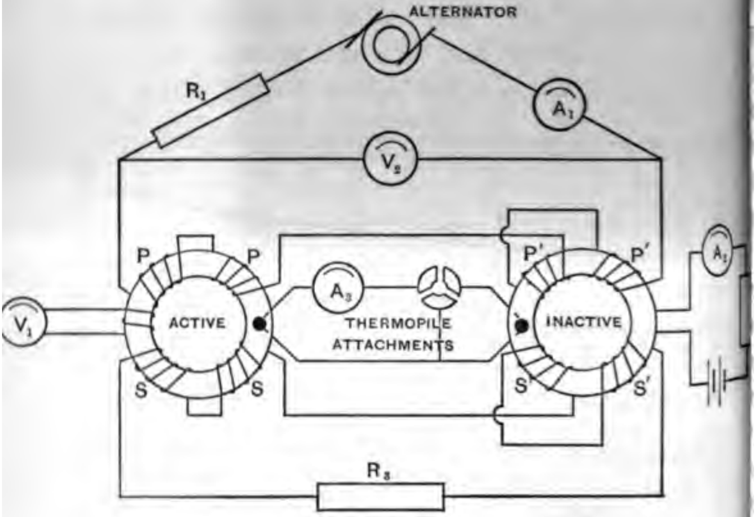

Fig. 227

[illegible] Ewing and Klaassen, Magnetic Qualities of Iron, *London Electrician*, [illegible]3.

of the instruments should be equal, and the difference between the reduced readings at any other load gives the corresponding drop of pressure in volts. The reduced readings are gained by multiplying the readings of the two voltmeters by their respective constants and dividing the reading of the voltmeter in the high pressure circuit by the ratio of transformation of the transformer. The drop of pressure, measured in this way, includes the CR drop in the windings and the drop due to magnetic leakage, which increases with the load. The magnetic leakage drop may be determined by subtracting from the total drop, the value of the CR drop which is calculated from measured resistances and currents.

A much more accurate regulation test may be made by using two transformers of equal transformation ratios and one voltmeter. The primaries are separately connected to the supply mains, and the secondary circuits are connected together on one side. A high resistance or electrostatic voltmeter is connected between the other legs of the secondary circuits. The reading of this voltmeter at any load on one transformer, when the other is unloaded, gives the total drop of pressure caused by loading the former.

Regulation tests are usually made with non-reactive loads, since good regulation is a matter of necessity in incandescent lighting circuits, which are practically non-reactive. The regulation of a transformer is changed for the worse by introducing inductance into the secondary circuit, and for the better by introducing capacity into the secondary, as has already been proven (Sects. 117 and 118). Regulation tests on a reactive load are of little service except from a comparative standpoint, or to determine whether a transformer will give a satisfactory service to both incandescent lamps and alternating-current motors or arc lamps. The combined service is seldom satisfactory on account of poor regulation which injures the incandescent lighting, though the defective regulation is not a matter of much importance to the power or arc-light service.

129. The All-Day Efficiency of Transformers and the Effect of Magnetic Reluctance.—The working efficiency of a transformer is by no means equal to its full-load efficiency. In the case of dynamos placed in a central station it is usual to divide the generating units so that the plant operating at any part of the day will be fairly loaded. In the same way the capacity of stationary motors, used for distributing electric power, may be chosen so that they seldom operate at very small partial loads. The way that transformers are usually operated, however, makes it quite difficult to keep a uniform load on them, and in fact, for a considerable portion of the day they are worked with the secondary circuit open. This being the case, the iron losses of transformers are of much greater influence on their all-day efficiency than are the copper losses, and it is desirable to reduce the iron losses to a minimum. For instance, suppose a transformer of 5000 watts capacity has an iron loss of 2.5 per cent or 125 watts, and a copper loss at full load of 2 per cent or 100 watts. Then the full-load efficiency of the transformer is 95.5 per cent, the half-load ficiency is 94.3 per cent, and the quarter-load efficiency 89.5 per cent. Supposing that the daily output of the ransformer is equivalent to 25,000 watts for one hour 25,000 watt-hours), which is equal to full-load operation or five hours and open-circuit operation for the remaining 19 hours, then the losses are equivalent in the iron core to 3000 watts for one hour, and in the copper to 500 watts for one hour, or a total of 3500 watts for one hour. The all-day efficiency is then 87.7 per cent. To increase this all-day efficiency, it is evidently necessary to decrease the iron losses. To do this with a fixed frequency, requires a decrease in the amount of iron in the magnetic circuit or a decrease of the maximum magnetic density. Either process calls for an increase in the windings and consequently in the copper losses. Suppose that decreasing the core losses to 1½ per cent makes it necessary to increase the copper losses to 3 per cent; then, other things being equal, the efficiencies become, full load, 95.5 per cent; half load, 95.7 per ent; quarter load, 93.7 per cent; and the all-day fficiency, on the assumption made above, is 90.7 per ent. There is a saving by the latter construction of 50 watt-hours in twenty-four hours, and in one year f 365 days the saving is nearly 350 kilowatt-hours. f one kilowatt-hour is worth 10 cents to the central tation, the difference in the amount of power wasted ach year by the two transformers has a value of nore than thirty-five dollars, which is several times he difference in the original cost of the two transformers. If the average load of the transformer were ess than that assumed, which is frequently the case in ractice, the iron losses would have a still greater influnce on the all-day efficiency. A still greater decrease in the iron loss with its attendant increase of copper loss would evidently raise the all-day efficiency to a higher point. Here, however, is met the question of regulation, which will not satisfactorily admit of too great a copper loss at full load on account of the attendant drop of secondary pressure, but this difficulty may be me ie cross-section of the copper. This an increase in the cost of the transfc sformer with small losses and good r more to the central station than one wı poor regulation. The all-day efficienc, umed basis, of the 17,500 watt transformer previously referred to (page 475) is 93.8 per cent, and of the 6500 watt transformer (page 479) is 91.3 per cent. The advantage of decreasing the iron losses, which is thus shown, led Swinburne to advocate and build transformers with a cylindrical iron core under the windings, but without closed iron magnetic circuits.* Decreasing the amount of iron in the magnetic circuit decreases the core losses but at the same time increases the reluctance and therefore increases the magnetizing current. This is a decided disadvantage if carried to an excess. While the true magnetizing current is wattless, and therefore requires the expenditure of no power in the transformer, yet it does result in a continuous C^2R loss in the conductors leading to the transformer and in the primary winding of the transformer itself. It also causes an extra demand for current from the dynamo supplying the circuit, so that extra generators must be operated at periods of light load in order to supply the wattless current. In other words, the power factor of the system as a whole is decreased, with an accompanying loss of **Plant Efficiency.** Finally, a large magnetic reluctance causes a considerable magnetic leakage and consequent increase in the secondary drop of a transformer, and therefore interferes with its regulation.

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* Swinburne, The Design of Transformers, *Proc. British Assoc. for the vancement of Science*, 1888, and *London Electrician*, Vol. 23, p. 492; nsformer Distribution, *Jour. Inst. E. E.*, Vol. 20, p. 163.

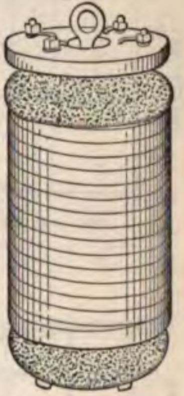

Fig. 228

The last two points may become very serious if the reluctance of the magnetic circuit of transformers is made excessive; consequently Swinburne decreased the reluctance in his transformers by making the core of iron wire, which was bent out into a spherical head (Fig. 228). From their prickly appearance these transformers have been called **Hedgehogs.** Figure 229 shows the various results of a test made by Fleming upon a 3000 watt hedgehog transformer with a ratio of transformation of 24:1, and Fig. 230 shows the power factor, at various loads, of a hedgehog transformer compared with two closed iron-circuit transformers. It has been claimed that the transformer tested by Fleming was defective and the results gained by Bedell, in testing a similar machine, were much better.*

Without entering a controversy regarding the exact point at which a high reluctance in the magnetic circuit of a transformer causes more disadvantage than is

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* *Trans. Amer. Inst. E. E.*, Vol. 10, p. 497; *Elect. World*, Vol. 22, p. 357.

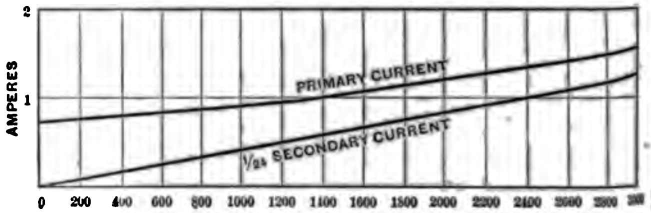

Fig. 229 a

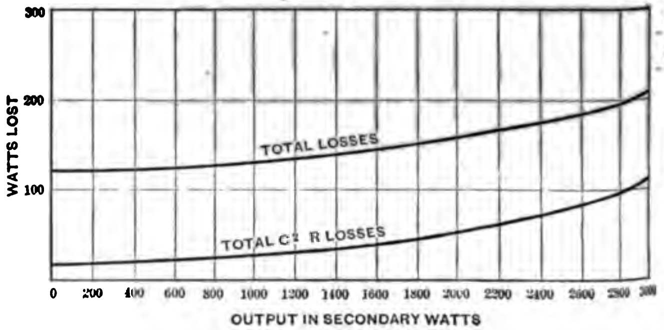

Fig. 229 b

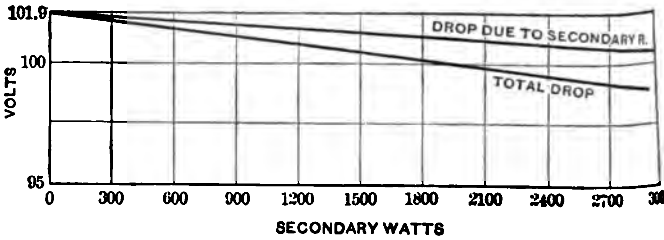

Fig. 229 c

counterbalanced by the decreased iron losses brought about by decreasing the amount of iron, the examples may serve to show the necessity of carefully designing ıe magnetic circuit to fit the conditions. Where a irly large proportion of the full load is carried by ıe transformer a great portion of the time, as may ɛ the case in many of the proposed power transmisons, the reluctance of the magnetic circuit of the ansformer should be made very small, so that the ɔpper losses may be small. On the other hand, where ıe average load is very small, as in the ordinary

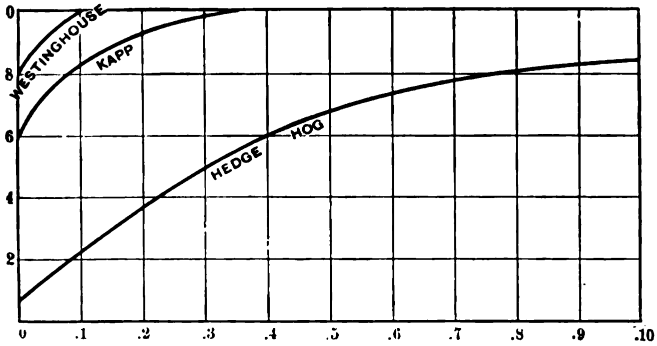

Fig. 230

rrangements of transformers for lighting service, the ${}^2R$ loss is of less moment than the iron losses, and very effort must be made to decrease the amount of iron, nd hence the iron losses, as far as the requirements f economy of construction, plant efficiency, and regulaon will permit. Transformers might even be built ithout iron in the core, were it not for the immoder:e cost caused by the large amount of copper which ould be required in their construction to gain a reaınably good degree of regulation.

The unsatisfactory features of transformers having cores of high magnetic reluctance may be largely neutralized by the use of condensers (Sect. 35). These may be put in parallel with each transformer of such a capacity as to practically supply the necessary wattless magnetizing current, or a group of condensers may be connected in parallel to each circuit of sufficient capacity to supply the wattless current for that circuit. In either case, the difficulties due to regulation and plant efficiency are obviated. Tests of condensers in parallel with transformers have shown that their use is entirely practical, provided that a cheap and durable dielectric can be found.*

130. Curves of Efficiency. — The curve showing the efficiencies of a good transformer at various loads is a very flat one. In Fig. 231 are given the curves of a 4000 watt Thomson-Houston transformer tested by Fleming, the 17,500 watt transformer tested by Ryan, which already has been referred to (p. 475), and the 3000 watt hedgehog transformer tested by Bedell which was referred to directly above (p. 495). Thes curves are flatter than the similar curves for dynamo (Vol. I., p. 270). To get the best all-day efficienc it is evident that every effort should be bent to mal ing the knee of the curve at the smallest possible loa

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* Swinburne, The Probable Use of Condensers in Electric Lighti *London Electrician*, Vol. 28, p. 227; Le Blanc, Application of Alternat Currents to the Transmission of Power, *Soc. Inter. des Électriciens*, 18 and *London Electrician*, Vol. 27, p. 383; Swinburne, Transformer Dis bution, *Jour. Inst. E. E.*, Vol. 20, p. 191, and *London Electrician*, Vol. p. 548: Bedell and others, Hedgehog Transformers and Condensers, Pt *Trans. Amer. Inst. E. E.*, Vol. 10, p. 513.

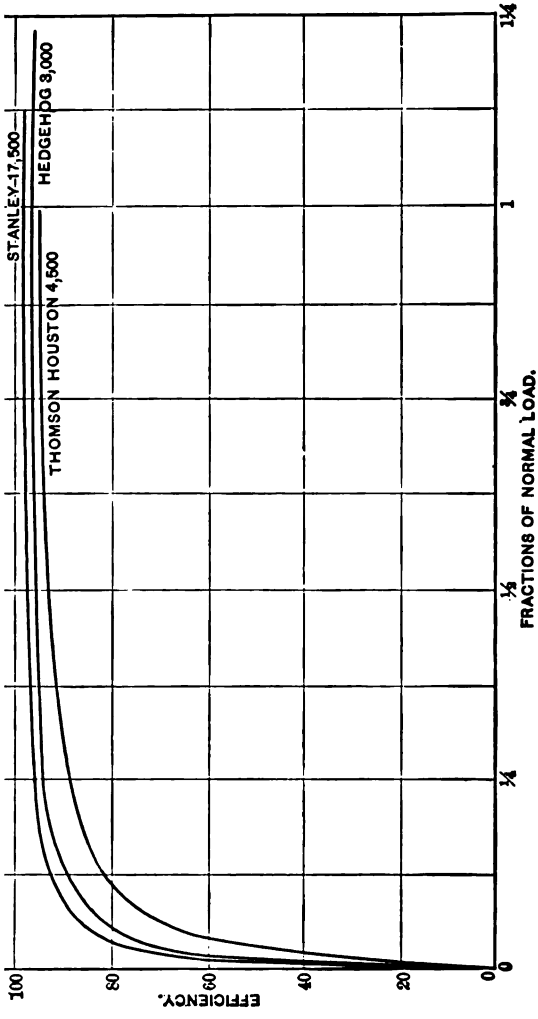
STANLEY-17,500
HEDGEHOG 3,000
THOMSON HOUSTON 4,500
100
80
60
40
20
0
EFFICIENCY.
0
¼
½
¾
1
1¼
FRACTIONS OF NORMAL LOAD.

Fig. 231

without at the same time increasing the exciting current too greatly. The maximum all-day efficiency for a given transformer comes at such a distribution of the load that the watt-hours represented by the copper and iron losses are equal. The maximum operating efficiency occurs at the load which causes copper and iron losses to be equal.

The full-load efficiency of average commercial transformers of different sizes is represented by the curve of Fig. 232. These efficiencies may be improved upon, but they represent average practice.

131. Weight Efficiency. — The total weight of transformers is exceedingly variable, as it depends not only upon the design of the machine, but also upon the containing case. The weights of copper and iron depend entirely upon the purpose of the designer, and the limits to which he has carried a desire to gain a high all-day efficiency. The total weights of transformers designed for frequencies not less than 100, nor greater than 135, will ordinarily vary between 75 and 100 pounds per kilowatt for transformers near one kilowatt in capacity, and from 40 to 60 pounds per kilowatt for transformers of 10 K.W. capacity. Figure 233 gives the total weights of the standard transformers of two well-known manufacturers, which are designed for a frequency of 135 and Fig. 234 for frequencies of 30 to 60.

132. Separation of Core Losses. — The hysteresis loss in a transformer may be considered constant for a given frequency and pressure, as indicated by Ewing's neat experiment (Sect. 127), but the foucault current loss will become less after the transformer has been run under load and thus become heated up; for, as the core rises

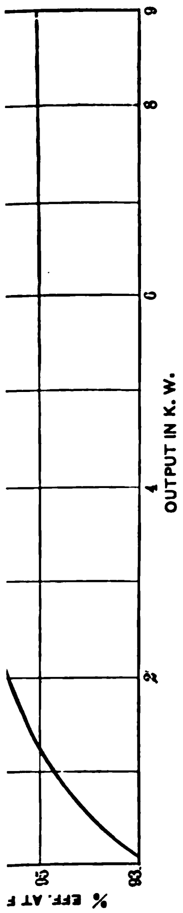

Fig. 232

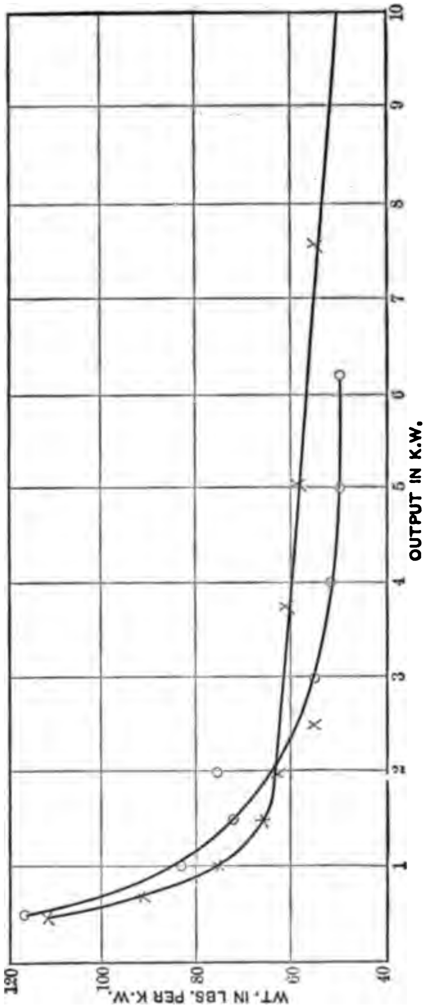

Fig. 233

in temperature, the resistance of the iron increase To separate the hysteresis from the foucault curre loss, the iron losses may be measured when the co is cold (Sect. 125 (8*a*)), and then when it has become h by being run under load. Let W_c be the first readin and W_h the second; also, let $t°$ be the difference temperatu the two readings. Then

d $W_h = H + F_h$

where H i s, and F_c and F_h the fouca current lo The foucault current loss $F = CE =$ are a current and pressu equivalent to the foucault currents and the pressur acting upon them, and R is a resistance equivalent that of the foucault current circuits combined. E constant during the test, as it depends only upon th magnetic density, the primary pressure, the frequenc and the dimensions of the core plates. Then

$$F_c = \frac{E^2}{R_c}, \text{ and } F_h = \frac{E^2}{R_c(1 + at°)},$$

where R_c is the resistance at the temperature of the cold measurement, and a is the temperature coefficient of the iron comprising the core discs. From the value of W_c and W_h, we have $W_c - W_h = F_c - F_h$, and substituting the values of F_c and F_h,

$$W_c - W_h = \frac{E^2}{R_c} - \frac{E^2}{R_c(1 + at°)} = \frac{E^2}{R_c}\left(\frac{at°}{1 + at°}\right);$$

and as $\frac{E^2}{R_c} = F_c$,

$$F_c = (W_c - W_h)\left(\frac{1 + at°}{at°}\right).$$

s all the quantities in the right-hand side of the equa- n are determined by measurement, the foucault cur- it loss may be thus separated from the hysteresis loss.

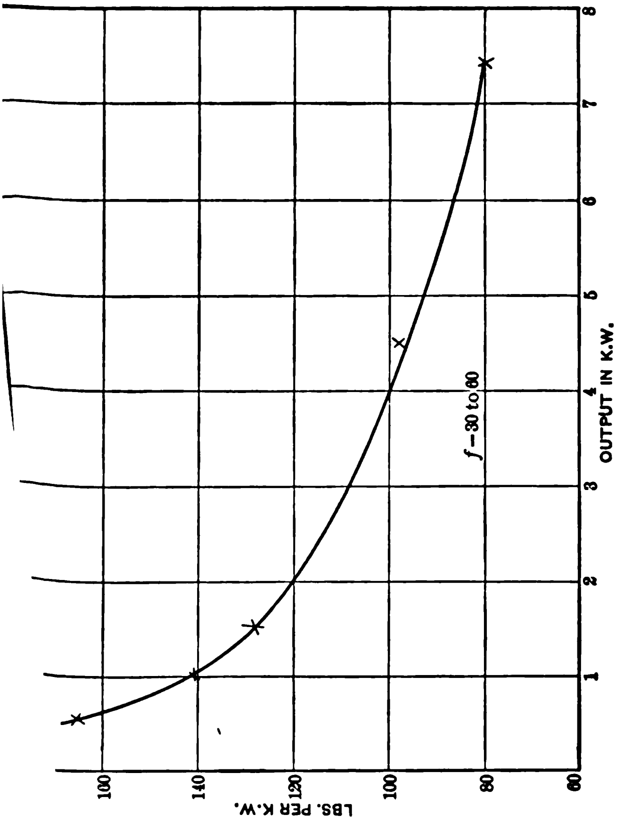

Fig. 234

·fficient *a* may be readily obtained by measuring stance between any two points on a core disc : core is at two different temperatures, when the desired will be the per cent change in resistance per degree change in temperature. In order that the results of these measurements may be reasonably reliable, the pressure applied to the transformer during the tests must be exceedingly constant. The foucault current loss in commercial transformers is but a small proportion of the total core loss. The hysteresis and foucault current losses may also be separated by measuring the core losses at two frequencies for the same pressure, then, by subtraction and an elimination similar to that given in Vol. I., p. 254, the value of the foucault current loss may be deduced.

133. Tests of American Transformers.—The most complete public test of recent American transformers is one made by Mr. A. H. Ford, in the laboratories of the University of Wisconsin, during the year 1895.*

The list included transformers of the following types: Bullard, Diamond, Fort Wayne, General Electric, Hornberger, National, Packard, Phœnix, Standard, Stanley, Wagner, and Westinghouse. The total number of transformers tested was over twenty.

In making the tests, Weston, Hoyt, Queen, and Cardew instruments were used, and their accuracy was frequently checked by comparison with Kelvin balances. Great care was exercised to eliminate errors. The tests were made at frequencies of 60 and 125; the pressure wave being rather peaked, especially at the frequency of 60.

The method used for finding the efficiency was that given in Section 125, No. 8 *a*, and the results were ecked by methods 8 *b* and 7. Method 8 *a* was found be the most accurate, but by using other methods as ecks serious errors could be detected and the test peated. As the method used measures the losses dictly, small errors of observation cause an inappreciable ror in the result. The all-day efficiencies were calcu:ted on the assumption that during each 24 hours the ansformer runs 5 hours at full load, and 19 hours at no ad. This assumption gives about the proper all-day ficiency to represent favorable conditions of present actice. The following table gives a synopsis of the sults obtained from twelve of the transformers:

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* Complete Tests of Modern American Transformers, *Bull. Univ. of Wisconsin*, Vol. 1, No. 11.

TABLE I.

Capacity in Watts.	Iron Loss.		Copper Loss.	Maximum Efficiency.		All-Day Efficiency.	
	f = 125.	*f* = 60.		*f* = 125.	*f* = 60.	*f* = 125.	*f* = 60.
1250	37.0	48.5	29.7	95.0	94.0	85.1	83.0
1500	50.5	70.6	45.2	94.6	94.0	84.8	80.6
1500	32.2	46.5	38.1	95.7	94.5	89.4	85.0
1500	57.0	82.0	35.3	93.7	92.0	83.0	77.8
1500	45.7	60.1	36.2	94.8	94.0	85.5	83.4
1500	126.0	206.0	14.8	91.5	87.4	70.8	60.0
450	23.4	38.6	6.2	91.0		76.5	
1800	53.5	108.7	66.6	94.0	91.7	84.7	75.5
2000	42.4	56.3	54.8	95.2	94.5	88.6	86.5
1500	97.5	125.0	38.5	91.7	90.1	76.5	70.2
1500	57.5	76.0	30.9	94.5	93.4	83.0	79.2
1500	43.2	55.5	28.5	95.3	94.6	86.5	83.7

Some of the transformers giving high maximum efficiencies do not give the highest all-day efficiencies, and such cases the figures would tend to indicate that an increase in the number of turns and a decrease in the iron and magnetic density would be of advantage.

The iron losses are so variable that a table was made up of the losses per cubic centimeter, from which rather better comparisons can be made. As the magnetic densities in the transformers are very different, this does n he relative qualities of the iron, hence is added of the hysteresis constants. are calculated from the formula of Ste $^{1.6}$ (see Vol. I., p. 74), where U is energy r cubic inch or per cubic centimeter c frequency, B the maximum induction (per square centimeter), and η is a constant of hysteresis which depends upon the quality of the iron. Table II. presents the reduced results.

TABLE II.

No.	Loss per cu. in.		Loss per cu. cm.		B_m		Constant of Hysteresis.	
	$f=125$.	$f=60$.	$f=125$.	$f=60$.	$f=125$.	$f=60$.	$f=125$.	$f=60$.
1	.13	.18	.008	.011	2050	4280	3.2×10^{-10}	2.9×10^{-10}
2	.21	.28	.013	.017	2600	5400	3.52×10^{-10}	3.04×10^{-10}
3								
4	.24	.34	.014	.021				
5	.24	.32	.015	.020	3640	7500	2.39×10^{-10}	2.10×10^{-10}
6	.68	1.10	.041	.067	3750	7720	6.26×10^{-10}	6.57×10^{-10}
7	.27	.45	.017	.027	3770	7870	2.59×10^{-10}	2.66×10^{-10}
8	.20	.40	.013	.025	5250	10950		
9	.26	.34	.017	.021	3540	7370	2.88×10^{-10}	2.86×10^{-10}
10	.40	.51	.024	.031				
11	.30	.39	.018	.024	4070	8480	2.42×10^{-10}	2.08×10^{-10}
12	.23	.30	.014	.018	3210	6650	2.72×10^{-10}	2.25×10^{-10}

In calculating the hysteresis constants, the core loss vere considered to be entirely due to hysteresis, whi oes not introduce a large error, as the foucault curent loss is only a small portion of the total loss. A lance indicates the great difference in the quality of on used in the different transformers, and shows very istinctly the necessity for testing each batch of iron efore it is made up into transformer cores. As the on losses are the most important factor in determinig the all-day efficiency, too much stress cannot be id upon this point. The table indicates that the practce of making such tests has not, by any means, been niversal.

Table III. gives the exciting currents and no-load ower factors of the transformers. The power factors

TABLE III.

o.	Exciting Current.		Power Factor, No Load.		B_m	
	$f = 125.$	$f = 60.$	$f = 125.$	$f = 60.$	$f = 125.$	$f = 60.$
	.043	.066	84.0	73.0	2050	4280
	.076	.124	64.6	56.5	2600	5400
	.052	.100	56.3	47.6		
	.085	.125	67.0	65.6		
	.054	.099	81.7	61.5		
	.173	.475	85.0	40.0	3750	7720
	.043	.079	64.0	58.0	3770	7870
	.076	.060	71.0	22.0	5250	10950
	.055	.091	78.5	63.0	3540	7370
	.124	.190	78.6	65.7		
	.072	.113	79.6	67.2	4070	8460
	.077	.144	55.6	38.4	3210	6650

e higher at the higher frequency, due to the fact that smaller magnetizing current is required on account the magnetic density being lower at that frequency. The exciting current is lower at the higher frequency, due to the above cause and to the lower iron losses.

Table IV. gives the results of an independent test of the regulation of the transformers. The total drop is the difference in volts between the secondary pressure at full load and at no load when the primary pressure is 1000 volts. In one of the transformers it is seen that the total drop is greater for a frequency of 60 than of 125, while in the others the opposite is the case. The leakage drop should be less at the lower frequency, as the total number of leakage lines does not change very much with the change in the frequency, but the inductive pressure varies directly

TABLE IV.

No.	Volts Total Drop.		Volts *CR* Drop in Secondary.		Volts *CR* Drop in Primary.		Volts Leakage Drop.	
	$f = 125$.	$f = 60$.	$f = 125$.	$f = 60$.	$f = 125$.	$f = 60$.	$f = 125$.	$f = 60$.
1	2.3	3.5	.80	1.00	10.9	12.4	.5	1.3
2	3.5	2.8	.85	.88	7.9	7.7	1.9	1.1
3	4.9	3.3	1.05	1.10	12.6	13.0	2.6	.9
4	4.6	3.4	1.35	1.35	8.5	8.5	2.4	1.2
5	2.9	2.0	1.04	.98	9.3	8.7	1.0	.1
6	1.8	1.2	.41	.42	3.4	3.9	1.1	.4
7	82.0	21.0	1.94	2.48	24.2	28.0	77.7	15.7
8	5.2	4.7	1.40	1.40	17.0	17.8	2.1	1.5
9	5.3	4.2	1.71	1.75	8.2	8.4	2.8	1.7
10	4.0	3.1						
11	2.8	2.2	1.04	1.04	8.2	7.9	1.0	.4
12	3.8	2.0	1.36	1.36	12.7	12.5	1.2	

with the frequency. This is shown very plainly in transformer No. 7, in which the magnetic leakage is sufficiently great to cause a current of almost constant value to flow in the secondary circuit when the frequency is 125, but when the frequency is 60 it fails of its purpose entirely. The characteristic curves of this transformer are given in Fig. 204.

The leakage drop is somewhat less for the transformers which have the apertures for windings with a larger dimension in the direction of the lines of force in the tongue of the transformer and with the coils wound one upon the other (see Figs. 240 and 241). A comparison of the *CR* drops shows that the transformers of high all-day efficiencies have a comparatively large fall of pressure from this cause.

Table V. gives the radiating surface in the transformers and the rise of temperature under various conditions measured in a special test. From the table it is evident that there is little uniformity in the watts radiated per square inch of core and coil, or case, per degree rise of temperature. Taking an average of the test, the watts radiated per square inch of the core and coil surface with the case on may be about .2, and with the case off .3, when the rise of temperature is about 40° C. This requires from 5 to 7 inches of core and coil surface per watt lost with the case on. In an excellent series of transformers of various capacities the product of rise of temperature and square inches of coil and core per watt radiated varies between about 270 and 360, with an average of about 310. This is nearly 50 per cent greater than the average result of

TABLE V.

No.	Total Watts Lost.	Watts Lost per sq. in. of Case.	Watts Lost per sq. in. of Coils and Core Surface.	Rise of Temperature in Case. Centigrade.	Frequency, *f*.	Current in Secondary.	Increase of Temperature Due to Case in Per Cent.
1	51	.105	.121	20.8	60	0	25
1	39	.080	.093	17.5	125	0	41.0
1	82	[illegible]	[illegible]	.2	60	13.1	51
1	65	[illegible]	[illegible]	.4	125	11.3	24
2	75.3	[illegible]	[illegible]	.2	60	0	Taken wit case.
2	53.8	[illegible]	[illegible]	.4	125	0	
2	115.3	[illegible]	[illegible]	.4	60	17.9	
2	93.8	[illegible]	[illegible]	.4	125	17.6	
3	49.6	[illegible]	[illegible]	.0	60	0	43
3	34.5	[illegible]	[illegible]	.8	125	0	178
3	85.6	[illegible]	[illegible]	.3	60	14.2	—
3	59.5	[illegible]	[illegible]	.0	125	12.5	89
a	233	[illegible]	[illegible]	.4	60	0	66
a	143	[illegible]	[illegible]	.1	125	0	110
a	275	.340	.466	100.0	60	18.7	55
a	198	.242	.334	70.0	125	19.5	63
5	64	.118	.166	21.8	60	0	—
5	49	.090	.127	19.1	125	0	—
5	93	.172	.242	40.8	60	13.7	—
5	78	.144	.203	40.6	125	13.6	—
6	210	.388	.547	62.4	60	0	25
6	133	.246	.346	52.3	125	0	114
6	223	.412	.580	86.8	60	14.7	20
6	246	.455	.640	72.2	125	14.6	23
7	41	.143	.175	31.4	60	0	12.5
7	25	.091	.107	24.3	125	0	14.6
7	46	.162	.198	57.4	125	8.6	12
8	115	.168	.266	43.4	60	0	49
8	56	.079	.130	32.1	125	0	33.8
8	171	.250	.396	101.8	60	17.1	5.3
8	102	.150	.234	76.9	125	15.7	—
9	60	.099	.150	25.4	60	0	1.2
9	45	.074	.112	21.2	125	0	48
9	102	.168	.255	67.5	60	16.9	10
9	90	.149	.225	51.6	125	17.6	—
b	29	.052	.110	20.1	60	0	37.6
b	26	.047	.098	15.2	125	0	11.8
b	57	.102	.214	47.8	60	9.4	15.5
b	47	.085	.190	30.8	125	8.3	—
11	76		.274	27.0	60	0	Taken wit case. Ar core and c 277 sq. in., of case 514
11	57.5		.208	18.9	125	0	
11	103.8		.372	52.2	60	14.3	
11	88.4		.320	50.4	125	15.1	
12	55.5	.086	.145	31.6	60	0	60
12	43.2	.067	.113	20.5	125	0	67
12	80.8	.125	.211	51.8	60	14.1	—
12	78.8	.122	.206	20.5	125	14.2	—

Mr. Ford's tests. These figures are very rough approximations, to be used merely as a guide in design; but fairly exact data for any particular type of transformer may be determined from measurements on two or three sizes of the type.

The transformer case increases the temperature on an average of about 43 per cent; but this effect is found o be quite variable. The form of the case does not eem to enter into the result, though when it closely ncases the core and coils the heating effect seems to e slightly less.

The table shows the results of four heating tests on ach transformer; a no-load and a full-load test at each f the two frequencies used; the temperatures being etermined from measurements of the resistance of the rimary coils, ample time being allowed in each test for he transformer to come to its maximum temperature, nd check measurements being made by thermometer.

The following table, from a test by Fleming,* made in 892, gives the principal data concerning a number of nglish transformers and two of American make.

A is the capacity in watts, *B* the exciting current in mperes, with a primary pressure of 2400 volts and frequency of 83, *C* the iron loss in watts, *D* the power ctor at no load, *E* the total drop in volts, *F* the coper drop in volts, and *G* the leakage drop in volts. The w power factor and high exciting current of the Swinurne hedgehog transformer is especially noteworthy. his is caused by the high reluctance of the open magetic circuit.

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* *Jour. Inst. E. E.*, Vol. 21, p. 594; *Electrical World*, Vol. 21, p. 15.

Name.	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Ferranti	1875	.18	288	.66			
“	3750	.34	540	.68	1.6	1.9	
“	7500	.25	444	.74			
“	11250	.34	578	.70			
“	15000	.57	1019	.75			
“	3750	.11	233	.88	2.4	2.1	.3
“	7500	.07	138	.77			
“	11250	.07	148	.81	3.4	2.7	.7
“	15000	.11	228	.85	2.1	1.6	.5
“	11205	.10	228	.92	2.2	1.8	.4
Swinburne Hedgehog	3000	.74	112	.06	3.2	2.2	1.0
“	6000	1.19	156	.05			
Westinghouse . . .	6500	.05	95	.79	2.4	1.4	1.0
Mordey-Brush . . .	6000	.08	140	.77	1.8	1.8	
“ . . .	750	.03	61	.81			
Thomson-Houston .	4500	.08	108	.54	3.3	2.5	.8
Kapp	4000	.14	152	.61	1.9	1.8	

CHAPTER XII.

DESIGN OF TRANSFORMERS.

134. Effect of Frequency. — The formulas giving the ressures developed in the coils of a transformer

$$E' = 2\,\pi f L' C = \frac{\sqrt{2}\,\pi n' N f}{10^8} \text{ and } E'' = \frac{n''}{n'} E' \quad \text{(Sect. 111)}$$

low that if the value of the magnetization in the core f a transformer is fixed, then the number of turns in le coils must vary inversely with the frequency. To onstruct transformers for all frequencies with a fixed alue of the total magnetization is not an economical lan, however, since the core loss per cubic centimeter f iron depends upon the frequency. If a certain core ss is determined upon as being that which will give le most satisfactory general results in the case of a ansformer of given size, the magnetic density in the ore must be made to vary inversely with the frequency. his may be seen from the fact that the foucault curnt loss varies with the square of the frequency, and e hysteresis loss directly with the frequency. Since e former is between one-tenth and one-fourth of the tter, this makes the total core losses vary somewhere tween the first and second powers of the frequency, if magnetic density is constant. The foucault current loss also varies with the square of the magnetic density, and hysteresis varies with some degree of approximation within the limits of transformer practice, as the 1.6 power of the induction. Consequently the total losses vary as some power of the magnetic density between the 1.6th and the second. It is thus shown that the core losses will not be greatly changed if the magnetic density, within reasonable limits, varies inversely with the frequency. Now it is seen from the formula that, if the turns and pressures remain unchanged, when the frequency changes, the induction will change inversely; hence a well-built transformer should give nearly the same efficiency for all frequencies reasonably near that for which it was designed, and the number of turns in the coils may be the same in a series of transformers of equal capacities designed for different frequencies. In support of this stands the fact that transformers properly built for 125 to 135 periods a second are capable of giving excellent results at a frequency of 60, as is indicated, for instance, in a number of the transformers as given in Table I. of the preceding section. Changing the frequency alters the wattless component of the exciting current in a proportion which depends upon the saturation of the core, and poorly designed transformers or those built of poor material may give unsatisfactory service and over-heat on lower frequencies than their normal, on account of an excessive magnetic density and an excessive exciting current.

Since the rates of variation of the hysteresis and foucault current losses with the frequency and with the procal of the magnetic density are not exactly the e, though they are quite approximately equal, it is to :xpected that the efficiency of a transformer will to some degree with the frequency, and that some icular frequency will give the best efficiency. The :r has been found to be the case by Mr. Mordey.*

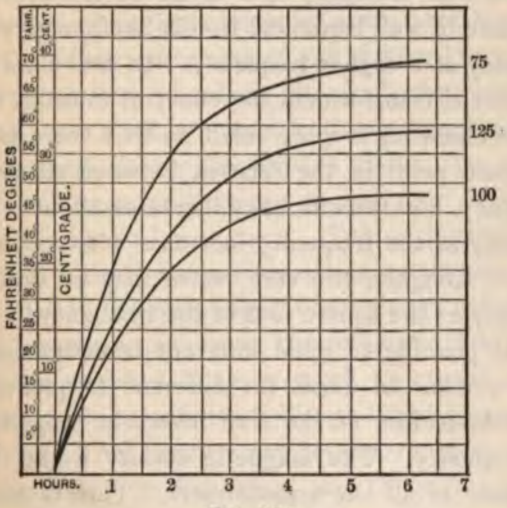

Fig. 235

ıre 235 shows the curves of rise in temperature of ordey transformer tested at three different frequen- showing that the best result is given for an interme- e value. The exact frequency at which a transformer give the highest efficiency must depend upon the ity of the iron and the effectiveness of the lamina-

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Alternate Current Working, *Jour. Inst. E. E.*, Vol. 18, p. 608; Dis- n of papers by Snell and Forbes, *Jour. Inst. E. E.*, Vol. 22, pp. nd 484.

tion, since these determine the rate of variation of the losses with the magnetic density and the frequency. The better the magnetic quality of the iron and the more thorough its lamination, the less difference is likely to occur in the efficiencies at various frequencies. A transformer made of poorly laminated iron is likely to give its best efficiency at a moderate frequency, while one made of well laminated iron is likely to give its best efficiency at a higher frequency. In first-class modern commercial transformers the effect of foucault currents on the efficiency is quite small, so that there is no real maximum point in the relation between efficiency and frequency, but the efficiency increases slightly but continuously as the frequency increases within commercial limits. Roughly, the core losses may be said to vary inversely as the square root of the frequency.

It is possible to build different transformers of the same number of turns for different frequencies with the cross-section of the core inversely proportional to the frequency. The magnetic density would then be the same in all the transformers. This is unsatisfactory, however, since it makes low frequency transformers large and expensive: and since the weight of iron in a core must vary more rapidly than the cross-section, this method of construction also causes comparatively large core losses in low frequency transformers and gives them a comparatively low efficiency.

The conclusion is therefore derived that the core and windings of a well designed and well constructed transformer will be almost equally satisfactory in performance on any frequency within the present commercial limits.

This conclusion is shown by experience to be approximately true for frequencies within the limits of 50 and 150. The table of satisfactory magnetic densities for various frequencies, already given (Sect. 97), shows that foreign manufacturers apparently prefer a mean path, and change both the cross-section of the core and the induction when building transformers for different frequencies. American manufacturers have heretofore ordinarily built transformers of one standard type, which are intended to be used on all usual commercial frequencies, but some are now building two types, one designed specially for frequencies above 100, and the other for frequencies between 50 and 100. The latter have a somewhat larger core. The outputs of transformers of a fixed size are almost in proportion to the square root of the frequency,* other things being equal. As the frequency decreases, the magnetic density may be allowed to increase to as much as 12,000 lines of force per square centimeter in large transformers of low frequency. This density may be satisfactorily used in transformers of 30 frequency, and those designed for lower frequencies must increase in weight in inverse proportion to the frequency.

135. Effect of the Form of the Impressed Pressure Curve. — Very few experimental data are at hand which show the effect of the form of the pressure curve, though a good deal has been written upon the subject. No matter what the primary pressure wave may be like, the secondary pressure wave must have practically the same form, the exciting current and magnetization curves varying to meet this requirement. When the pressure curve is peaked, the magnetization curve will be flat and not reach so high a maximum as for an equivalent sine curve. When the pressure curve is flat the magnetization curve will be peaked (Sect. 77). Since the iron loss of a transformer depends upon the maximum value of the magnetism, it is to be expected that a peaked pressure curve will cause a minimum iron loss in a transformer. Steinmetz gives the appended table in support of this:

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* Steinmetz, Effect of Frequency on the Output of Transformers, *Elect. Engineer*, Vol. 15, p. 260; *Trans. Amer. Inst. E.E.*, Vol. 11, p. 38.

HYSTERESIS LOSS IN WATTS.

Transformer Number.	Sine Wave.	Peaked Wave.	Difference.
35992	42.3	38.6	9.8 %
36067	41.1	37.8	8.95 %
36668	36.8	33.9	8.7 %
35799	36.6	33.7	8.7 %

Steinmetz also gives another case where the loss in a 200 K.W. transformer was 13 per cent less when worked on a distorted curve than when a sine curve was impressed. The change in efficiency is, however, very slight. Experience has shown that alternating current arc lights give the best results when worked on a flat-topped curve, while Ryan, Duncan, and others claim that a sine curve gives the best efficiency in the operation of induction motors.* The latter is shown to be theoretically correct in Section 188, and if the deduction proves to be of much importance in practice, an approximate sine form will probably give the best general satisfaction. Roessler has lately shown that the iron loss of a transformer depends upon the ratio of the mean value of the pressure wave to its effective value.

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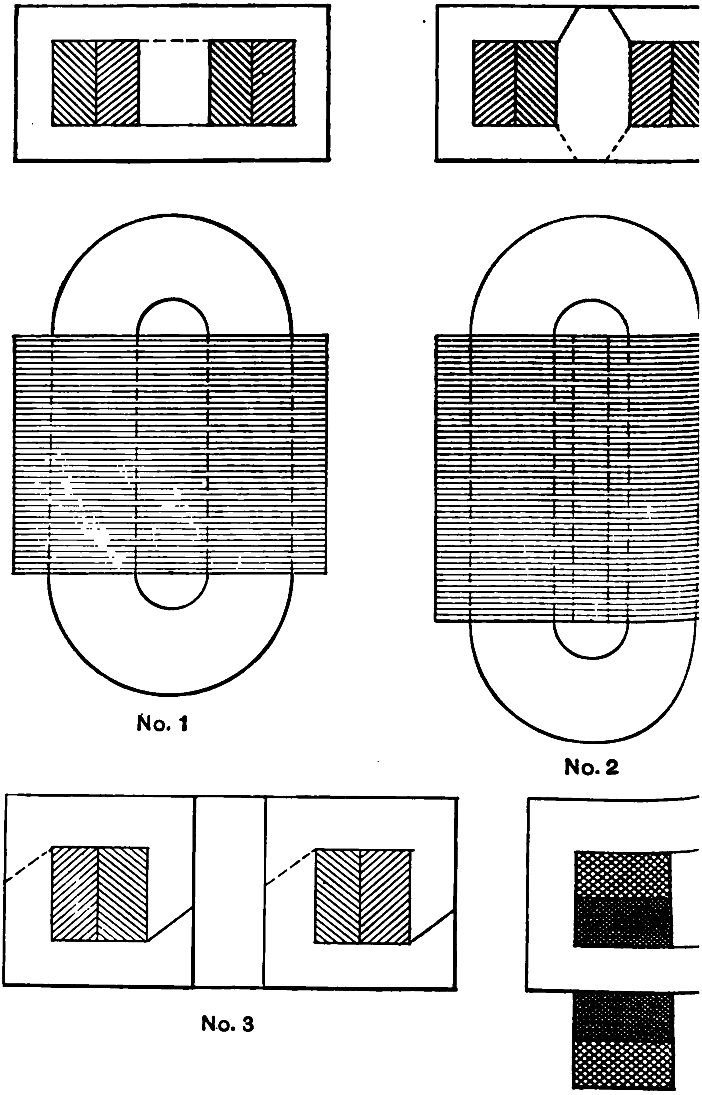
* *Electrical World*, Vol. 24, pp. 107, 155, and 178.

136. Example of Transformer Calculations. — The formula $E' = \frac{\sqrt{2}\,\pi n' N f}{10^8}$ in any practical case contains only two unknown quantities, and these may be suitably chosen by a designer to suit his conditions. Thus, suppose it be desired to design a 10 : 1 transformer of 1650 watts capacity, for a primary pressure of 1100 volts and a frequency of 125. This is equivalent to a 1500 watt transformer which is rated on the usual basis of 1000 volts. By solving the equation, the product of $n'N$ is given, thus

$$n'N = \frac{10^8\,E'}{\sqrt{2}\,\pi f} = 200{,}000{,}000 \text{ approximately.}$$

Calculation of Copper and Core Dimensions. — Assuming the number of turns in the primary coil to be 650, makes the maximum magnetization $N = 308{,}000$. Now taking the maximum magnetic density, $B_m = 3000$, gives the core cross-section 102.7 sq. cm. The forms taken by the cores and coils of transformers which are intended for ordinary single-phase work are quite limited, and the more important ones are shown in Fig. 236. Choosing whichever one of these lends itself to the requirements (in this case, say, the first) the dimensions of the core may be quickly determined. The apertures

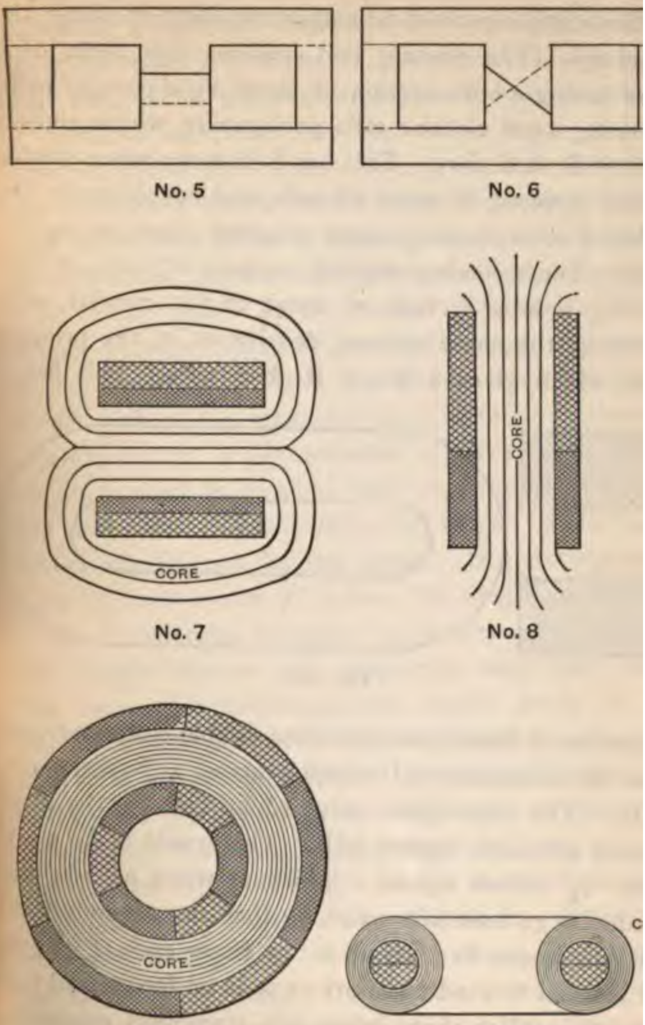

No. 1

No. 2

No. 3

No. 4

Fig. 236



No. 5 No. 6

No. 7 No. 8

No. 9 No. 10

Fig. 236

mine whether this transformer will serve its purpose, it is necessary to calculate the core loss, the copper loss, the magnetic leakage, and also the radiating surface per watt. The hysteresis loss in the core is readily calculated by the usual hysteresis formula Vol. I, p. 74. The number of cubic centimeters of iron in the core is 3000. This gives a loss by hys-

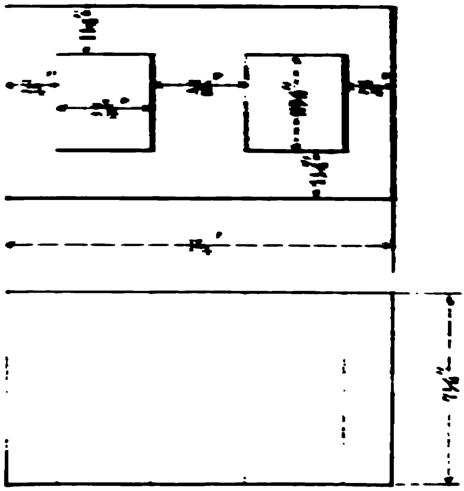


Fig. 238

teresis at a frequency of 125 of 37.5 watts for iron in which the hysteresis constant is 21×10^{-11} based on the cubic centimeter and the cycle per second. This value is 35×10^{-13} when based on the cubic centimeter and cle per minute, as the constants are given on page 75 Vol. I. That 21×10^{-11} is a fair average value for hysteresis constant of the best transformer iron own by a series of tests made by Steinmetz, the verage of which gives 17×10^{-11}.* Where a series of ransformers is to be designed with the same kind f iron in the core, it is advantageous to plot a curve which gives the hysteresis loss in watts per cubic entimeter, or per pound, of iron at different magnetic ensities. Such a curve for a very good quality of iron hysteresis constant $= 21 \times 10^{-11}$) is shown in Fig. 239 or a frequency of 100. The loss in the iron when sub- ected to other frequencies may be determined by multiplying the values shown by the ratio $\frac{f}{100}$. On account f the differences in the quality of iron it is never safe o depend upon curves drawn from tests of one brand o represent the quality of another brand, and in fact ome transformer iron shows losses not much more than two-thirds as great as those indicated in Fig. 239; but a etter grade than that shown cannot be uniformly btained in the market.

Hysteresis testing may be carried out on such instruments as those of Professor Ewing † or Mr. Holden,‡ or t may be carried out by punching the sample into trans- ormer plates, inserting them in a coil, and measuring he losses by a wattmeter. The value of B_m may be determined from the number of turns in the coil and the readings of a voltmeter at its terminals. For the results to be absolute, the wave of impressed pressure must approximate closely to a sinusoid, but satisfactory comparative results may be had independently of the

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* *Trans. Amer. Inst. E. E.*, Vol. 11, p. 705.

† *Jour. Inst. E. E.*, Vol. 24, p. 398; *London Electrician*, Vol. 34, p. 786.

‡ *Electrical World*, Vol. 25, p. 687.

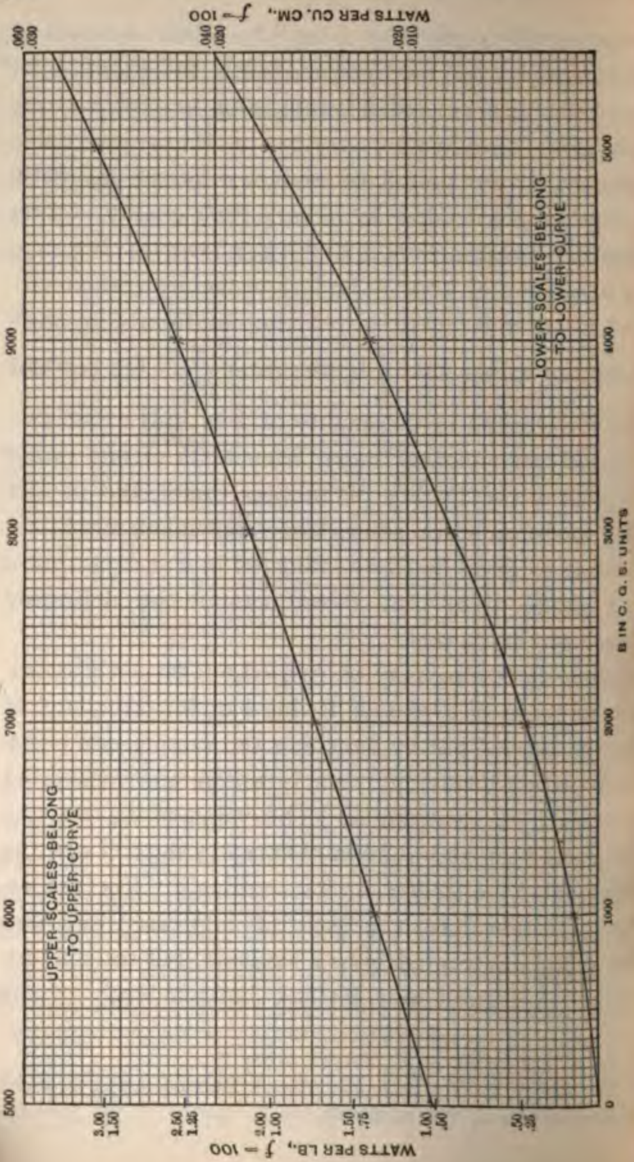
WATTS PER CU. CM., f = 100
.060
.030
.040
.020
.020
.010
5000
6000
7000
8000
9000
UPPER SCALES BELONG TO UPPER CURVE
LOWER SCALES BELONG TO LOWER CURVE
0
1000
2000
3000
4000
5000
B IN C.G.S. UNITS
3.00
1.50
2.50
1.25
2.00
1.00
1.50
.75
1.00
.50
.50
.25
WATTS PER LB., f = 100

Fig. 230

rm of the pressure wave, provided the same form of ave is used in all tests.

Calculation of Foucault Current Loss.—The exact alculation of the foucault current loss in the core of a ansformer is more difficult than that of the hysteresis ss; but equal exactness is not essential since the ucault current loss very seldom exceeds 25 per cent the core losses and often constitutes only from 10 15 per cent.

A formula is developed by Mr. Steinmetz[*] which for minated iron is $u = (dfB_m)^2\ 10^{-16}$, where u is the watts st per cubic centimeter of iron, d is the thickness of he plates in mils, f the frequency of magnetic reversals, and B_m the maximum magnetic density. The total ss in watts for M cubic centimeters of iron is

$$U = M(dfB_m)^2\ 10^{-16}.$$

educing this to the loss per cubic inch of iron simply ntroduces the constant coefficient 16.4 into the formula. Reducing the formula to the watts lost per ound of iron changes the constant to nearly

$$U' = 6M'(dfB_m)^2\ 10^{-15}.$$

A similar formula is deduced by Thomson and wing.[†]

For the transformer under consideration, iron 12.5 ils thick is used and the foucault current loss is

$$3900 \times (12.5 \times 125 \times 3000)^2 \times 10^{-16} = 8.5 \text{ watts}.$$

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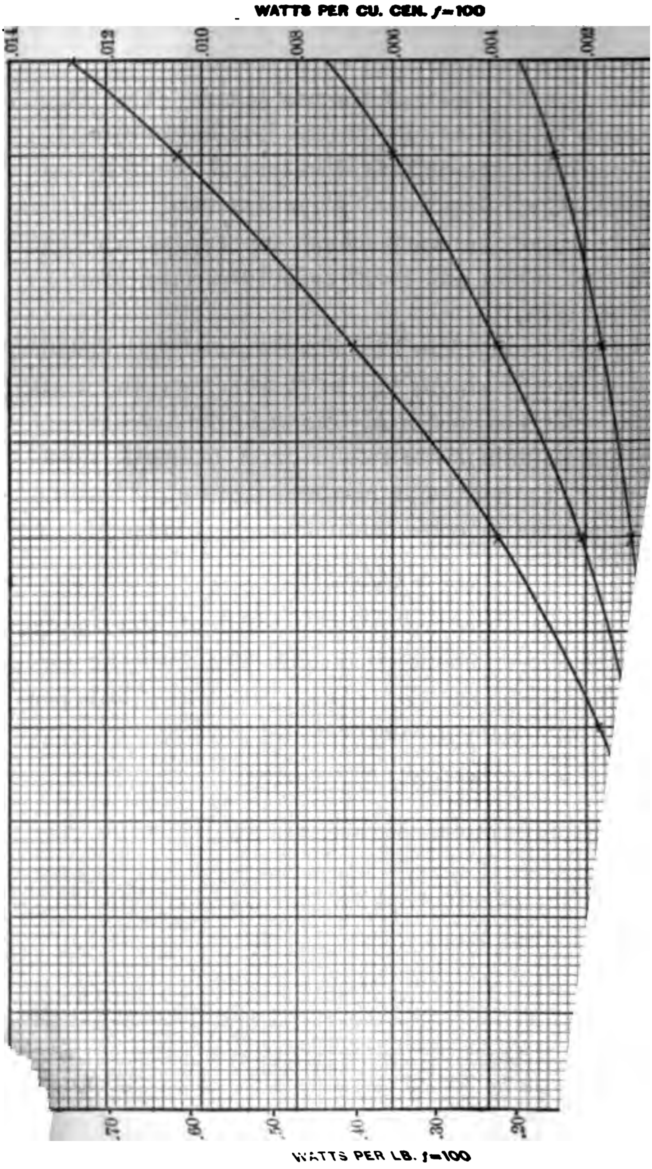
[*] *Trans. Amer. Inst. E. E.*, Vol. 11, p. 600.

[†] *London Electrician*, Vol. 28, p. 631; Weekes' *Alternating Current Transformers*, p. 21.


WATTS PER CU. CEN. f=100
.014
.012
.010
.008
.006
.004
.002
.70
.60
.50
.40
.30
.20
WATTS PER LB. f=100

The total core loss for the transformer is therefore 6.0 watts or 2.8 per cent, which is a little high for a transformer of this size, and the number of turns in the coils might be increased and the cross-section of the iron proportionally decreased with advantage. The thickness of iron which is here taken is a usual one, though transformer stampings are made of various thicknesses between 10 and 18 mils. The commonest thicknesses are 12.5 and 15 mils. It is possible to plot a curve of the losses due to foucault currents in sheets of fixed thickness similar to that already given for hysteresis, or a curve may be plotted which gives the combined losses which are found in stampings of a given thickness and made of a fixed quality of iron. Figure 239*a* shows a curve for the calculated foucault current loss in stampings of the three thicknesses, 10, 15, and 20 mils, at a frequency of 100. The loss at any other frequency may be found by multiplying the values from the curves by $\left(\frac{f}{100}\right)^2$.

Calculation of Copper Loss.—The copper losses of the transformer come out as follows: The mean length of a turn is practically 30 inches. The secondary coil has 65 turns or 165 feet of No. 7 wire, which makes the cold resistance .08 ohms; allowing a rise of 50° C. in temperature gives at full load a loss of 21.6 watts. The primary coil consists of 650 turns, or 1650 feet of No. 16 wire, which makes the cold resistance 6.35 ohms. The copper loss at full load with the transformer hot is therefore practically 40 watts or 2.42 per cent, which is reasonable.

Calculation of Exciting Current.—The final tion to be determined in relation to the trans is the exciting current. This is made up of two (Sect. 113) which are in quadrature, *i.e.* the ma ing component and the loss component. The la evidently equal to the core loss in watts divided primary pressure, or $\frac{46}{1100} = .042$, and is in ste the primary pressure. The magnetizing compon is found from the formula

$$\sqrt{2}\, n' C'_{\mu} = \frac{N_m P}{1.25} = \frac{N_m l}{1.25\, A \mu} = \frac{B_m l}{1.25\, \mu},$$

or

$$C'_{\mu} = \frac{B_m l}{1.75\, n' \mu},$$

when l is the length of the lines of force in the co A the area, which gives for C'_{μ} in the case und sideration,

$$C'_{\mu} = \frac{3000 \times 37.5}{1.75 \times 650 \times 2000} = \frac{113,000}{2,300,000} = .04$$

The total exciting current is therefore,

$$C_1' = \sqrt{.049^2 + .042^2} = .064,$$

which is a satisfactory value. The value $\mu =$ which is here used, is a fair value for transformer

The value of the magnetizing component i calculated without considering the effect of the in the magnetic circuit upon its reluctance. T impossible to estimate, and it may cause a consi increase in the exciting current. Its effect n experimentally determined from measurements ı one transformer, and the correction applied in :signing other transformers of the same type.

Arrangement of Conductors.—The primary conduct- 's of transformers, except those of very large size, 'e usually of wire, and the secondary conductors are : wire or ribbon. Wires which are larger than No. 7 · No. 8 B. & S. gauge are not often used, and con- ıctors of larger cross-section are made of single bbons, or of two or more wires or ribbons in parallel. 'he thickness of ribbon conductor which is used :ldom exceeds 75 mils, since the insulation of thicker ›nductors is likely to be injured in winding around the nall radii at the ends of the coils. The insulation ›mmonly consists of two or three cotton coverings ‹actly as in armature windings, and the finished coils re very thoroughly insulated with rubber tape, mica, tc. Sometimes tape or cord wrappings are used on bbon conductors instead of cotton coverings, and bare bbons may be wound up with tape between; but the itter is not advisable. The primary coil should always e broken into sections by inserting oiled paper or ırnished muslin between the layers, and where the ›ils are divided for the purpose of sandwiching to :duce magnetic leakage, the primary coil should have ıe least number of divisions so that its insulation may : less interfered with.

Checks.—The total radiating surface from which :at may leave this transformer is nearly 400 square ches, and the total losses at full load make 86 watts, we have nearly five square inches per watt lost. ıe efficiency of the transformer at full, half, and quarter load, is 95.0 per cent, 93.6 per cent, 89.4 per cent. The all-day efficiency, based on five hours full load and 19 hours no load, is 86.4 per cent. The number of turns per volt in the windings is $\frac{24}{\sqrt{\text{Output}}}$ in this transformer. In transformers of the best makes intended for frequencies between 60 and 135, the number of turns per volt in the windings seems to vary between $\frac{15}{\sqrt{\text{Output}}}$ and $\frac{40}{\sqrt{\text{Output}}}$, when the output is given in watts, and the numerator seems usually to be less than 25 for the best transformers. This ratio gives a guide to the choice of the number of turns which shall be used in a transformer, and the number of lines of force in the magnetic circuit is then fixed by the formula. In determining the size of the plates and the length of the core, the ratio of the over-all area of the plates to the area of the apertures may be used as a guide. In the above example this ratio is 4.00, and in a large number of commercial transformers of capacities from 500 watts to 30,000 watts the ratio is found to vary from 2.75 to 4.25, with an average of about 3.00. A final check upon any design may be made to depend upon the calculated core loss per pound or per cubic centimeter of iron, which varies in commercial transformers from .80 to 2.80 watts per pound, or .012 to .042 watts per cubic centimeter, and averages in first-class transformers about 1.00 watt per pound, or .015 watts per cubic centimeter. In any case, the determination of the most economical design in a particular form depends upon working out several designs with different constants. The best design may then be chosen.

137. Dimensions of Various Commercial Transformers. — The data for 1500 watt transformers of various manufacturers are given on page 534 for comparison. All the data are based on a primary pressure of 1100 volts, frequency of 125, and ratio of transformation of 10 : 1.

138. Calculation of Magnetic Leakage. — In the preceding example no attempt has been made to calculate the magnetic leakage. By properly placing the coils with respect to each other and to the magnetic circuit, the magnetic leakage may be made almost or quite negligible. Thus in Fig. 240 the coils are so placed that a short-circuiting of lines of force along the path indicated by the dotted lines is to be expected; but when the coils are arranged as shown in Fig. 241, the leakage is not likely to be great; while if the coils are divided and the parts sandwiched together, the leakage may be made very small. The leakage may be calculated quite approximately by the method indicated below. Since the currents in the primary and secondary coils of the transformer are in practical opposition of phase, their magnetizing effects are opposite. This tends to cause

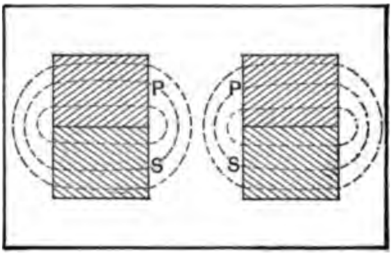


Fig. 240

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.
Watts	1400	1650	1650	2200	1650	1650	1650
n'	760	640	630	720	560	1080	760
n''	76	64	63	72	[illegible]	108	76
$B_{\max}$	2300	4200	2690	[illegible]	[illegible]	3900	2000
$N_{\max}$	263000	314000	517500	[illegible]	[illegible]	[illegible]	263000
$A = \frac{N}{B}$	116	75	118	[illegible]	[illegible]	46.5	130
CR in watts	30	14	45	[illegible]	[illegible]	36	
CR in per cent	2.1	.9	2.7	[illegible]	[illegible]	2.2	
Core loss in watts	37		56.	[illegible]	[illegible]	45	
Core loss in per cent	2.6		3.4	[illegible]	[illegible]	2.73	
Per Cent efficiency full load	95.		95.	[illegible]	[illegible]	96	
All-day efficiency per cent	85.		85.	89		89	
Primary wire	17 B. & S.	16 B. & S.	15 B. & S.	14 B. & S.	17 B. & S.	16 B. & S.	2 × 20 B. & S.
Resistance of primary	7.00				7.17	8.9	
Secondary wire	7 B. & S.	8 B. & S.	7 B. & S.	7 B. & S.	7 B. & S.	7 B. & S.	.183 × .077
Resistance of secondary	.069				.072	.085	
Thickness of stampings	.015	.0125	.0125	.0125	.017	.014	.014
Number of stampings		930	420		574	180	
Pounds of iron		46		43	57	30	106
Pounds of copper					18	27.5	16
Exciting current	.043		.076	.055		.052	
Total drop in pressure, per cent	2.3		3.5	5.		3.9	
Magnetic leakage drop	.2		0.8	2.5		1.7	

lines of force to short-circuit through the coils, as shown in Fig. 240, the tendency being greatest at the plane where the coils touch each other, and falling off to zero at the outer edges of the coils, so that the magnetic leakage will differ for each layer of wire in the coils. The effect of leakage must therefore be calculated for each layer, and the total effect may then be summed up. In Fig. 242 the ordinates of the line $A'BA''$ are proportional to the ampere-turns acting at any point to cause leakage lines to pass through the coils. These

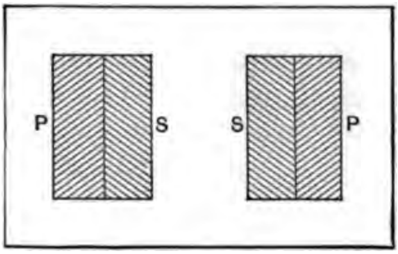


Fig. 241

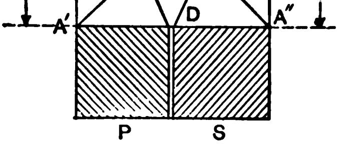


Fig. 242

ordinates are equal to the number of turns in the coils between the foot of the ordinate and the outer edge of the coils, multiplied by the current flowing in the turns. The ordinate is evidently zero at the outer edges of the coils, and a maximum equal to practically $n''C''$ at the plane between the coils. The number of the leakage lines of force enclosed by any layer is proportional

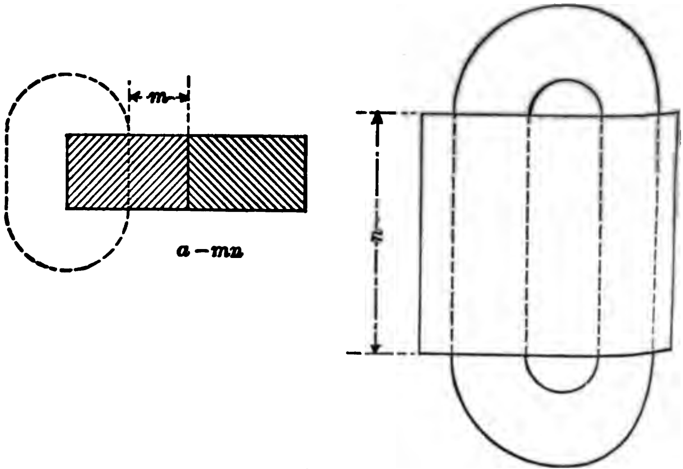


Fig. 243

to the corresponding ordinate of the lines $C'DC''$. These ordinates are respectively equal to

$$x = \frac{1.25\sqrt{2}ya}{l},$$

where x is the desired ordinate, y is the mean ordinate of the line $A'BA''$ taken from the neutral plane at D to the point under consideration, a is the area of a coil between the neutral plane and the point under consideration, and l is the length of the lines of force through coils (Fig. 243). The maximum value of x falls at outer edges of the coils and is

$$x_m = \frac{1.25\sqrt{2}\,n''C''A_1}{l},$$

re A_1 is the total iron surface presented to a coil which leakage lines emerge, and the average numof leakage lines enclosed by the different layers at instant of maximum leakage is

$$\frac{x_m}{2} = \frac{1.25\,n''C''A_1}{\sqrt{2}\,l} = \frac{.90\,n''C''A_1}{l}.$$

inductive effect of this leakage on the secondary ling is equal to

$$\frac{\sqrt{2}\,\pi n''\frac{x_m}{2}f}{10^8},$$

an equivalent effect is produced on the secondary ccount of the leakage of lines of force through the ary coil. If A be taken to represent the total area on from which leakage lines emerge, which is pre ed to both coils, the formulas become

$$= \frac{1.25\,n''C''A}{\sqrt{2}\,l} \quad \text{and} \quad E_l = \frac{\sqrt{2}\,\pi n''N_l f}{10^8} = \frac{4\,n''^2C''Af}{10^8\,l}.$$

inductive effect due to leakage is, as has already shown (Sect. 112), to be in quadrature with the e pressure in the secondary circuit, E''. Consetly, the drop in secondary pressure caused by mag leakage is

$$E'' - \sqrt{E''^2 - E_l^2},$$

and the total drop of secondary pressure in the transformer is

$$E'' - (\sqrt{E''^2 - E_1^2} + C''R'' + \frac{1}{k}C'R').$$

The value for E_1 in the transformer designed above, when the coils are wound one inside of the other, is

$$E_1 = \text{[illegible]}\ 18 \times 125 = 10 \text{ volts};$$

when the co[illegible] by side, this becomes

$$E_1 = 4\ \text{[illegible]} \times 142 \times 125 \over \text{[illegible]} \times 10^8 = 12 \text{ volts}.$$

The leakage drop is, therefore,

$$100 - \sqrt{10000 - 100} = .5 \text{ volt}$$

or

$$100 - \sqrt{10000 - 144} = .72 \text{ volts}.$$

This may be made practically negligible with either arrangement of the coils by dividing one of the windings into two parts and sandwiching the other between them. This reduces N_1 to one-fourth and the leakage drop in a still greater ratio.

139. Joints in the Magnetic Circuit. — In this discussion no account has been taken of the position of the joints in the stampings, which really have a marked effect on both the magnetic leakage and exciting current. Every effort is made to reduce the magnetic reluctance of the joints by making them as few in number as possi- e, and arranging them so that the joints of adjoining es come in different positions in the core. The ts in the magnetic circuit are usually one or two, l should never be more than two. Lapping joints really essential to the best performance, and while t joints have been used in transformers, their effect always been detrimental. The arrangement of ıts is shown very plainly in Fig. 236. The last four ws shown are of antiquated forms. The arrangent of the joints may be made in various ways with h form of built-up core, as the position of the joints ›ends only on the punching.

40. Ageing of Transformer Cores. — Experience has wn that the core loss in some transformers increases a very considerable extent during the first few nths of operation. The increase in loss is due to an :eased hysteresis loss per cycle, and was originally ribed by Ewing* to magnetic fatigue of the iron. has, however, been quite conclusively proved by eximents † and the records of transformer manufactrs to be caused by the continuous condition of high ıperature at which the iron is operated. The ageing ms to have the greatest effect upon poor qualities ot n, hastily and imperfectly annealed, and the least :ct upon the best grades of iron which have been ıealed with great care. The conditions under which : annealing of the transformer plates is performed, ›ecially with reference to temperature and duration the process, have much to do with the extent of the :ing effect, and by proper annealing it can be ren'ed very small in cores made of proper qualities of ı. The iron now generally used for transformer cores is a very mild steel made by either the bessemer or open-hearth processes, though puddled iron sheets are still used to some extent.

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* *London Electrician*, Vol. 34, p. 161.

† *Ibid.*, pp. 160, 190, 191, 219, 297, 498.

141. Current Rushes. — It has been shown in Section 25 that the exponential term in the complete equation for an alternating current in an inductive circuit is ordinarily negligible, but under certain conditions its effect for a few periods after the current is started in a circuit may be considerable. This question was investigated by Fleming * and others † with especial reference to the action of transformers when first switched onto an alternating-current circuit. If a transformer is switched onto a circuit, the current does not instantly assume the final form of the wave, but rises gradually through a short interval to its final form. The length of the interval and the magnitude of the early current depends upon the reactance of the circuit, the frequency, and the point in the pressure wave at which the connection is made. If the instant of switching onto the circuit is that at which the impressed pressure is passing through zero the current in the transformer is less during the early interval than its final value, while if, at the instant of switching on, the impressed pressure is passing through its maximum value there may be quite an excess of current flow through the circuit for a short time, on account of the relations which exist between the instantaneous impressed and counter electric pressures during the first half period. The abnormal state of the urrent can only exist for a very short time unless the eactance of the circuit approaches a condition of resnance (Appendix D), which is very exceptional. As ar as the operation under ordinary conditions of transormers or other commercial alternating-current applinces is concerned, the phenomena of current rushes ay be entirely neglected.

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* *Jour. Inst. E. E.*, Vol. 21, p. 677.

† Hay, On Impulsive Current-rushes in Inductive Circuits, *London Electrician*, Vol. 33, pp. 229, 277, and 305.

142. Impedance Coils, Compensators, etc.—The design f *impedance coils*, *reactance coils*, *choking coils*, or *econ-*

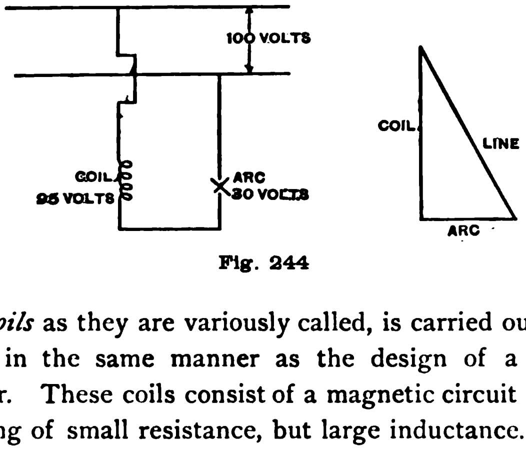


Fig. 244

my coils as they are variously called, is carried out very uch in the same manner as the design of a transormer. These coils consist of a magnetic circuit with a inding of small resistance, but large inductance. The agnetic circuit and winding are proportioned in exactly he same manner as the primary winding of a transormer, using the formula $E = \frac{\sqrt{2}\,\pi n N_m f}{10^8}$. Coils of this pe are used for a variety of purposes where it is esired to throttle the flow of current without the tendant loss of power which always follows the use of sistances. Where arc lamps are used on constantressure alternating-current circuits, an economy coil

netic circuit, to which both the primary and secondary circuits are connected. Figure 248 shows the connections of a 220 volt compensator which feeds two 110 volt secondary circuits. In this case the function of the compensator is to equalize the pressure between the two secondary circuits regardless of their relative loads. This purpose is fulfilled fairly well, but the regulation is not as satisfactory as that of a transformer. Figure 249 shows the connections of a 220 volt compensator which supplies a 1000 volt secondary circuit. When one of the secondary terminals of a compensator is

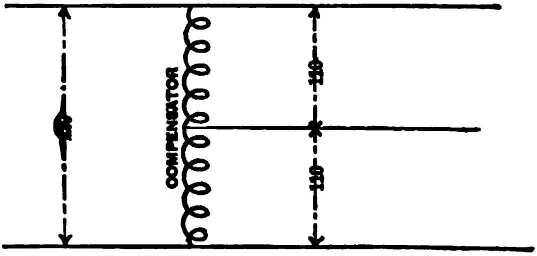


Fig. 248

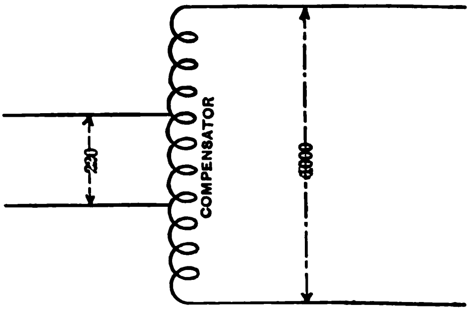


Fig. 249

arranged so that the position of its connection to the winding may be varied, the machine is called an **Auto-transformer.** The secondary pressure and current of an auto-transformer may be arranged to vary through any desired range, while the primary current changes only so far as is required by any change in the power absorbed by the secondary circuit.

CHAPTER XIII.

POLYPHASE CONDUCTING SYSTEMS AND THE MEASUREMENT OF POWER IN POLYPHASE CIRCUITS.

143. Polyphase Conducting Systems. — A full discussion of conducting systems has no place in this book, but a brief explanation of the methods of connecting the coils of polyphase machines and the wires of polyphase circuits is essential for the purposes of the following chapters.

Polyphase systems are usually operated with either two currents with approximately 90° difference of phase, or three currents with approximately 120° phase difference. Polyphase machines arranged for two currents are called **Two-phase Machines, or Two-phasers.** Those arranged for three currents are called **Three-phase** or **Tri-phase Machines, Three-phasers** or **Tri-phasers** (see page 386).

The transmission circuits for two-phase currents may be arranged to be entirely independent of each other, four wires being then required (Fig. 250); or, three wires may be used, in which case one of them is common to the two currents (Fig. 251); the current in the third or common wire, at any instant, is equal to the algebraic sum of the currents in the other two, and *the algebraic sum of the instantaneous currents in the* *ree wires is always equal to zero.* The effective cur-nt in the common return wire is equal to the vector m of the two circuit currents; and is, therefore, $\sqrt{2}\,C$, here C is the effective current in one circuit, pro-ided the currents are equal in the two circuits and ave a phase difference of 90°, which is the condition

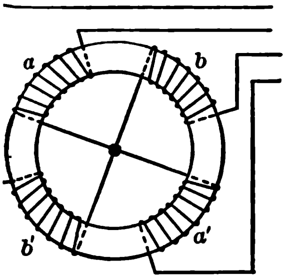


Fig. 250

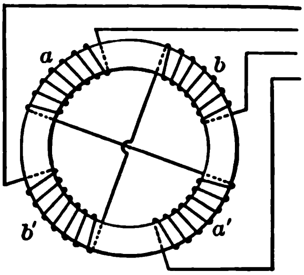


Fig. 250 a

hen the system is properly designed and symmetri-lly loaded or **Balanced.** The pressure between the vo outside wires of the two-phase system with com-on return is the vector sum of the two circuit press-res, and is, therefore, $\sqrt{2}\,E$ in a balanced system, here E is the pressure between one side and the mmon return. The com-on current and pressure in balanced system are 45° om the phase of the cur-ent in either of the inde-endent wires. Figure 252 hows the graphical compo-ition of the pressures. A nd B are the two line press-

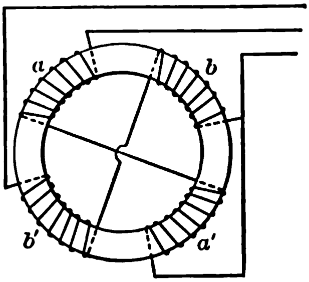


Fig. 251

ures, and R is the resultant pressure measured across the outside wires.

The coils of two-phase machines may be entirely independent of each other, in which case four collector rings are required, or the circuits may be joined so as to require only three collector rings. In some two-phase machines the armature is wound with the equivalent of a series-path continuous-current winding, and four collector rings and independent circuits are then required to avoid short-circuiting portions of the armature. Figure 253 shows, by diagram, various ways of connecting the coils of two-phase machines (see Sect. 102*a*).

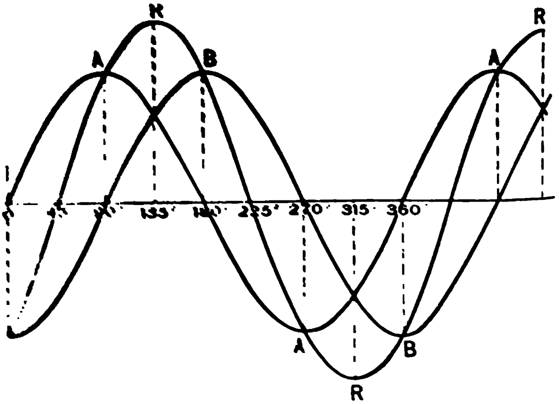


Fig. 252

It is possible, in three-phase systems, to use three entirely independent circuits, each consisting of two wires, and carrying currents of 120° difference of phase; but in practice the circuits are almost invariably combined so as to use three wires, and the current in each wire is then equal to the vector sum of two circuit currents.

The coils of three-phase machines may be connected ether so that they form the three sides of a triangle

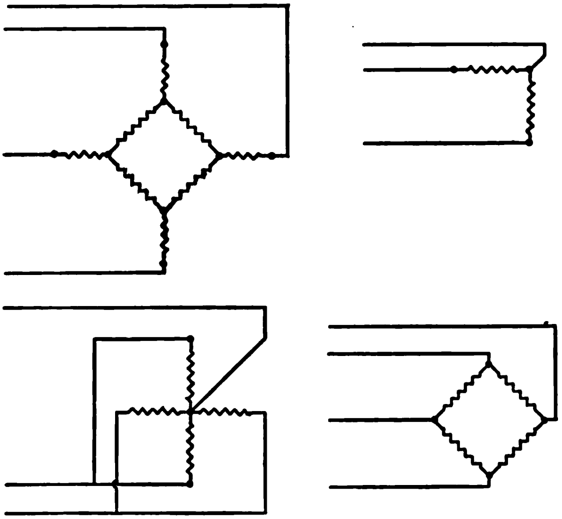


Fig. 253

th the transmission wires connected to the three corrs of the triangle (Fig. 254), or one end of each coil

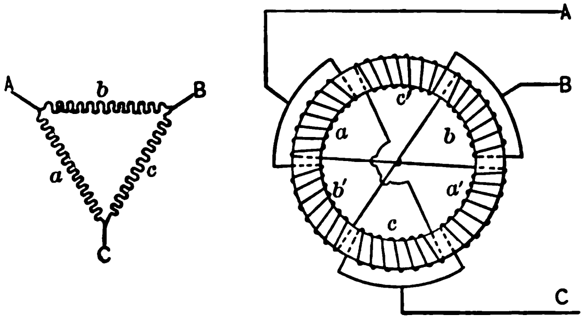


Fig. 254

y be individually connected to the transmission wires,

the free ends of the coils being connected together (
255).* In either case the number of transmission w

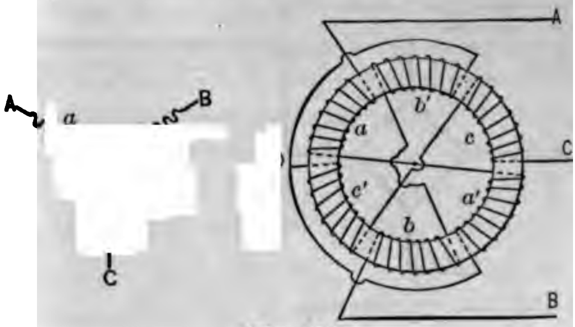


Fig. 255

is three, and *the algebraic sum of their instantaneous rents is always equal to zero.* In the latter or
arrangement, which is often represented by the syn

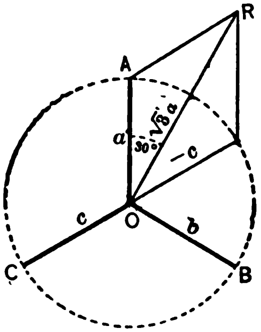


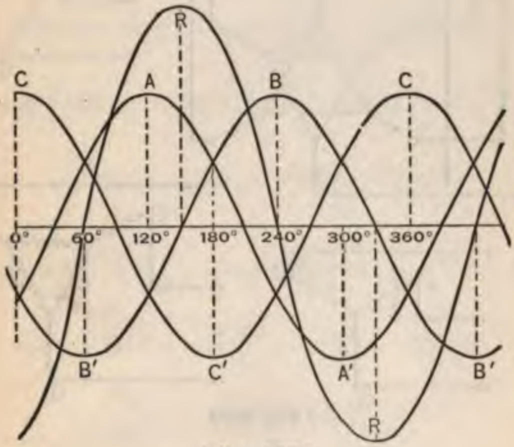
Fig. 256

Y, the pressure between any two line wires in a balar
system is $\sqrt{3}$ E, where E is the pressure in one co
the machine. Thus in Fig. 256, if a, b, and c are the pr

---

* See Section 102 *a*.

in a vector diagram, the pressure between A and C
R, which is $\sqrt{3}\,E$ in magnitude and has 30° differ-
of phase from a or $-c$. In Fig. 257 the curve R
rs the potential difference between A and C. The
current must in this case be the same as that pass-
through the coil to which it is attached. In the
ngle or **Mesh** winding, which is often represented
he symbol △, the pressure between wires is evi-



[Fig. 257]

ly that generated by one coil, and the current in the
wire is the resultant of that in two adjacent coils,
$\sqrt{3}\,C$ in a balanced system, where C is the current in
l. This may be obtained from Fig. 256 by consid-
g a, b, and c to be currents. If a is the initial cur-
from which the phase is measured, b must be taken
ward, as this current must flow in the opposite
tion from a, in order to get to the line which is

connected to the junction of *a* and *b*. This makes the resultant current 30° from the current *a*. Figure 258 shows various ways in which the coils of three-phase machines may be connected. The arrangements are either of the star or mesh connection or a combination thereof.

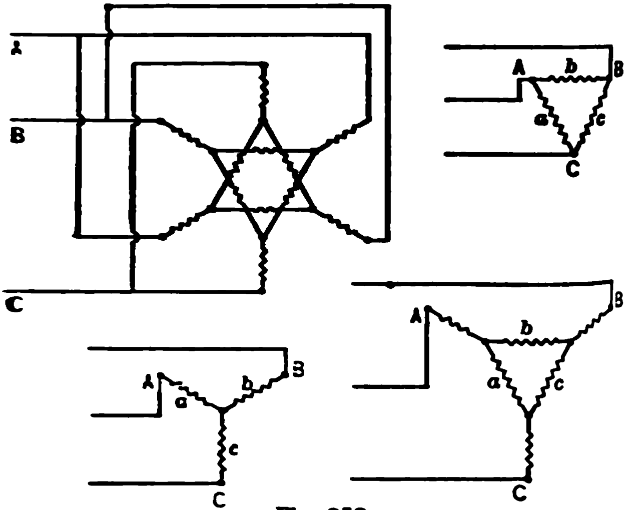


Fig. 258

144. Uniform Power in Polyphase Systems. — In general, the power transferred in a balanced polyphase circuit is uniform throughout each period, and the torque exerted by balanced polyphase machinery is uniform. This is different from the conditions in single-phase circuits, where the power has been shown to vary from a maximum to a minimum during every quarter period (Sect. 43). In the case of a single-phase circuit the power at any instant is $c_m e_m \sin(a - \phi) \sin a$

$= c_m e_m \sin^2 a \cos\phi - c_m e_m \sin a \cos a \sin\phi$, which varies vith a. In a balanced two-phase circuit the instantaneous power is $c_m e_m \sin^2 a \cos\phi + c_m e_m \sin^2 (a - 90°)$ os $\phi = c_m e_m \cos\phi (\sin^2 a + \cos^2 a) = c_m e_m \cos\phi$, which is onstant. In the same way the power in a balanced hree-phase circuit is $c_m e_m \cos\phi \{\sin^2 a + \sin^2 (a - 120°) + \sin^2 (a - 240°)\} = \frac{3}{2} c_m e_m \cos\phi$, which is constant; and, n general, the power in any balanced polyphase circuit n which the phase differences are equal to $\frac{\pi}{m}$ or $\frac{2\pi}{m}$, vhere m is the even or odd number of phases, is,

$${}_m e_m \cos\phi \left\{ \sin^2 a + \sin^2\left(a - \frac{2\pi}{m}\right) + \sin^2\left(a - \frac{4\pi}{m}\right) + \cdots + \sin^2\left(a - \frac{2(m-1)\pi}{m}\right) \right\},$$

vhich is equal to $\left(\frac{m}{2}\right) c_m e_m \cos\phi$, and is constant, since

$$\sin^2 a + \sin^2\left(a - \frac{2\pi}{m}\right) + \sin^2\left(a - \frac{4\pi}{m}\right) + \cdots + \sin^2\left(a - \frac{2(m-1)\pi}{m}\right) = \frac{m}{2}.^*$$

The uniformity of power in a balanced polyphase ircuit may also be directly deduced from the proposiion that the resultant of m equal harmonic motions cting in lines having an angular difference of $\frac{2\pi}{m}$ is a niform circular motion with an amplitude equal to $\frac{m}{2}$ imes the amplitude of the components.

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[*] Todhunter's *Plane Trigonometry*, p. 243.

145. Algebraic Sum of Instantaneous Currents is Zero. — It is also easily proved that the algebraic sum of the instantaneous currents in a balanced polyphase circuit of any number of phases, *m*, is always equal to zero. Thus

$$c_1 = c_{max} \sin a ;$$

$$a - \frac{2\pi}{m}\Big);$$

$$- \frac{4\pi}{m}\Big);$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$c_m = c_{max} \quad a - 2\frac{(m-1)\pi}{m}\Big).$$

Hence, $c_1 + c_2 + c_3 + \cdots + c_m = c_{max} \Big\{ \sin a + \sin\Big(a - \frac{2\pi}{m}\Big)$

$$+ \sin\Big(a - \frac{4\pi}{m}\Big) + \cdots + \sin\Big(a - 2\frac{(m-1)\pi}{m}\Big)\Big\};$$

but evidently, $\sin a + \sin\Big(a - \frac{2\pi}{m}\Big) + \sin\Big(a - \frac{4\pi}{m}\Big) + \cdots$

$$+ \sin\Big(a - 2\frac{(m-1)\pi}{m}\Big) = 0,\ ^*$$

and therefore $c_1 + c_2 + c_3 + \cdots + c_m = 0$. When polyphase circuits have an odd number of phases, the number of line wires may be equal to the number of phases, but when the number of phases is even, the number of line wires must be one greater than the number of phases. †

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* Todhunter's *Plane Trigonometry*, p. 243.

† Blondel, Elementary Theory of Rotary Field Apparatus, *La Lumière Électrique*, Vol. 50, p. 351.

146. Relations between Currents and Pressures. — The following are the relations between the currents and the pressures in the lines and coils of a balanced three-phase system developed from the earlier discussion (Sect. 143):

1. *Star Connection.* $C_l = C_c$; $E_{AB} = E_{AC} = E_{BC} = \sqrt{3}\,E_c$. ✓ Line pressure E_{AB} is the vector sum of coil pressures E_a and E_b and is 30° behind the phase of coil pressure E_b. Line pressure E_{AC} is the vector sum of coil pressures E_a and E_c and is 30° behind the phase of coil pressure E_a. Similar relations hold for the other two corners. Y

2. *Mesh Connections.* $E_{AB} = E_c$, $C_l = \sqrt{3}\,C_c$. Line current C_A is the vector sum of coil currents C_a and C_b and is 30° ahead of the phase of coil current C_a and 30° behind the phase of coil current C_b. Similar relations hold for the other corners. ∇

The subscripts applied to the letters C and E in the paragraphs above have the following meanings: l, line; c, coil; a, b, specific coils; A, B, specific lines; AB, AC, BC, measurements between the respective corners of the circuits. Figures 254, 255, 256, and 257 should be used for reference.

If the circuits of utilization in a polyphase system are not machines (for instance incandescent lamps), the devices must be connected exactly as would be the coils of a machine, unless transformers intervene, in which case the secondary circuits may be independent; but the load should be uniformly distributed to keep the system balanced. In three-phase circuits in which the generator coils are connected in star fashion, a fourth wire may be introduced which runs from a common junction of the three branches of the load to the neutral point of the generator, but this method is not used commercially.

147. Effect of Mutual- and Self-Induction between Circuits. — The effects of self- and mutual-induction in polyphase circuits may be determined by using the principles already set forth (Sects. 47 and 110), provided the resultant effect of the differing phases is always properly considered. In unbalanced systems the mutually inductive influence of the circuits tends to increase their defects in balance.* In order to regulate phase pressures independently, pressure regulators such as those described in Section 82, or rheostats, must be introduced in each phase; or, if the generator armature is stationary, the number of active conductors on each phase may be varied by a commutator. (Example: Stanley alternator.) In some polyphase alternators where the armatures of the different phases are influenced by different field frames, the regulation may be effected by varying the field magnetism (Example: large Westinghouse alternators), but this is an unusual construction.

148. Measurement of Power in Two- and Three-phase Circuits. — The principles underlying the methods of measuring power in polyphase circuits differ in no respect from those already deduced in relation to single-phase circuits (Sect. 44), but it is desirable to apply them in such a way as to reduce the number of necessary readings to a minimum. For satisfactory measurements, non-inductive wattmeters are of essential importance, and very satisfactory commercial portable wattmeters are now to be had for a reasonable price.

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* *Electrical World*, Vol. 25, p. 302.

A. Two-Phase Systems.

A I. **Independent Circuits.** In a two-phase system with separate circuits, independent wattmeter readings are taken in each circuit and the total power is the sum of the readings. One wattmeter placed in each circuit (Fig. 259), from which simultaneous readings are taken, is the best arrangement; but if two wattmeters are not to be had, one may be inserted successively in the two circuits, and the sum of the readings is equal to the power in the system, provided the load does not vary while the readings are being taken. If the circuit is perfectly balanced, twice the reading of a wattmeter in

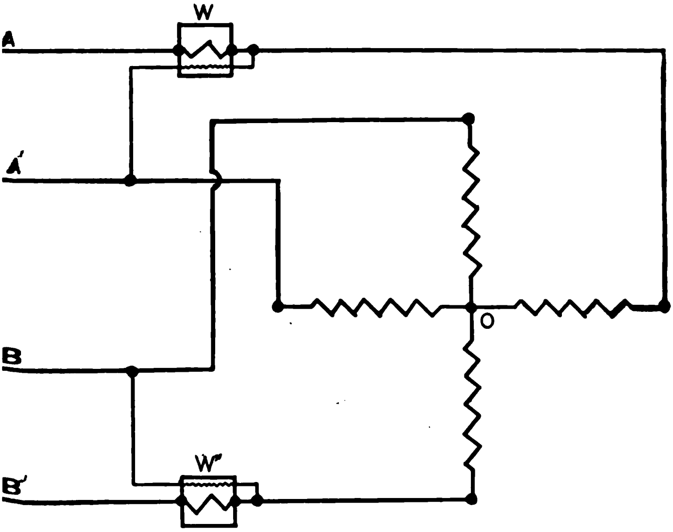

Fig. 259

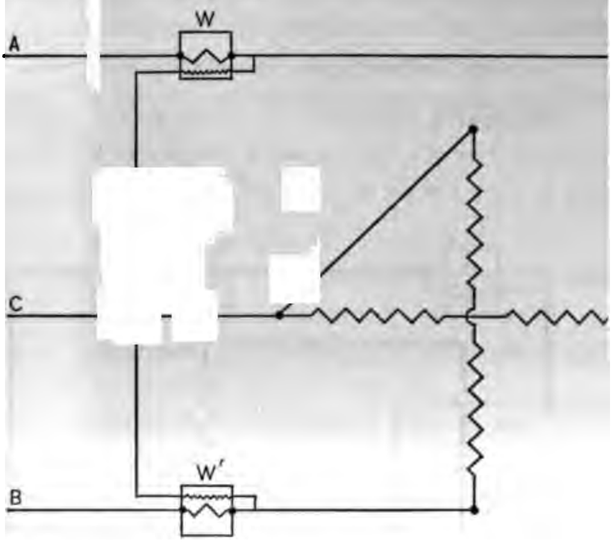

Fig. 260

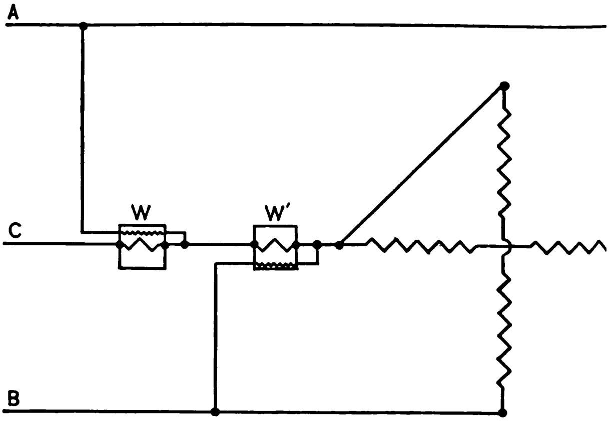

Fig. 260 a

one circuit is equal to the power, but this is a condition which cannot be relied upon.

A 2. **Circuits with Common Return.** — Two wattmeters may here be used, one for each circuit, connected in the way shown in Fig. 260. The arrangement shown in Fig. 260*a* is equivalent to a single wattmeter connected as in Fig. 261, and is only correct for a system in exact balance. When the single wattmeter is used in a balanced system, the current coil is placed in the common wire, and a reading is taken with the free end of the pressure coil connected to one outside wire. The pressure coil terminal is then quickly transferred to the other outside wire and a new reading taken. The condition of exact balance is not to be relied upon, so that the arrangement of Fig. 260 must ordinarily be used.

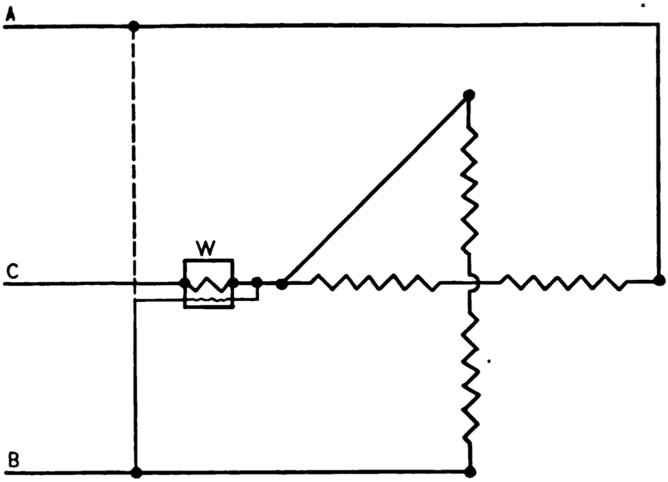

Fig. 261

The sum of the readings of the two wattmeters then gives the power in the system.

B. Three-Phase Systems.

***B* 1. Three Wattmeters.**—*a.* If the power delivered by a generat[illegible]y a motor or other device, which is co[illegible]shion, is to be measured, three wattme[illegible]l, connected as shown in

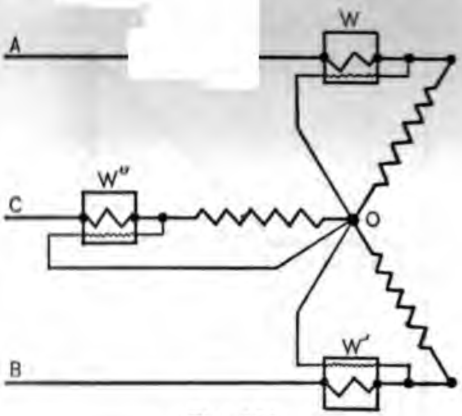

Fig. 262

Fig. 262, provided the common or neutral point is accessible. It is evident that each wattmeter measures the power in one coil so that the sum of the readings gives the power in the system. If the system is exactly balanced, three times the reading of one wattmeter gives the power.

b. If the devices are connected mesh fashion, three attmeters may still be used, provided the current coils the wattmeters can be inserted directly into the coil

·cuits as shown in Fig. 263. The power in the circuit equal to the sum of the three wattmeter readings, d if the circuit is exactly balanced, three times the ading of one wattmeter gives the power.

c. When it is impossible to insert the wattmeters in e coil circuits of a device with mesh connection, the ree-wattmeter method may still be used by the creaɔn of an artificial neutral point as shown in Fig. 264. ɔr this purpose, three equal non-reactive resistances

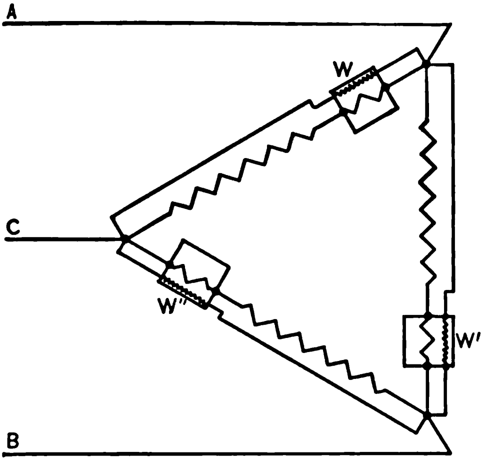

Fig. 263

re connected together at one end, and the other ends re connected to the respective corners of the mesh ircuit. The pressure between the neutral point and ither corner is equal to $\frac{E}{\sqrt{3}}$, where E is the pressure of one coil, and the phase of this pressure is ϕ degrees n advance of the current entering the corner. A wattneter with its current coil inserted in the circuit wire leading to the corner carries a current equal to $\sqrt{3}\,C$, and if the free end of the pressure coil is connected to the neutral point, the power reading of the wattmeter is

$$\frac{\sqrt{3}\,CE\cos\phi}{\sqrt{3}} = CE\cos\phi,$$

which is the il. Care must be taken that the resi ttmeter pressure coils are

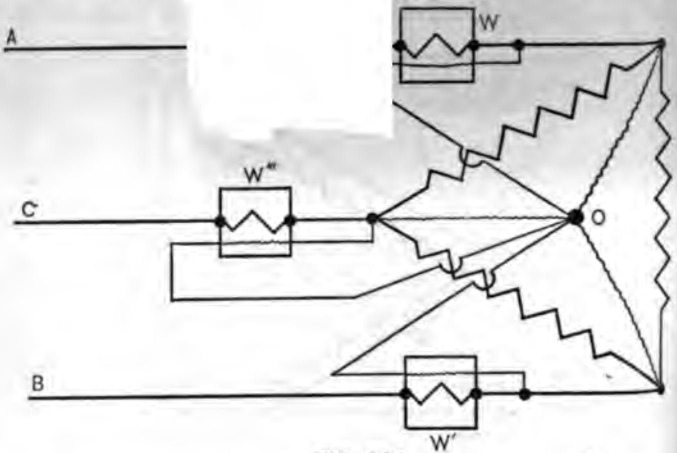

Fig. 264

so large compared with the three auxiliary resistances that connecting them in circuit does not disturb the pressure of the neutral point. If the resistances of the wattmeter pressure coils are exactly equal, auxiliary resistances are unnecessary, and the measurement may be made by joining the free ends of the three pressure coils.

These methods are independent of the condition of lance in the system or the current lag.

. **Two Wattmeters.** — The algebraic sum of the gs of two wattmeters, inserted in a three-phase t as shown in Fig. 265, gives the power in the n with entire independence of the balance of the n or current lag. When the current lag in the t is less than 60°, or the power factor is greater .50, the arithmetical sum of the readings is equal power in the circuit; but if the lag is greater

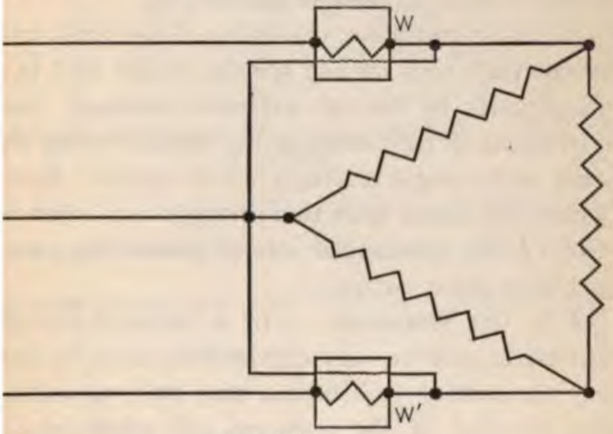

Fig. 265

60° (the power factor is less than .50), the relaf the currents in the current and pressure coils e of the wattmeters causes it to have a negative g, and the arithmetical difference of the readf the two instruments gives the power. There ne difficulty in distinguishing which condition in many cases, especially when the power ab- d by partially loaded induction motors, in which ower factor is low, is under measurement. As a general rule, if the conditions do not make the case evident, the truth may be discovered by interchanging the positions of the instruments without altering the relative connections of their main and pressure coils. If the deflections of both needles are reversed, the difference of th gs represents the power, but if the ı the same direction as before, the s gs is correct. The proof of this theore tion 149.

A double w ing of two fixed coils and two movable dle, can be used in measuring power by the two wattmeter method. Such an instrument of itself sums up the double reading algebraically, and a single reading gives the power. Recording wattmeters based upon this principle can be made very useful in the commercial sale of power from two-phase and three-phase circuits.

B 3. **One Wattmeter.** — In a balanced circuit one wattmeter may be very conveniently used by connecting the current coil in one wire and connecting the free terminal of the pressure coil alternately to the other two leads (Fig. 266), when the sum of the readings gives the power. For, the power reading of the wattmeter in its first position is $\sqrt{3}\,CE\cos(\phi+30^\circ)$, and in its second position, $\sqrt{3}\,CE\cos(\phi-30^\circ)$, and the sum of the readings,

$$\sqrt{3}\,CE\{\cos(\phi+30^\circ)+\cos(\phi-30^\circ)\}=3\,CE\cos\phi,$$

where C, E, and ϕ are the current, pressure, and lag in a coil; but $CE\cos\phi$ is the power in one coil and $CE\cos\phi$ is the total power of the three coils, hence one wattmeter gives correct indications provided the same for all the coils, and the load is uniformly

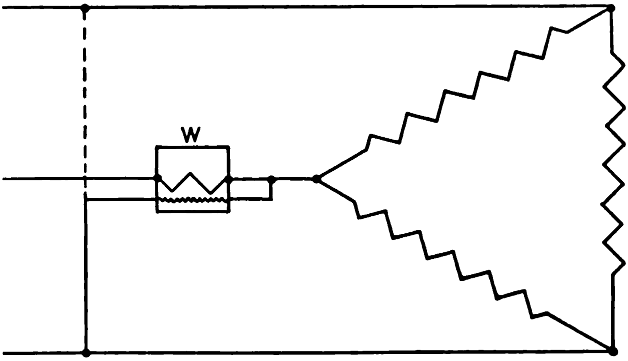

Fig. 266

ibuted. A wattmeter having two independent sure coils could be used as a direct reading instru-

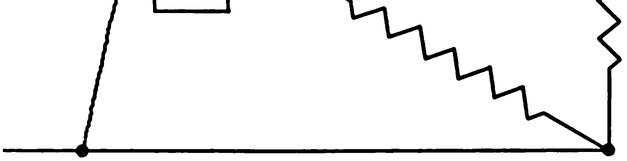

Fig. 266 a

t for this purpose. A similar wattmeter could also sed in one-wattmeter measurements of power in two-phase circuits. An ordinary wattmeter with one pressure coil may be used for the double measurement at one observation by connecting the free end of the pressure coil to the middle of a high non-inductive resistance which connects the two lines opposite to the one in which the current coil is inserted (Fig. 266*a*). This reading to the sum of the readings with the ent, but the wattmeter constant must ith one-half of the high resistance in

149. Measu Any Polyphase Circuit.* — In the case of a po system of m phases and m conductors, the power in the circuit may be measured by $m - 1$ wattmeters. Supposing A, B, C, D, etc., are points where the m conductors of a polyphase supply circuit connect to the circuits under test, then, as has been already proved (Sect. 145), $\Sigma c = 0$, if c represents the instantaneous current in any branch. The power supplied through the A conductor at any instant is equal to $\frac{q_a v_a}{dt} = c_a v_a$, where q_a is the quantity of electricity brought to A during a time dt, and v_a is the absolute electrical potential of A.

The average power transferred through conductor A during a complete period is $\frac{1}{T}\int_0^T c_a v_a dt$, and the total power in the circuit, $W = \Sigma \frac{1}{T}\int_0^T cvdt$.

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[*] Blondel, Measurement of the Energy of Polyphase Currents, *Proc. Elect. Congress held at Chicago*, p. 112; Lunt, Measurement of the Power of Polyphase Currents, *Electrical World*, Vol. 23, p. 771.

The absolute potentials of the points are inconvenient measure, and it is desirable to introduce into the nula the difference between the pressures at the nts and some fixed point of potential v'. Since $= 0$, we also have $\Sigma cv' = 0$, and $\Sigma cv'$ may therefore directly inserted in the formula without destroying equality, or

$$W = \Sigma \frac{1}{T}\int_0^T (cv - cv')\,dt = \Sigma \frac{1}{T}\int_0^T c\,(v - v')\,dt.$$

iting e for $v - v'$ (the instantaneous difference of ssure between the fixed point and any given point the system) gives

$$W = \Sigma \frac{1}{T}\int_0^T ce\,dt.$$

fixed point may be taken at one of the corners of circuit, A for instance, since $\Sigma cv' = 0$ holds equally it, and the power formula becomes

$$V = \frac{1}{T}\int_0^T c_b e_{ab}\,dt + \frac{1}{T}\int_0^T c_c e_{ac}\,dt + \text{etc.} + \frac{1}{T}\int_0^T c_m e_{am}\,dt,$$

$$W = C_b E_{ab} \cos\theta' + C_c E_{ac} \cos\theta'' + \text{etc.}$$

re θ', θ'', etc., are the angular differences in the ses of the pressures and currents. The terms on right of the equation are the familiar forms repnting the power readings of a wattmeter, so that $- 1$ wattmeters are inserted in circuit with their rent coils respectively in the $m - 1$ conductors, B, D, etc., and the free ends of their pressure coils all connected to A, the algebraic sum of their readings is equal to the power in the system.

When the circuit includes three phases only, the formula is $W = C_bE_{ab} \cos \theta' + C_cE_{ac} \cos \theta''$, and only two wattmeters, connected as in Fig. 265, are required to give the power in the circuit, but due regard must be had to the relative signs of $\cos \theta'$ and $\cos \theta''$. From the relative phases of the currents C_b and C_c and pressure E_{ab} (Sect. 146) it is easy to see that in a balanced system $\theta' = \phi' - 30°$, and also that $\theta'' = \phi'' + 30°$, and therefore

$$W = C_bE_{ab} \cos (\phi' - 30)° + C_cE_{ac} \cos (\phi'' + 30°),$$

in which ϕ' and ϕ'' are the angles of lag of the circuit currents. The formula shows that the first term at the right, which represents the reading of one wattmeter, is positive within the limits $\phi' = +90°$ and $\phi' = -60°$, and that its value is negative between the limits $\phi' = -60°$ and $\phi' = -90°$. The second term, which represents the reading of the second wattmeter, is positive between the limits $-90°$ and $+60°$ and negative between $+60°$ and $+90°$. Consequently, *if the current lags equally in the circuits*, or $\phi' = \phi''$, both wattmeters have a positive reading, and *the power in the circuit is the sum of the readings, for angles of lag between* $+60°$ *and* $-60°$. *If the angle of lag is* $+60°$ (the current lags behind the pressure), the second wattmeter reading is zero, and *the power in the circuit is equal to the reading of the first wattmeter. If the lag is greater than* $+60°$, the reading of the first wattmeter is positive and the second is negative, and *the power in the* *uit is equal to the difference of the two readings.* in, *if the angle of lag is* −60° (the current leads pressure), the first wattmeter reading is zero, and *ower in the circuit is equal to the reading of the second rument. If the lead is more than* 60°, the reading of first instrument is negative and of the second posi-

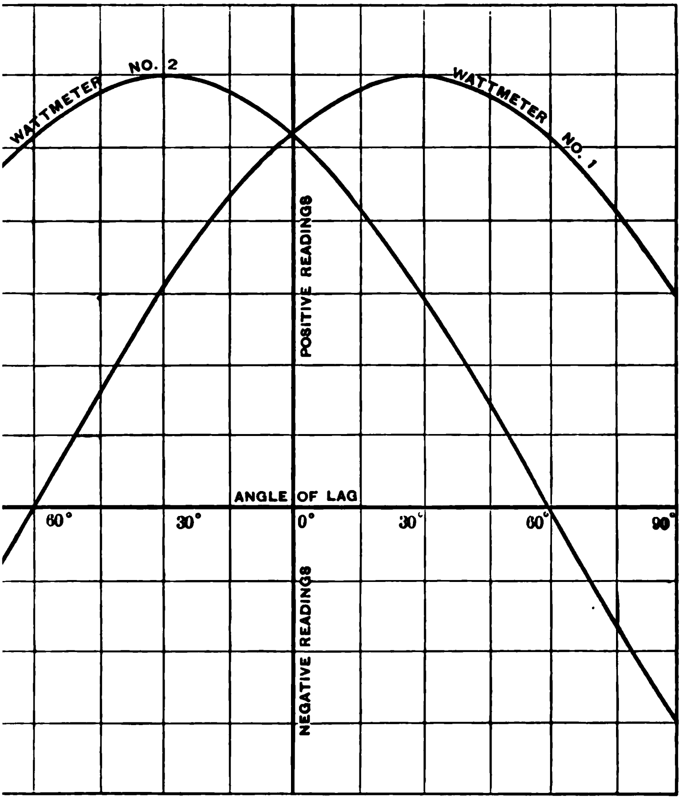

Fig. 267

tive, and *the power in the circuit is equal to the difference of the two readings.* When the angle of lag is ±90°, the readings of the two instruments are equal, but one is positive and the other negative. The relation of the wattmeter readings to the angle of lag between +90° and −90° are shown by the curves in Fig. 267. These are two equal sinusoids with a phase difference equal to 60°. The readings of two wattmeters in a balanced three-phase circuit at any value of the lag are in the proportion of the corresponding ordinates of the two curves. The same conditions obtain in an unbalanced circuit, provided equivalent angles of lag are considered

CHAPTER XIV.

ALTERNATING-CURRENT MOTORS.

50. Alternators as Synchronous Motors.[*] — Any ernator may be run as a motor, provided it is ught up to synchronous speed and into step be- e it is thrown into circuit. The motor will then in complete synchronism if left to itself. If it is erloaded, or by other means is thrown out of synchro- m, it will stop. In general, the action of an alter- tor used as a synchronous motor is quite similar to t of an alternator operated in parallel with another. great disadvantage of single-phase synchronous tors is the fact that they are not self-starting, but st be brought up to speed before they will operate; d while polyphase synchronous motors may be made start themselves without load, the operation is uneco- mical. The starting of single-phasers may be done a small series-wound auxiliary motor made with lam- ted fields. Such a motor will run when placed in alternating-current circuit, since the magnetism of fields and armature will reverse together as the current changes direction; but very little power can be developed by such a machine on account of its enormous self-inductance. A small two-phase motor (Sect. 182), with a device for splitting the current into two phases, may be used (see Fig. 268); or the exciter of

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[*] Picou, Transmission de Force par Moteurs alternatifs synchrones, *ll. Soc. Int. Électriciens*, Vol. 12, p. 60; Blondel, Couplage et chronization des Alternateurs, *La Lumière Électrique*, Vol. 45, pp. , 563; Rhodes, A Theory of Synchronous Motors, *Phil. Mag.*, July, 5; Alt. Current Motors, *Lond. Elect. Review*, Vol. 37, pp. 182, 222.

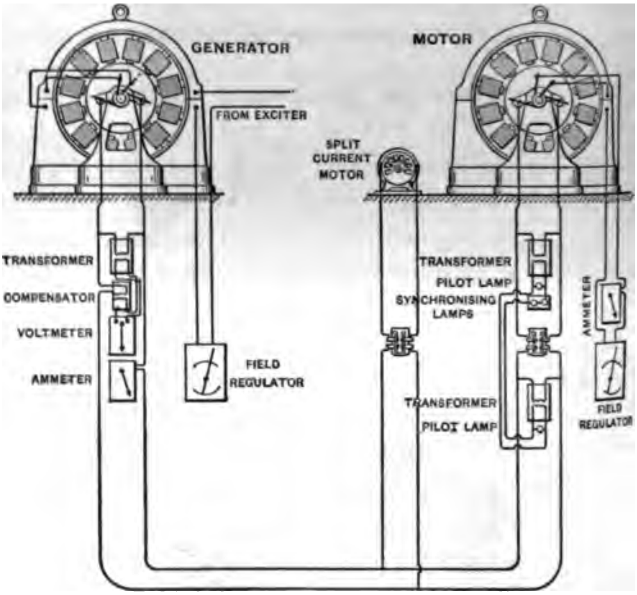

Fig 268

the alternator may be run as a motor by a storage battery and used to bring the alternator into synchronism, the storage battery being recharged by current from the exciter after the alternator is operating on the circuit. Polyphase synchronous motors may be tarted by an ordinary polyphase induction motor, such s is described in later sections.

While the practical disadvantage of synchronous motors, due to the fact that they are not self-starting, may be overcome by these special devices, the expense of motor equipment is increased, and, at the best, the motor cannot be started under load. Consequently, synchronous motors are not satisfactory for general power distribution. They have been used with considerable satisfaction in certain special plants for the long-distance transmission of power, and may be said to be destined to play an important part for such work; but for general power transmission and distribution purposes, they cannot be satisfactorily used.

151. Relation of Field Strength to the Working of Synchronous Motors. — When a synchronous motor is put in the circuit, a peculiar relation exists between the strength of the field of the motor and the current in its armature. In continuous-current motors, if the strength of the field is slightly changed without altering any of the other conditions, the speed of the motor changes inversely, and the current in the armature remains practically unchanged; but the speed of a synchronous motor cannot change permanently, and, consequently, upon first consideration, it would appear that the field of a synchronous motor must be exactly adjusted, in order that the machine may operate satisfactorily. This, however, is proven not to be the case in practice, on account of the effect which may be gained through variations of the relative phases of the current and of the impressed and counter pressures. The active pressure, which at any instant causes current to flow through the armature of a motor, is equal to the difference of the corresponding instantaneous values of the impressed and counter pressures. If the field strength of a motor is so adjusted that the values of the impressed and counter pressures are equal, and the motor armature is brought into exact step with the impressed pressure curve, then, when the motor is switched on the supply main, it will fall back in phase with respect to the impressed pressure, sufficiently to permit the proper load current to pass through the armature. Now suppose that at some instant the load is increased, the difference of instantaneous pressures at that instant will be insufficient to pass the current, which is necessary for the new load, through the armature. The motor, therefore, falls back in its phase without losing synchronism, if the load is not too great, and then continues operating in synchronism, but with a greater lag in step. When a motor lags in step behind the phase of impressed electromotive force, its counter pressure lags to an equal extent. The armature current ordinarily takes an intermediate phase, so that it is behind the resultant pressure, but in advance of opposition to the counter pressure.

Were it not for the effect of the current lag with respect to the resultant pressure, caused by self-inductance, it would be necessary to adjust the field excitation of a synchronous motor, so that its counter pressure would be less than the impressed pressure, and the range of load carried with a given excitation would be small. The effects due to the current lag, however, make it possible to adjust the field excitation ce for all, so that the motor may be operated on a dely varying load. It is even possible, on account the automatic adjustment of the pressure phases, to erate a motor when its excitation is much greater or ich less than its normal value. The adjustment is sisted by the effect of armature reactions on the otor, in which a lagging current tends to strengthen e fields and a leading current to weaken them (Sect.). When a single motor is operated from an altertor of about its own size, the automatic adjustment the machines is still more marked, since the current ich strengthens the field of the motor tends to aken that of the alternator as the load is varied, d *vice versa*, which is desired.[*]

It is evident from the preceding that the armature rrent of a motor must have a wattless component ich depends directly upon the phase differences of e impressed and counter pressures and the angle of g, and it may readily be seen that the most economical citation of a synchronous motor field is that which duces the armature current to a minimum (or makes e power factor a maximum) when the motor is carryg the average load.

152. Graphical Illustrations showing the Relations Pressure and Current in a Synchronous Motor Armare. — In order to bring out more clearly the facts just ven, recourse may be had to a diagram in which relations of current and pressures are shown much as in parallel working. It was shown (Sect. 87) that in parallel working the machines were held in step by a motor action, and that if one machine was cut off from its prime mover it would continue to run in synchronism as a motor, its pressure being unchanged. In Fig. 269 let OC be the [illegible] through two alternators, one acting as [illegible] t OL be the pressure of self-induction [illegible] OS the active pressure

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[*] Compare Ryan, The Behaviour of Single-Phase Synchronous Motors, *ley Journal of Engineering*, May, 1894; Scott, Long-Distance Transsion for Electric Lighting and Power, *Trans. Amer. Inst. Elect. Eng.*, l. 9, p. 425.

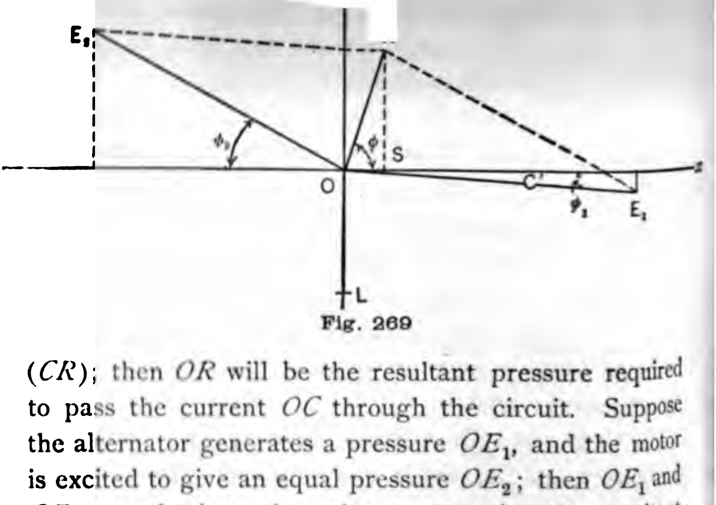

Fig. 269

(CR); then OR will be the resultant pressure required to pass the current OC through the circuit. Suppose the alternator generates a pressure OE_1, and the motor is excited to give an equal pressure OE_2; then OE_1 and OE_2 must be in such a phase as to give the resultant OR, while the elements of pressure resolved upon the current line OC must be such that the element of OE_1 s the same direction as the current and the element E_2 is in opposition. The work delivered by the rator is $OC \times OE_1 \cos\phi_1$, and that utilized by the motor armature in furnishing power and overcoming the magnetic and friction losses is $OC \times OE_2 \cos \phi_2$; while that lost, due to resistance, is their difference, and is equal to

$$OC \times OR \cos \phi = OS \times OC.$$

It will be seen that for small loads the current may lead the generator pressure as shown in Fig. 269, but that as the load increases (and the length of OR therefore increases), the generator pressure is caused to swing forward so that the current takes a lagging position, as shown in Fig. 269 *a*. The construction indicates that the current is always in the lead of direct opposition to the counter pressure when the impressed and counter pressures are equal, and that the value of ϕ_2 increases when the load on the motor is increased. The value of C is one-half greater in Fig. 269 *a* than

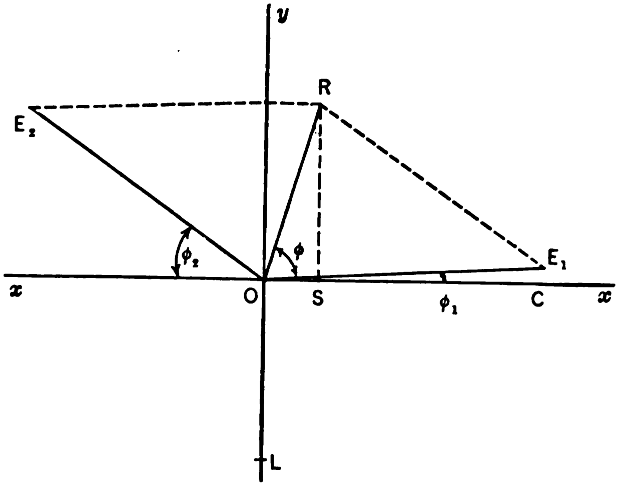

Fig. 269 a

in Fig. 269, the input of the motor is 50 per cent greater, and the output about 45 per cent greater. The latter is increased in a smaller proportion because the C^2R loss increases directly as C^2, while the output increases less rapidly than C. The motor will continue to operate, as the load is increased, until ϕ_2 has attained such a value that $E_2 \cos \phi_2 \times C$ becomes a maximum; then, if an additional load is put on the motor, the corresponding increase of ϕ_2 will cause $CE_2 \cos \phi_2$ to decrease, and the motor will fall out of synchronism and stop, because the maximum value of its torque is not sufficient to pull the load. In the case under consideration (when the impressed and counter pressures are equal) this will not occur with well-designed alternators until a load much above the normal is reached.

153. Impressed and Counter Pressures Unequal. — As was stated in a preceding section (Sect. 151), the pressure at which the motor is run, and therefore its excitation, has an important bearing upon the stability of operation and the efficiency of transmission. If the motor pressure is made larger than that of the generator, the current and pressure relations may be shown by a construction similar to that used in Fig. 269. Let OK in Fig. 270*a* represent the magnitude and direction of the resultant pressure, and the impressed and counter pressures have magnitudes OE_1 and OE_2; then the parallelogram can be completed with but one value of the angles ϕ_1 and ϕ_2, *i.e.* that shown in the figure. In this case the counter pressure is greater than the impressed pressure. Now suppose the counter pressure is made OE'_2, having the same horizontal projection as while OR and the impressed pressure have the values as before; then the phase relations are own by the lines OR, OE_1', and OE_2'. The values $E_2 \cos \phi_2$ and $OE_2' \cos \phi_2'$ are equal by the contion, since the points E_2 and E_2' are in the same :al line, and since OR has the same magnitude and

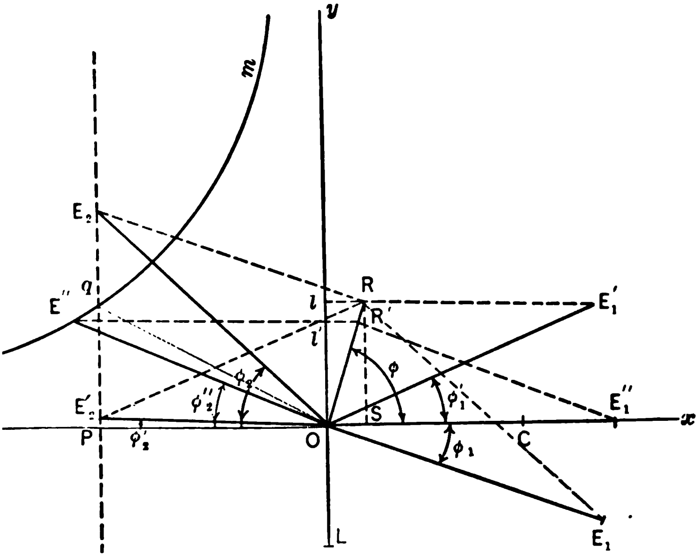

Fig. 270 a

ion in the two cases the current is the same, and $\cos \phi_1$ and $OE_1' \cos \phi_1'$ are equal; but in the first the counter pressure is greater than the impressed ure and the current leads the impressed pressure, n the second case the impressed pressure is greater the current lags. This construction shows that for / load on the motor, except that corresponding to a zero angle of lag, there are two values of the excitation which cause the same armature current to flow, the current leading the impressed pressure with one excitation and lagging by an equal angle with the other excitation: hence an over-excited motor acts upon the line current very much like a condenser, and an under-excited motor acts like an inductance coil. When the counter pressure is much less than the impressed pressure, the current may lag with respect to opposition to the counter pressure, but under no other conditions, and this is not a practical condition.

154. Excitation which gives Greatest Power Factor.— The excitation at which the motor will do the most work with a given current flowing, or will carry a given load with the least current (and hence do it most efficiently), is that which causes the current to come into phase with the impressed pressure. In this case the current for a given load is a minimum, and $E_2 \cos \phi_2$ is a maximum. If the value of OE_2 in Fig. 270*a* is increased (by increasing the excitation of the motor), either the length of OR which is proportional to C, or the impressed pressure, must be increased, provided $CE_2 \cos \phi_2$, which is equal to the motor load, is constant. On the other hand, if OE_2 decreases while OE_1 remains constant, the angle of lag, ϕ_1, and the current, decrease until the current and impressed pressure come into phase, after which a further decrease of OE_2 causes the current to increase again, as is shown by the relations between E_1, C, and E_2 in the figure. The value of $CE_2 \cos \phi_2$ is proportional to the area of the rectangle $Olqp$, since Ol is proportional to C. Now if the motor load is kept constant, but its excitation is changed, the corner of the corresponding rectangle must be different from q, but the locus of the motion of the corner is a hyperbola with its origin at O and the rectangular axes x and y, since the rectangles included between the axes and the ordinates and abscissas of all points must be of equal area. Consequently, the point of the vector representing E_2 at the least current for a given load must be in the rectangular hyperbola, *mm*, which passes through q. This point, which is E_2'' for the given load, is found by laying off the horizontal line equal in length to OE_1 which will just reach between the hyperbola and the line OR. This cuts OR at R', and $C = \frac{E_i''(= OI')}{2\pi fL}$, which is the minimum current for the load. The impressed pressure and current are, under these conditions, in phase with each other. At this point $E_2'' \sin \phi_2'' = E_i''$. If a larger or smaller pressure than OE_2'' is used, such as OE_2 or OE_2', the impressed pressure and current are thrown out of phase, and the current in the circuit is therefore increased, which causes increased C^2R losses and armature reactions. The excitation of the motor which brings the current and impressed pressure into step, depends upon the relation of the impedance of the motor circuit to its resistance. If $I = 2R$, the counter pressure is equal to the impressed pressure at zero angle of lag, and if $I \gtrless 2R$, the counter pressure is respectively greater or less than the impressed pressure at zero lag.* The expression $I = 2R$ is equivalent to, reactance equals $\sqrt{3}\,R$ ($I = \sqrt{R^2 + 4\pi^2 f^2 L^2} = 2R$, and, therefore, $2\pi fL = \sqrt{3}\,R$). In the machines which are now commonly built, the impedance of the armature circuit is commonly equal to or larger than twice the resistance, so that a maximum power factor is gained in such synchronous motors by an excitation which gives a counter pressure that is equal to or greater than the impressed pressure.

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* Steinmetz, Theory of the Synchronous Motor, *Trans. Amer. Inst. E. E.*, Vol. 11, p. 767.

155. Curve showing the Relation of Armature Current to Excitation. — We may plot the relation of armature current to field excitation for a motor operating under the conditions considered above, by taking the corre-

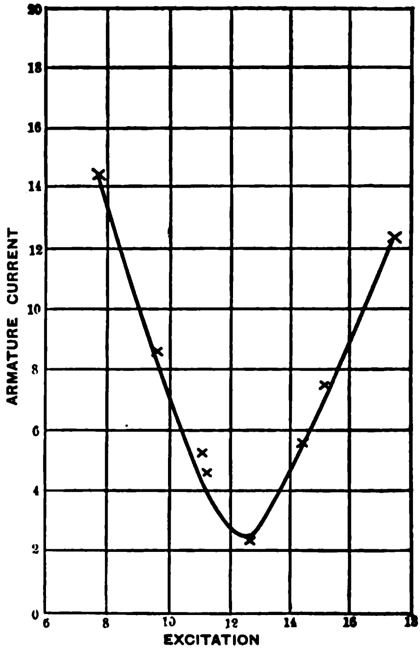

Fig. 270 b

ponding values of E_2 and C from a chart made like ig. 270*a*. This gives a curve like Fig. 270*b*, which has vo values of its abscissas for every value of the ordiate except the lowest; or for each value of the armaure current there may be two values of the excitation, ne being greater and the other less than the impressed ressure, except at the point of minimum armature urrent, which corresponds to but one excitation, as has lready been explained (Sect. 153). The way in which ig. 270*a* is constructed shows that the smaller the angle ϕ, or the smaller the armature self-inductance, the less vill be the difference in the two excitations correspondng to any armature current; and hence the curve showng the relation of excitation to current in a machine aving a large time constant is broad and rounded, but e curve for an armature having a small time constant sharp and narrow. In an ideal machine without selfductance, the two values of excitation for a leading nd lagging impressed pressure are equal, and the curve ecomes a straight line.

156. Maximum Load. — When a synchronous motor s operated with a fixed excitation on a variable load, he step of the motor will automatically adjust itself, when the load changes, to the changed conditions, until $CE_2 \cos \phi_2$ is equal to the new load, provided a certain maximum limit is not exceeded. As the load increases, he current must increase, whence it is evident from Fig. 270*a* that the change in phase of the motor pressure must be in the direction which increases ϕ_2, and that $CE_2 \cos \phi_2$ will therefore reach a maximum point beyond which the motor cannot work, since $\cos \phi_2$ decreases as

ϕ_2 increases. This point depends upon the self-inductance of the armature, which controls ϕ_2, when the impressed and counter pressures are constant. If the load on the motor is made greater than the maximum value of $CE_2 \cos \phi_2$, the motor must fall out of synchronism and stop. The smaller the impedance of the motor circuit, the less will the angle ϕ_2 change with any change of current, when E_2 remains constant, as is seen by the construction of Fig. 270 *a*, and therefore the maximum load which the motor will carry depends inversely on its impedance; but the greater the angle of lag, ϕ, the less will be the value of ϕ_2 for fixed values of the pressures and impedance, and consequently the less rapidly will $\cos \phi_2$ vary with a given variation of ϕ_2. So that the maximum load which a motor will carry depends inversely upon the impedance of its armature circuit, and directly upon the angle by which the current lags behind the resultant pressure.* In practice, armature reactions always tend to weaken the field of the motor as the current is in the lead of opposition to the counter pressure; and it is therefore advisable to excite the machine rather above that pressure corresponding to the least current for normal load, as the armature reactions then tend to decrease the field strength and thus modify the motor pressure so as to cause a decrease in the amount of armature current. If the excitation is made smaller than that corresponding to minimum current, the armature reactions cause the deviation from minimum current to become still greater. To decrease the effect of armature reactions as well as make the machine capable of carrying considerable overloads without being dragged out of synchronism, it is advisable to use strong fields, and armatures with the least number of conductors compatible with an economical design. There is ordinarily no danger to the motor if it stops, as the armature inductance cannot be economically reduced below a value sufficient to prevent a destructive flow of current, as was shown in the example in Section 94.

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* Mordey, Alternate-Current Working, *Jour. Inst. E. E.*, Vol. 18, p. 595; Kolben, *Elektrotechnische Zeitschrift*, Vol. 16, p. 802; *London Electrical Engineer*, 1895; Kapp's *Electrical Transmission of Energy*, 4th ed., p. 278.

As showing the gain in stability of operation by exciting the motor somewhat above that which would result in the minimum current, were there no armature reactions present, Mr. Kapp gives the following theoretical table.*

TABLE SHOWING WORKING CONDITION OF TRANSMISSION PLANT.

Total resistance in circuit, 1 ohm; total reactance, 4 ohms.

Generator excited to give	1100	1100	1100 volts
Motor excited to give	1200	1300	1350 volts
Normal power given off by motor . . .	125	125	125 H.P.
Maximum power given off by motor before breaking from synchronism . .	200	250	268 H.P.
Margin of excess load causing breakdown of the system	60	100	134 per cent

With a smaller impedance in circuit, the possible overload before the motor breaks from synchronism would be greater, as is shown by the considerations just discussed, the experiments of Kolben,* and the experience in American plants.

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* *Electrical Transmission of Energy*, 4th ed., p. 277.

157. Experiments of Bedell and Ryan. — Bedell and Ryan† made a series of experiments on a pair of diminutive eight-pole, smooth-core Westinghouse alternators, giving a frequency of 139, one of which was run as a motor, and the other as a generator. (The machines were built to each supply ten 16 C.P. lamps.) The resistance of the machine circuit was .31 ohm, and the self-inductance of the motor armature .32 millihenry.

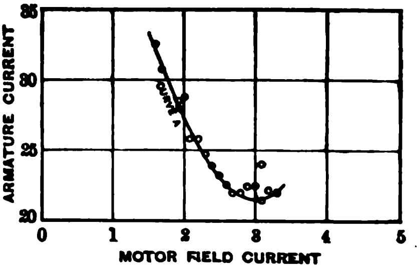

Fig. 271

The curve of magnetization of the motor was practically a straight line. It was found that the motor would operate only under field excitations varying from 1.5 to 3.5 amperes, and required an abnormal armature current to carry its load, the minimum current being at an excitation of 3 amperes (Fig. 271); also, that a very small ase of load would throw it out of synchronism. load consisted of the friction of a $\frac{1}{2}$ horse-power on dynamo. A variable inductance consisting of l with a movable iron core was then inserted in ircuit. By moving the core of this coil it was found when its inductance was 1.68 millihenrys the motor red a minimum armature current for a given load, with stability through a wide range of load, and

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* *Elektrotechnische Zeitschrift*, Vol. 16, p. 802; *London Electrical Engineer*, 1895.

† Action of a Single-Phase Synchronous Motor, *Jour. Franklin Inst.*, March, 1895.

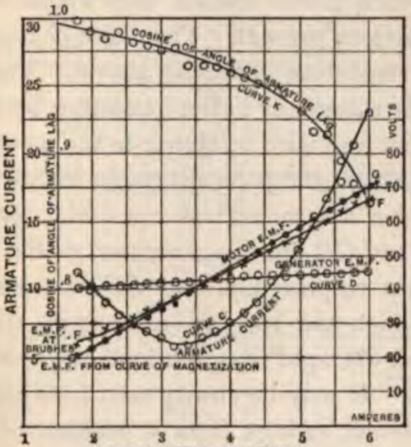
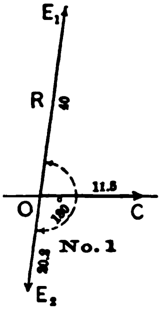


Fig. 272

ated at excitations of from 1.8 to 6 amperes. The ation was not carried over 6 amperes as there was er of springing the motor shaft, which was weak. e *C*, in Fig. 272, shows the armature current for rent excitations when the motor was under the small constant load, as in the trial without ex- l inductance. Curves *D*, *F*, and *G* show plainly endency of the armature reaction referred to above to hold the generator and motor excitations at the point of maximum efficiency. Curve *D* represents the generator pressure, and, although the excitation was constant, the generator pressure rises as the motor pressure is increased. This is due to the reaction caused by the current swinging from a lag to a lead with reference to the generator pressure. At the same time the motor pressure, which is represented by curve *F*, at first is larger and then grows smaller than would be the case were no reactions present. The curve *G* represents the pressure, considering reactions absent. The effect in the motor is caused, as in the generator, by the current increasing its lead with reference to the motor pressure. Figure 273 shows the polar diagrams for various excitations at which the motor was run under constant load. OE_1, OE_2, and OR are the generator, motor, and resultant pressures respectively, and OC the current. The angle by which the current lags behind the resultant pressure was obtained from the impedance of the armature circuit. It may be clearly seen from the diagrams that the current swings from a position of large lag, with reference to the generator pressure, at a small excitation of the motor, into phase with it, — the point of minimum current for the given load on the motor, — and finally into a position of large lead when the motor is greatly over-excited.

This series of experiments and the preceding discussions (Sect. 156) show that there was some foundation for the statement of earlier experimenters, that alternators must have self-inductance in their armature circuits if they are designed to be run in parallel. The appli-



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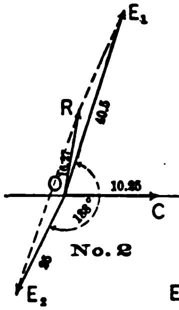

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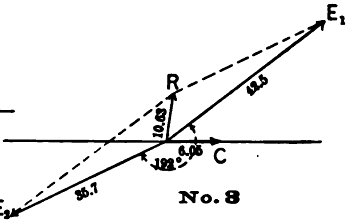

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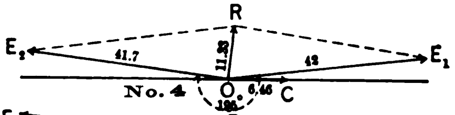

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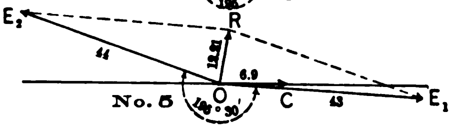

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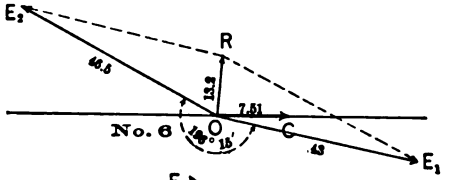

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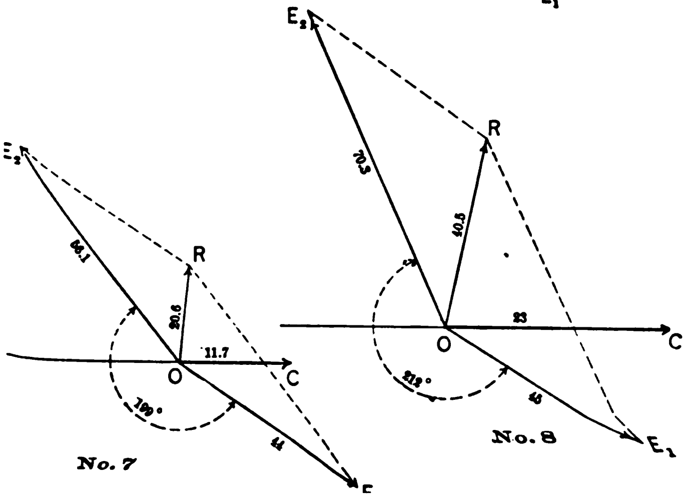

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No. 8

cation of that statement to the case, however, is fallacious, since alternators operating in parallel should require much less than the torque of normal load to hold them in step, so that the synchronizing tendency of armatures with small inductance is ample to make them run in parallel; and *for either parallel working or for operation as synchronous motors, a small armature impedance is of the greatest importance.*

158. Effect of Wattless Current on Torque. — Since a synchronous motor seldom operates at the exact load for which its excitation is adjusted, the armature current is likely to have a large wattless component. Hence, during a portion of each half period the motor armature must return to the circuit some of the energy which was delivered to it during the remainder of the half period. This causes the torque of a single-phase armature to vary from a large positive value to a small negative value in each half period, and in order that this effort to return the energy represented by the wattless current may not break it from synchronism, it is well for the armature to be very solidly built, or to have a fly-wheel attached to its shaft. Since the torque of a polyphase armature is uniform throughout the period (Sect. 144), polyphase synchronous motors are likely to run more satisfactorily than single-phasers.

The magnitude of the wattless component depends directly upon the armature self-inductance and the amount of excitation given the motor. When the armature self-inductance is small, the armature current does not differ greatly with different excitations, and hence the wattless current in average operation is reduced.

is an additional advantage of motors having arma- with a minimum impedance.

I. **Rotary-Field Induction Motor.** — The well-known iples which cause the rotation of a disc of copper ed above a rotating horseshoe magnet have been nto use through the discoveries of Ferraris, Tesla, lwander, Dobrowolsky, and many others. The gements proposed by Tesla were doubtless the direct applications of these principles to commer- se, in which they are destined to play a large part e transmission and distribution of power.* An st simultaneous publication of a series of scientific iments by Ferraris shows the operation of similar atus,† and various experiments of a similar nature a similar purpose are on record. Each of these iments caused an iron or copper armature to rotate placed within the region of a rotating magnetic

I. **A Rotating Magnetic Field.** — If two coils of are arranged at right angles so as to enclose a lrical iron core, or if two pairs of coils are placed ht angles on a ring core (Fig. 250), the magnetism in the core when a current is passed through the is the resultant of the magnetization due to the oils.

the magnetizing currents are two sinusoidal alter- g currents with 90° difference of phase, then, at

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New System of Alternate-Current Motors and Transformers, *Amer. Inst. E. E.*, Vol. 5, p. 308.

ectro-dynamic Rotation by Means of Alternating Currents, *Lon- ctrician*, Vol. 21, p. 86.

any instant, the magnetizing force due to one of the coils is $H_1 = H_m \sin a$, and of the other coil is

$$H_2 = H_m \sin (a - 90^\circ) = H_m \cos a,$$

where H_m is the maximum magnetizing force of either coil (Fig. 274). The resultant magnetizing force is then $H_R = \sqrt{H_1^2 + H_2^2} = H_m$, and is therefore constant in magnitude. The direction of this constant magnetizing force is variable. When $a = 0^\circ$, H_R lies in the plane of the first coil, and when $a = 90^\circ$, H_R lies in the plane of the other coil. The magnetizing force of each coil has a sinusoidal or harmonic variation, and the resultant magnetizing force is the resultant of two harmonic variations with 90° difference of phase. As is well known, such a resultant has a uniform magnitude and a uniformly varying direction. The instantaneous values of the resultant may therefore be diagrammatically represented by the instantaneous positions of a line of fixed length, rotating at a uniform rate around one end, such as OH_R in Fig. 274.

If the maximum ampere-turns of one coil are greater than those of the other coil, the magnitude of the resultant magnetizing force varies. The rotating field, in this case, may be diagrammatically represented by a uniformly rotating line, which varies in length, so that its tip traces an ellipse whose minor and major axes are respectively in the planes of the stronger and weaker coils. If the windings of the coils are similar, and the currents equal, but the phase difference is not 90°, a variable field again results.

If the phases of the two currents are in unison,

$$H_R = \sqrt{H_1^2 + H_1^2} = \sqrt{2}\, H_m \sin a.$$

s shows that when the two currents are in unison varies with sin *a*, and therefore varies from zero to aximum of $\sqrt{2}\, H_m$, but its direction must be constant, e the values of its two components are equal at every ant. Its direction evidently lies in a plane between planes of the two coils. The diagrammatic represen- on of the resultant, here, is a line of fixed direction

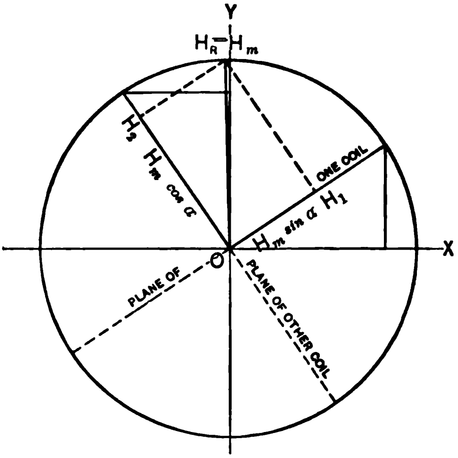

Fig. 274

ch harmonically varies in length, the total range of ation being from $-\sqrt{2}\, H_m$ to $+\sqrt{2}\, H_m$.

'or any difference of the current phases between and 90°, both the magnitude and direction of H_R in vary, and the diagrammatic representation is again ie with its tip tracing an ellipse. The ratio of the axes depends upon the phase difference of the cur- s. If the currents have 90° phase difference, but planes of the coils are not 90° apart, the effect on resultant magnetizing force is evidently the same as if the conditions were reversed. If the currents are not sinusoidal, the value of the resultant magnetizing force, H_r, varies in a more or less irregular manner. Thus, Fig. 275*a* indicates in a general manner the strength of the field at different angular positions when a peaked current is applied to two coils having 90° difference of position, and Fig. 275*b* is the same for a flat-topped current curve having the same maximum value. The dotted circles in each case represent the rotating magnetizing force due to sinusoidal currents in the same coils.

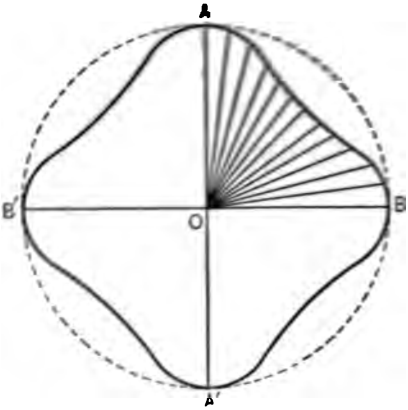

Fig. 275 a

The same argument may be readily seen to apply to the resultant magnetizing force due to any number of coils surrounding a core. When equal coils are at equal angular distances, and equal currents in the individual coils differ in phase by an amount equal to the angular distance of the coils from each other, the resultant magnetizing force is always uniform in mag-

: and rotates at a uniform rate, provided the curare sinusoidal, and its value is $H_R = \frac{m}{2} H_m$, where he number of phases* (compare Sect. 144). The

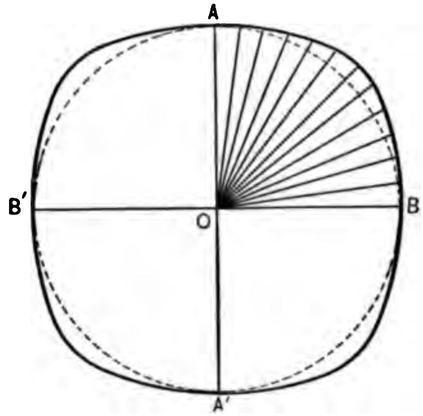

Fig. 275 b

:tness of these deductions is directly indicated periment.

: Germans call the rotating field **Drehfelde,** and olyphase currents which set up a rotating field **trom,** or rotating current.

Action of a Short-circuited Armature Winding ı a Rotating Field.—If a drum core of laminated e properly pivoted within a ring, on which coils are ıated that the field rotates, it will be dragged into n by the magnetic pull. If the pivoted core be per, it will be dragged into rotation by the res of the foucault currents which are developed : core. This is directly analogous to the experiment with the Arago disc, to which reference has already been made (Sect. 159).

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* E. Arnold, *Elektrotechnische Zeitschrift,* Vol. 14, p. 42.

In the case of either a solid core or Arago disc, the foucault currents are not constrained in position, and therefore take the path of least resistance. The result is that much of the effectiveness of the currents in bringing about a rotation is lost, and the efficiency of the device is small. If, in the disc experiment, the disc be cut up into an indefinitely large number of fine radiating wires which are connected together at their inner and outer ends, the useless or parasitic eddies may in a large measure be done away with, and the efficiency of the device be considerably raised. In the same way the drum core may be made of laminated iron in order that the magnetic circuit shall be of small reluctance, and embedded in this may be copper wires which cross the face of the core and are all short-circuited by copper rings at the ends (Fig. 276). These make constrained paths for the induced currents, and, if the core is sufficiently laminated and the copper conductors are not too thick, the

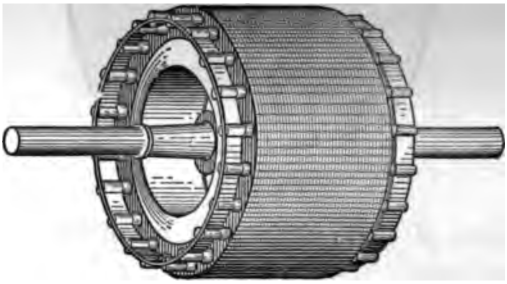

Fig. 276

parasitic eddies are largely done away with, and the efficiency of such a motor may be made quite large.

162. Variation in a Rotating Field. — There has been considerable dispute regarding the uniformity of the strength of the rotating field in motors of this class. The question at issue being whether the effective magnetizing force at each instant is equal to the sum of the ampere-turns on the coils, or the ampere-turns are compounded to gain the resultant effect according to the parallelogram of forces. The latter assumption is made in the discussion given above (Sect. 160). Dobrowolsky, Pupin,* and others have taken the other view, and have determined from that standpoint that there is a fluctuation of about 40 per cent in the strength of the field due to two sinusoidal currents with 90° difference of phase, and about 14 per cent fluctuation in the field due to three sinusoidal currents with 120° difference of phase.

With a view of experimentally determining which assumption is correct, Messrs. Hanson and Webster undertook, in the electrical laboratories of the University of Wisconsin, the experimental measurement of the strength of the rotating field of a three-phase motor, when magnetized with three sine currents with phase differences of 120°. For this purpose they placed a test coil on the surface of the motor armature and arranged the armature so that it could be readily rotated through a small arc of fixed value. The reading of a ballistic galvanometer connected to the test coil was therefore proportional to the number of lines of force cut by the coil when the armature was rotated. The magnetization was effected by continuous currents in the windings of the motor fields, which were so adjusted as to give the proper phase relation to each other. Thus, calling the coils *a*, *b*, and *c*, and supposing the current in *a* is desired to be the instantaneous zero value of the current, then the current in *b* must be adjusted so that

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* *Trans. Amer. Inst. E. E.*, Vol. 8, p. 562.

$$C_b = c_{max} \sin 120^\circ,$$

and the current in *c* must be adjusted so that

$$C_c = c_{max} \sin 240^\circ.$$

The resultant magnetism thus produced is equal to the instantaneous magnetization due to an alternating current taken at a corresponding instant. To get the instantaneous magnetization for any other phase of the alternating currents, the test currents must be so adjusted that

$$C_a = c_{max} \sin a,$$
$$C_b = c_{max} \sin (a + 120^\circ),$$
$$C_c = c_{max} \sin (a + 240^\circ).$$

The algebraic sum of the currents must always be equal to zero. The apparatus was arranged somewhat as in Fig. 277. By the method thus outlined it was found that the magnetization due to the field windings advanced uniformly as a wave of fixed magnitude, as closely as the limits of error of the experiment would show. As these errors were well within 2 or 3 per cent, the experiments prove:

1. That the resultant magnetizing force due to the

eral coils arranged as in the rotary-field motor is, practical purposes, equal to the magnetizing effects all the coils compounded according to the ordinary thods of composition of harmonic variation.

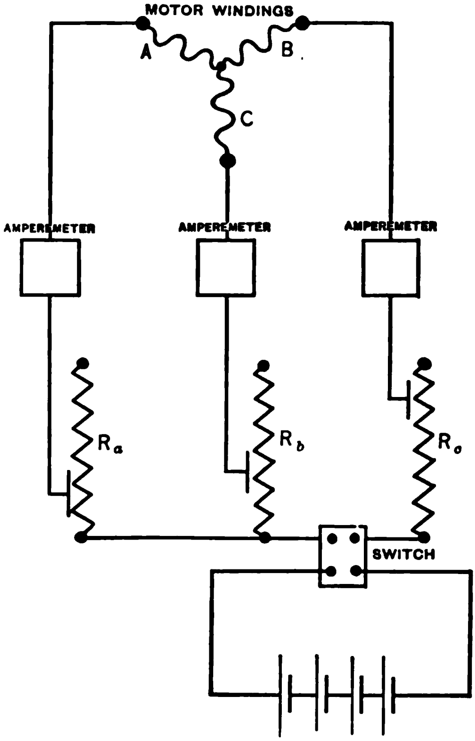

Fig. 277.

:. That the magnetization set up is practically protional to the magnetizing force when the induction .ot pushed too high.

'o determine to what extent the saturation of the ı in the magnetic circuit affects the latter deduction, Hanson and Webster made tests which covered a considerable range of maximum currents, and which were carried above the bend in the curve of magnetization of the motor. The deductions given above appeared to be practically correct within the limits of the experiments.[*] Similar experiments have been proposed and carried out by du Bois-Reymond,[†] Blondel,[‡] Behn-Eschenburg, and others.

163. Distinction between Armature and Field. — There is some ambiguity in the designation of the armature and fields of induction motors, since it is not uncommon to make them with revolving field cores, and both fields and armature carry an alternating current, but the following definitions avoid all ambiguities. The **Field** *is the core upon which are placed windings connected to the external circuit.* The current in the fields is therefore due to the impressed pressure of the external circuit. The **Armature** *is the part of the motor in the conductors of which current is induced by the revolving magnetism of the fields.* Since the armature current is wholly induced by action of the fields, these motors are called **Induction Motors.** With these definitions, it is readily seen that the induction motor acts, in many respects, like a transformer, the primary winding of which is on the fields, and the secondary winding on the armature.

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[*] Jackson, Three-Phase Rotary Field, *Electrical Journal*, Vol. 1, p. 185. Also see Pupin, *Trans. Amer. Inst. E. E.*, Vol. 11, p. 549.

[†] Theoretical and Experimental Study of Polyphase Currents, *Elektrotechnische Zeitschrift*, Vol. 12, p. 303; *Electrical World*, Vol. 17, p. 477.

[‡] Elementary Theory of Rotary-Field Apparatus, *La Lumière Électrique*, Vol. 50, p. 358.

The Germans call polyphase induction motors **Drehstrom Motors.**

The energy developed in the secondary circuit of the induction motor is expended in causing rotation of the revolving part instead of causing heat and light in the external circuit, as is the case of the ordinary transformer. The same general methods apply, in designing these motors, that apply in designing transformers.

164. Wattless Magnetizing Current. — Since an air space must be made in the magnetic circuit to allow the motors to operate, it is evident that the wattless magnetizing current of induction motors must be materially greater than that of transformers. In fact, the no-load current of some comparatively small motors of this type, which show quite a high efficiency, is entirely comparable to the full-load current. To reduce the wattless current to a reasonable limit, every effort must be bent to decrease the reluctance of the air space. As the armature conductors may be embedded in the armature core, it is possible to make the air space simply that required for mechanical clearance, and, by care in the workmanship, this may be made very small compared with the air space of dynamos built according to the ordinary methods.

165. Motor Speeds and Slip. — The velocity of rotation of the magnetic field depends upon the frequency of the current supplied to the motor, and the number of pairs of poles in the field. In two-pole machines, the number of rotations which the field makes per second, or the **Field Frequency,** is equal to the current frequency, and, in multipolar machines, the field frequency is equal to the current frequency divided by the number of pairs of poles, or $\frac{f}{p}$. The number of pairs of poles which is referred to is the number in the rotating field. This is equal to the number of pairs of poles set up by the windings in fields with a smooth magnetic surface, but is equal to $\frac{1}{2m}$ times the number of salient poles in salient-pole machines (m being the number of phases). The latter can scarcely be said to give a uniformly rotating field unless there are m crowns of poles.

The velocity of rotation of the armature can never equal the velocity of rotation of the field magnetism, since the armature conductors must be cut by the lines of force of the fields in order that an electrical pressure may be developed in the armature; that is, the field magnetism must always have a relative velocity of rotation with reference to the armature conductors. In any machine, the relative velocity is $v = V - V'$, where V and V' are respectively the number of revolutions per minute of the field magnetism and the armature conductors. This relative velocity is called the armature **Slip**, and is small, seldom exceeding 5 per cent of the speed of the motor. Since the current in the armature must be proportional to the work done by the motor, it must vary with the load, and v must increase as the load is increased. A little consideration shows that, if the magnetism remains constant, the variation of v with the load must be just sufficient to counterbalance the drop of pressure caused by the current flowing in the armature conductors.

A variation of v demands a variation of V' of equal magnitude, since V is fixed by the frequency of the current delivered to the motor; consequently, the speed regulation of a rotary-field motor is directly dependent upon the loss of pressure in the armature conductors if we neglect the effect of armature reactions and drop of pressure in the primary windings. This is entirely analogous to the case of continuous-current shunt-wound motors.

At starting, the relative velocity of the field magnetism and the armature is evidently V, since V' is zero. The armature current is therefore very great, and the starting torque may also be very great provided the armature reactions do not too greatly disturb the field. To avoid injury to the armature from the current at starting, means must be taken to prevent its becoming excessive, exactly as in the case of continuous-current machines worked on constant pressure.

166. Graphical Illustration of Relations in Induction Motors. — The reactions of the polyphase induction motor may be set forth very clearly by graphical representation. Suppose we have under consideration a two-phase motor, as shown diagrammatically in Fig. 278, where aa' and bb' are two pairs of coils in series, each pair being connected to a pair of two-phase feeders. The armature we will suppose for convenience is of the squirrel-cage or short-circuited bar type. That is, the conductors are embedded in the face of the armature and are short-circuited by rings extending around the armature at each end (see Fig. 276). From the foregoing discussion (Sect. 160) it is evident that a magnetic north pole on one side and a magnetic south pole just opposite, will rotate around the field core with the frequency of the alternating current (f). This magnetic field will induce under it, in the conductors of the armature which it cuts, a pressure which causes a cur- ıt to flow in the conduc- rs. This current, if the nature is free from self- luctance, will set up a ıgnetic field lagging 90° hind the magnetism set up by the field windings (Fig. 278*a*). Therefore let OB in Fig. 279 be the strength of the rotary field which produces in the armature the resultant current, OC_e. Then OA may be represented as the field due to this armature

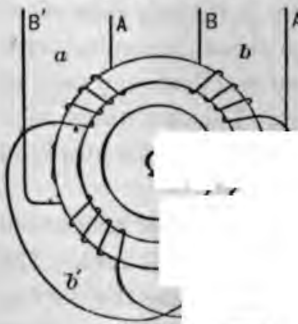

Fig. 2'

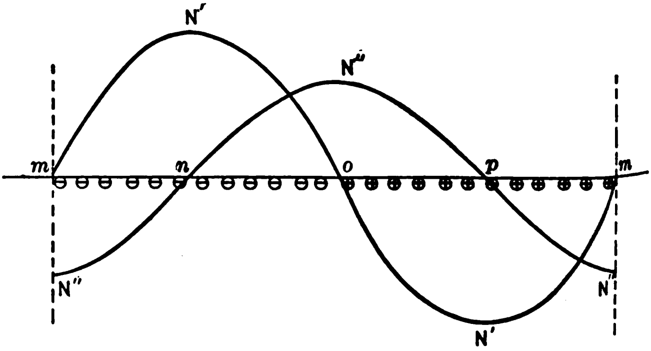

Fig. 278 a

current. The impressed magnetizing force must be ufficient to supply the field OB and overcome OA, or ıst be sufficient to set up a field OM. OC_f repre- ıts the relative phase and magnitude of the field curovided the number of primary conductors is the number of secondary conductors. This tion requires us to consider the polyphase combined at every instant into a resultant

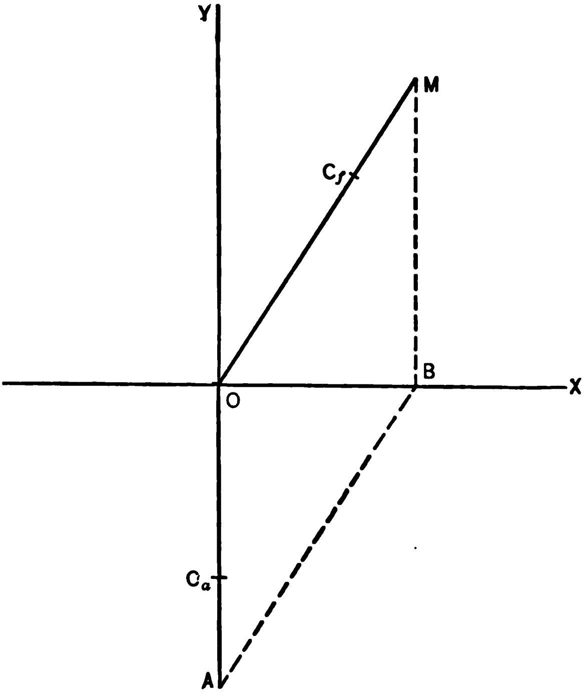

Fig. 279

ay be represented by the sum of the vertical ns of the polyphase current vectors. This ion simplifies the construction very much and the use of exactly the same method as that transformers (Sect. 118); namely, in Fig. 280, if OC_μ is the magnetizing ampere-turns required to set up the desired field, and OC_a the armature ampere-turns, then the field ampere-turns must be OC_r. Also the self-induced pressure in the fields will be OE_1, drawn to the proper scale, while OE is the pressure that must be applied to the fields when OA is the element of pressure which multiplied by the current furnishes the motor losses.

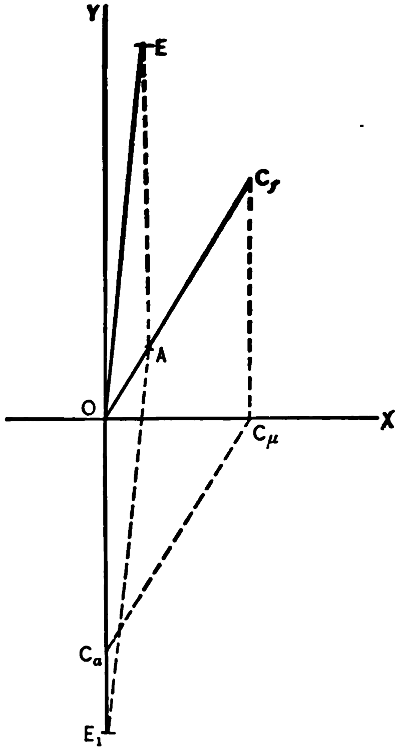

Fig. 280

167. Torque of Ideal Motor. — It is evident that in a ıotor giving this diagram, the starting torque will be normous for a low-resistance armature, as the current ıduced will be enormous, since the wattless magnetizing urrent, and hence the resultant magnetic field, remains ractically constant if there are no armature reactions. 'he torque is of course proportional to the field mulplied by the current, or to $OB \times OC_a$ (Fig. 279). As ıe motor comes up to speed, the current will decrease irectly as the speed increases, since the relative speed r slip of the armature with reference to the rotating eld decreases directly as the speed increases. Hence, eglecting armature reactions and self-inductance, the ›rque will be a maximum with the armature standing :ill and will gradually decrease to zero as the armature ›eed increases towards its limit, which is synchronism 'ith the rotating field.

168. Effect of Magnetic Leakage. — As there must be learance between the armature and field, a path is ıade for magnetic leakage, which, on account of the pposing action of the armature and field magnetism, ecomes of very considerable magnitude at large loads. .ines of force set up by the armature and leaking ırough this clearance space cause the armature current › lag exactly as though self-inductance were introduced ıto the windings, as is also the case in the fields. This ffect materially alters the diagram, as is shown in 'ig. 281, where OE_{2a} and OE_{1a} are respectively the ɾmature and field reactive pressures. The diagram in his case is also drawn exactly as in the case of transormers, where there is self-inductance in the primary

and secondary coils, and gives an impressed field p
ure OE and a field current OC_f lagging by the a
ϕ_f. The angle of lag ϕ_a in the armature evidently

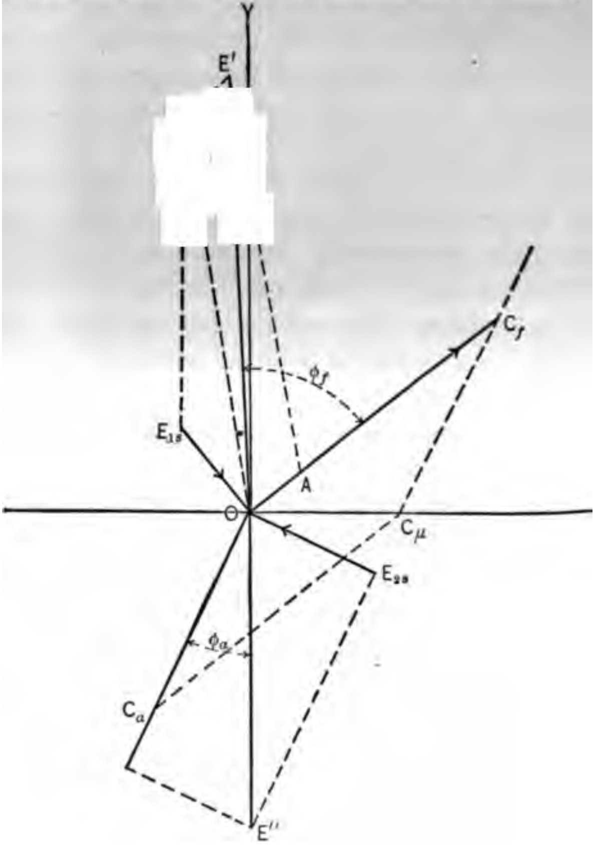

Fig. 281

cause a serious decrease in the motor torque from
auses; first by decreasing the armature current f
en induced pressure, and second by retarding the ph

:he current with respect to the magnetic field. For this son, added to its effect on the slip, induction motors built to give as small a leakage field as possible. ;ure 282 indicates a method of winding frequently ployed, which makes it possible to reduce the leakage d to a minimum by reducing the air space. The ıdings are placed in evenly distributed slots, thus

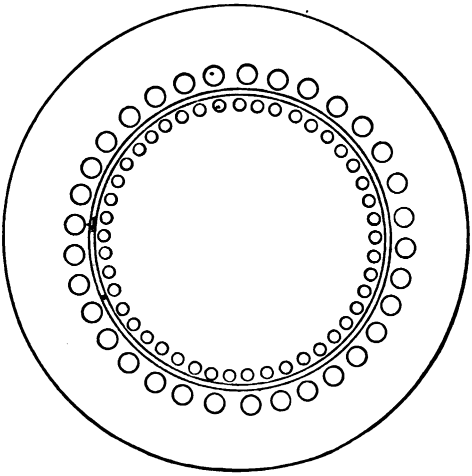

Fig. 282

›iding polar projections. As the frequency of alter: ion in an armature bar is a maximum when the tor is at rest, its reactance is then a maximum, and armature current is caused to have a maximum lag h respect to the induced pressure; and the torque a given current is reduced in proportion with the ine of the lag, $\cos \phi_a$. Since

$$\tan \phi_a = \frac{2\pi f L_a}{R_a}, \text{ or } \cos \phi_a = \frac{R_a}{I_a},$$

it is evident that by increasing the resistance of the armature conductors at starting, cos ϕ_a may be increased, and the starting torque, which is equal to

$$m\frac{C_a E \cos\phi_a}{2\pi V'} = K\frac{m v_1 \cos\phi_a}{\text{Impedance}},\text{*}$$

may be increased to a maximum.

The constant, K, in this formula depends upon the winding and dimensions of the armature and the strength of the magnetic field. In practice an external resistance is usually introduced by some mechanical device into the armature windings, at starting, which serves both to increase the torque at starting and to avoid the excessive rush of current which might occur with the armature stationary. Figure 283 shows the relation of torque to slip for an armature having a reactance of .18 and resistances of .02, .045, .18, and .75 ohms.† This shows plainly that the torque can be caused to have a maximum value up to a slip equal to 10 per cent of the field frequency by gradually reducing the resistance of the armature circuit from .18 to .02 ohms as the speed of the armature increases. The relations are as follows:

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* $C_a = \frac{e}{I} = k\frac{v}{I}$, where k is a constant depending on the strength of the field and the number of armature conductors; $E = kV'$; $K = \frac{k}{2\pi}$. E is the pressure that would be induced in the armature windings if the armature were rotated at a speed of V' revolutions per minute in a stationary field equal in magnitude to the rotating field; e is the pressure that would be induced under similar conditions at a speed v; v_1 is the frequency with which the field cuts the armature conductors when the slip is v, and, therefore, $v_1 = \frac{pv}{60}$. m is the number of phases.

† Steinmetz, *Trans. Amer.* Inst. E. E., Vol. XI., p. 760.

$$\frac{\cos\phi_a}{I_a} = \frac{R_a}{I_a^2} = \frac{R_a}{R_a^2 + 4\pi^2 v_1^2 L_a^2}$$

the torque is therefore equal to

$$mK\frac{v_1 R_a}{R_a^2 + 4\pi^2 v_1^2 L_a^2},$$

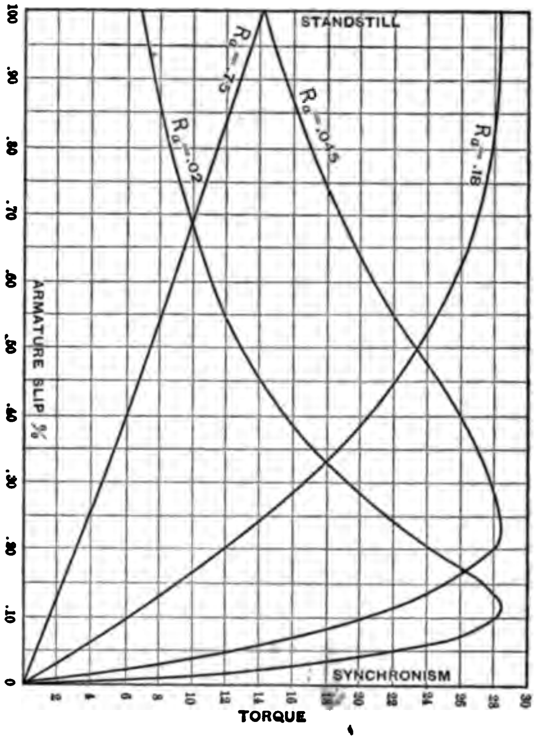

Fig. 283

ich is a maximum when $2\pi v_1 L_a = R_a$. When the arma-
e is at rest, $v_1 = f$ and to give maximum torque R_a
st be equal to $2\pi f L_a$. If R_a is either greater or less,

the torque is reduced, for armature at rest. If R_a is greater than $2\pi f L_a$, the torque continuously decreases as the speed of the armature rises; while if R_a is less than $2\pi f L_a$, the torque reaches a maximum when v has such a value that $R_a = 2\pi v_1 L_a$. If R_a is very small compared to $2\pi f L_a$, the starting torque is very small, and the torque increases to a maximum which occurs at a slip very near to synchronism.

Induction motors are usually designed to run at a speed which is between synchronism and the speed giving the greatest torque. In designing them, L_a is made the least possible, and R_a is then given such a value that the slip at normal full load is sufficient to give a value of the torque, which is from one-half to three-fourths of its maximum value. Such motors can therefore carry considerable overloads, but if the resisting moment of the load is increased beyond the maximum torque, the motor stops. In this respect, induction motors differ from continuous-current motors operated on a constant pressure, in which the torque increases in direct proportion with the armature current and therefore with the resisting moment of the load, provided the total magnetism passing through the armature remains constant. Increasing the load on a continuous-current motor will not stop it until the armature burns up or the drop of pressure due to current flowing through the armature conductors is equal to the impressed pressure.

169. Forms of Armature Windings. — The armature windings of induction motors may be of either drum or ring types, though the drum type is most commonly used. The arrangement of the windings may be of three forms:

1. *Squirrel-cage form*, in which single embedded bar conductors are placed on the armature core and all connected together at each end by a copper ring, thus making a conductor system similar in form to the revolving cylinder of a squirrel cage (Fig. 276). The conductors are insulated from the core.

2. *Independent short-circuited coils.* In this form of winding the armature conductors are of insulated wire wound in independent short-circuited coils, or of insulated bars connected by end connectors in such a way as to make independent short-circuited coils.

3. *Independent coils short-circuited in common.* Here the coils are wound as in the preceding form, but instead of being short-circuited independently, all the ends are brought to a common point, or pair of points, one of which may be at the front end and the other at the back end of the armature.

It is evident that the pitch of the coils of the second and third forms of drum windings must be equal to an odd number of times the pitch of the field poles, in order that the electrical pressure set up in the conductors may be additive, and coils may therefore be diametral or chordal in machines with an odd number of pairs of poles, but cannot be diametral in machines with an even number of pairs of poles. The actual number of coils is a matter of perfect freedom, provided it is a multiple of two or three and the connections of conductors be properly made so that the armature surface may be uniformly covered. A three-coil winding for a drum armature, which is intended to surround the revolving field of an eight-pole machine, is shown diagrammatically in Fig. 284, and a three-coil armature which is intended to revolve within a six-pole field, is shown in Fig. 285.

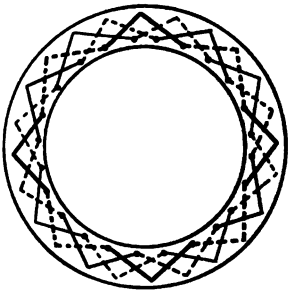

Fig. 284

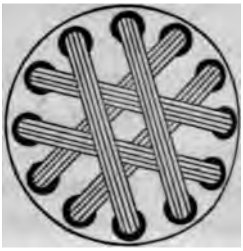

Fig. 285

170. Field Windings.— The field windings of induction motors are almost always arranged to produce more than two poles, in order to bring the machine to a reasonable speed. The field frequency in revolutions per second, as already shown (Sect. 165), is equal to $\frac{f}{p}$, where f is the frequency of the alternating current and p the number of pairs of poles, and in revolutions per minute this becomes $V=\frac{60f}{p}$, and we therefore have the following table of motor speeds for the frequencies common in this country.

This table shows the futility of attempting to build satisfactory induction motors of small size, intended for use on even the lowest frequencies commonly used in this country, with less than six poles; and on the higher

mber of poles of motor.	Frequencies in common use.				V, when $f = 25$.
	V, when $f = 60$.	V, when $f = 66\frac{2}{3}$.	V, when $f = 125$.	V, when $f = 133$.	
2	3600	4000	7500	8000	1500
4	1800	2000	3750	4000	750
6	1200	1333	2500	2666	500
8	900	1000	1875	2000	375
10	720	800	1500	1600	300
12	600	666	1250	1333	250
16	450	500	937	1000	187
20	360	400	750	800	150
24	300	333	625	666	125

quencies not less than twelve poles are required to e a reasonable speed. Motors of greater output than horse power should have a sufficiently large number

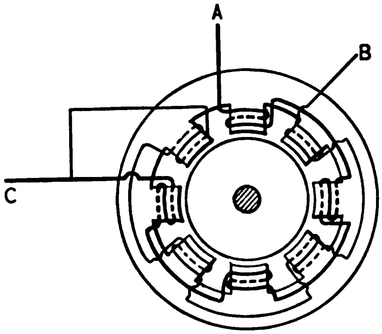

Fig. 286

poles to give a field velocity which does not exceed 750 800 revolutions per minute. The windings for this rpose may be either placed directly upon polar projecns of the *field frame*, as in Fig. 286, which shows a four-pole two-phase machine, or they may be arranged as embedded conductors in a frame of uniform magnetic surface, as in Fig. 287, which shows a four-pole three-phase machine. The embedded conductors may be

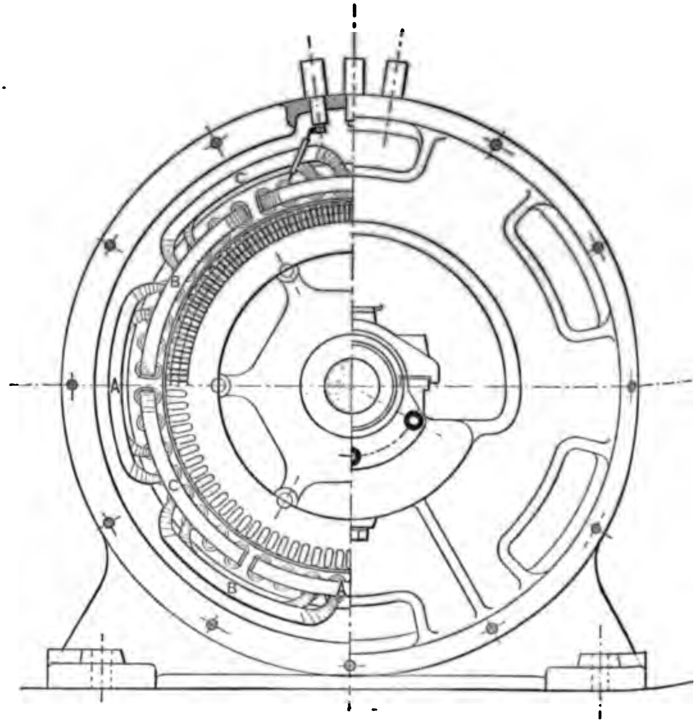

Fig. 287

wound as either a drum (Fig. 287), or ring (Fig. 287*a*) field, and they give the most approved arrangement, since embedding serves to reduce the reluctance of the magnetic circuit and therefore to increase the power factor of the motor. Since the actual magnet poles are

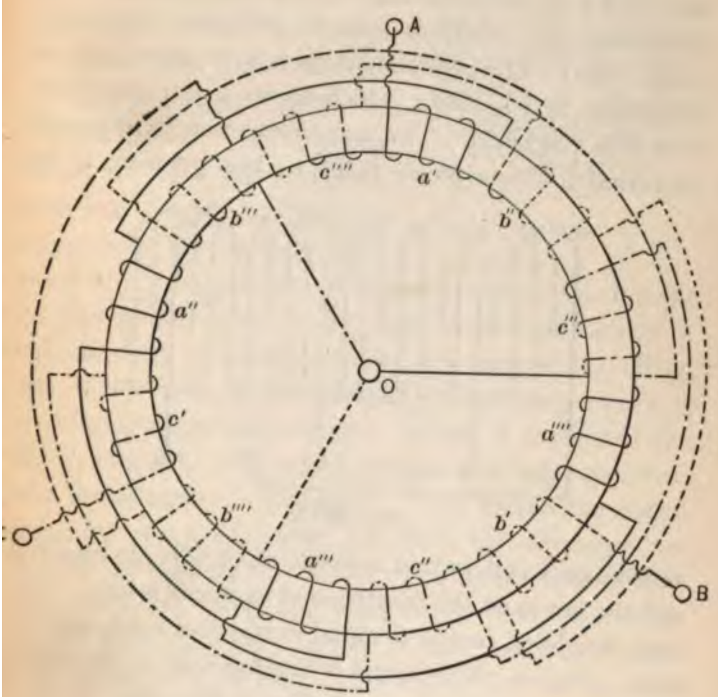

Fig. 287 a

produced by the resultant effects of the polyphase currents, it requires two-coil sections to produce each magnet pole in a two-phase machine, and three-coil sections in a three-phase machine. The connections of the field coils may be traced out according to the instructions given for connecting the armature coils of polyphase generators (Sect. 102*a*). The connections for a three-phase field are illustrated by Fig. 287*a*, which shows a star-connected, four-pole ring field. The star connection is ordinarily preferred for three-phase fields, as less pressure is impressed on a coil, so that fewer turns of wire are required and the strain on the insulation of each coil is less. The total weight of copper is equal in star and mesh connections. Figure 287*b* shows a development of the eight-pole, star-connected drum winding of Fig. 288.

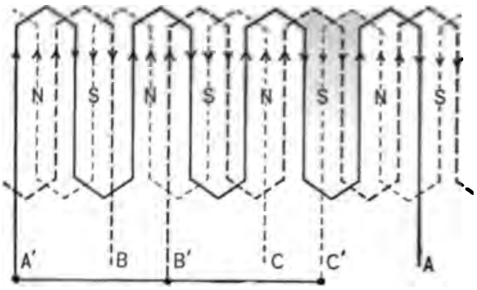

Fig. 287 b

The frequency of the magnetic cycles in the iron of the fields is equal to the frequency of the current flowing in the magnetizing coils, but in the armature it is equal to the motor slip; and the hysteresis and foucault current losses per pound of iron are therefore many times greater in the fields. In this respect the characteristics of an induction motor are exactly the reverse of those of a continuous-current machine; where the field loss consists of the C^2R loss only, while the armature loss is the sum of the C^2R and core losses, of which the latter may be the larger portion. In the induction motor, the field-core losses are large and the armature-core losses are scarcely appreciable in a well-designed machine; it is therefore desirable to reduce the amount of iron in the fields to the least volume possible, if it can be done without increasing the magnetic density. For this reason, in machines of considerable size, it is usual to arrange the armature so that it surrounds the fields, as in Fig. 288, in which case the latter revolves and the armature is stationary. This makes the use of collector rings necessary, but their disadvantages are usually inconsiderable. In all cases where squirrel-cage armatures are used, the number of field conductors should be an uneven multiple of the number of armature conductors, in order that there may be no dead points in starting.

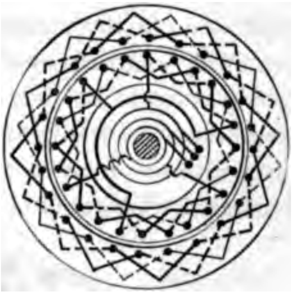

Fig. 288

171. Starting and Regulating Devices. — Four different arrangements may be used for starting polyphase induction motors.

1. *Small machines* are commonly connected directly to the circuit without the intervention of any special starting devices. This is not a safe proceeding for

large machines, as when the armature is at rest and the fields are directly connected to the supply circuit, the machine is in the condition of a transformer with the secondary short-circuited, and is liable to burn up before getting under way. The Allgemeine Elektricitäts Gesellschaft of Berlin arrange their smaller motor armatures with two rows of conductors, making two independent squirrel-cages (Fig. 289), one considerably farther from the armature surface than the other, which is reported to reduce the starting current of the machines.

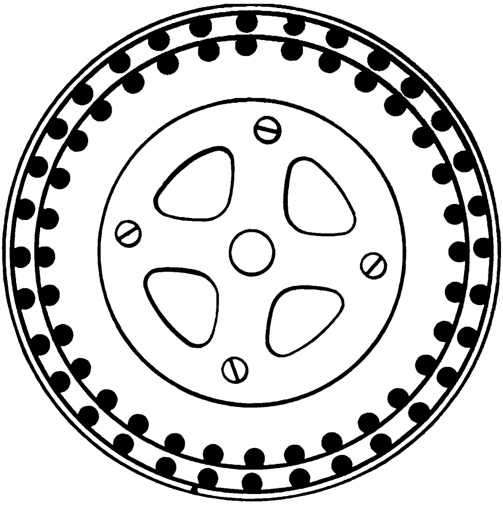

Fig. 289

2. (a) *Resistances in Field.* Resistances may be inserted in the circuits leading to the motor fields, to be used in much the same manner as starting resistances are used in starting continuous-current, constant-pressure motors. Resistances arranged in this way in each cir-

cuit must be manipulated simultaneously, and therefore must be mechanically coupled. Starting rheostats similar to continuous-current motor starting boxes, and liquid resistances arranged to be varied by dipping plates in a bath, have been used. Three rheostats are required for a three-phase motor, and two for a two-phase motor operated on independent circuits. Two-phase motors on three-wire circuits may be started with a single resistance inserted in the common wire, or by two resistances inserted respectively in the independent wires.

The insertion of resistance in the field circuits of induction motors serves to reduce the starting current on ight loads by reducing the pressure at the field terminals; this also causes a reduction of the magnetism, which is equivalent to reducing K in the expression for he starting torque $\left(mK\frac{v_1R_a}{R_a^2+4\pi^2v_1^2L_a^2}\right)$, which shows hat the starting torque under these conditions is mateially smaller than the maximum torque of the armature, ince $4\pi^2v_1^2L_a^2$ is bound to be considerably larger than R_a^2 if the armature C^2R losses are not excessive; and he motor must therefore take an excessive current to tart a heavy load. This plan has been used quite exensively by European manufacturers, especially for large machines which may be started with belt on loose pulley.

(b) *Variable Compensator or "Auto-transformer."* The pressure at the terminals of the fields may be reduced at starting by introducing an impedance coil across the supply circuits and feeding the motor from variable points on its windings. This arrangement may be caused to supply a large starting current without interfering with the supply circuits, but it has the same effect on the motor torque as a resistance in series with the fields.

3. *Resistance in Armature.* The torque of the armature at starting may be made equal to the maximum running torque by inserting resistances in the armature circuits which increase the total armature resistance at starting in the ratio $\frac{R_e+R_a}{R_a}=\frac{R_t}{R_a}=\frac{f}{v_m}$, where R_e is the external or starting resistance and $\frac{60\,v_m}{p}$ the slip at full maximum torque. As already shown (Sect. 168), $v_m=\frac{R_t}{2\pi L_a}$, and therefore to get a maximum torque when $v_m=f$, requires that $\frac{R_a+R_e}{2\,\pi L_a}=f$, or $R_e=2\,\pi f L_a-R_a$, or the external resistance in the armature circuit required at starting in order to give maximum torque must equal the difference between the reactance and the resistance of the armature. The total resistance used should be larger and arrangements made to reduce it gradually. As the maximum torque is usually designed to occur, in running with natural armature circuits, at a slip between one and one-third and two times that corresponding to the normal full-load torque, the armature and field currents at maximum torque do not exceed twice the full-load currents, so that resistances inserted in the armature circuits serve the double purpose of increasing the starting torque and keeping the starting current within bounds.

This plan has been largely used by Siemens and Halske, the General Electric Company, the Stanley Electric Manufacturing Company, the Westinghouse Company, and others. The arrangement of the start-

; rheostat depends largely upon the type of the
nature windings to which it is applied. In arma-
es of the squirrel-cage type the conductors may be
ped at one end with tapered german silver strips,
ich come in contact with a sliding copper ring. At
rting, this ring may just touch the german silver
s, and as the machine speeds up the ring may be
l along until the tips are cut out and the copper
nature conductors are directly connected together
ough the ring.

[f the armature revolves, it is evident that the ring
ıst be arranged to slide on a spline on the shaft, and
be controlled by a grooved sliding collar and loose
er. The same device may be used for armatures
h coils having a common short-circuiting point. In
case one set of coil terminals are permanently con-
ed together, and the other set are connected into
ıan silver strips, which may be short-circuited by a
ıg ring, as already explained. This arrangement
een used by Siemens and Halske, the General
ic Company, and others.

he armature windings are arranged so as to have
e coil for each phase, the introduction of resist-
very simple, since only one resistance coil for
ase is required. In this case, if the armature is
y, the connections of the rheostat are made
into the armature circuits, or between the ar-
ircuits and one point of common connection;
the armature revolves, three collector rings
laced on the shaft, and stationary rheostats
ed to control the resistance of the coils which

are properly connected to the rings, or the resistance may be placed inside the armature spider and controlled by a sliding collar and loose lever. Such arrangements are used by the Stanley Electric Company, the General Electric Company, and others.

4. *Commutated Armature.* The armature may be wound with the coils so arranged that their conductors are in series when starting and in parallel when running. If x is the number of parallels in such an armature, the resistance at the start is x^2 times the running resistance, and the reactance at start is x^2 times the running reactance. The starting current in the armature with the conductors in series is therefore $\frac{1}{x}$ times as great as it would be with the conductors in parallel, but the field current at starting is the same with either arrangement of the armature conductors. On the other hand, since placing the conductors in series increases the resistance and reactance in the same proportion, the starting torque of the armature is the same with the conductors in series or parallel. The armature winding may also be so arranged that, instead of starting with all conductors in series, a portion of the conductors are connected in opposition to the others at the start, and are then reversed and connected properly in series with the others after the machine is in operation. The opposition arrangement affects the current of both armature and field.

The third and fourth arrangements may be combined by making the connectors, by means of which the armature conductors are placed in series, of high resistance material; these connectors are cut out when the conductors are short-circuited together.

This device has been used in various forms by Siemens and Halske and the Westinghouse Electric Company.

172. Effect of Rotary Field on Field Windings. — The distribution of magnetism in the air space of an induction motor has been found to be approximately sinusoidal, when the motor is fed by sinusoidal currents,* and the counter electric pressure set up in the field windings by the rotating magnetism is exactly the same as though the windings moved with an equal angular velocity in a stationary uniform field. The electrical pressure developed in a conductor depends, in other words, upon the relative angular velocity of conductor and magnetism, and it makes no difference which moves. Therefore, the formula at the bottom of page 80, Vol. I., applies to this case, or

$$e' = \frac{2\,\pi n' N V}{10^8 \times 60} \sin a,$$

in which e' is the instantaneous pressure in the coil, n' the number of turns in the coil, N the total magnetization from one pole, V revolutions per minute, and a angular position of the coil. In the case under consideration,

$$N = \frac{2}{\pi}\,\frac{2\,\pi r l B_m}{2p} = \frac{2\,r l B_m}{p},$$

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* Blondel, Notes sur la théorie élémentaire des appareils à champ tournant, *La Lumière Électrique*, Vol. 50, p. 351; Jackson, Three-phase Rotary Field, *Electrical Journal*, Vol. 1, p. 185.

where r and l are the inner radius and the length of the field core, B_m the maximum magnetic density in the air space, and p the number of pairs of poles in the field. Since $\frac{V}{60}=\frac{f}{p}$, and the magnetism per magnetic circuit in a multipolar machine must be multiplied by the number of pairs of poles to get the total number of lines of force cut per conductor per revolution (Vol. I, p. 278), the formula may be written, generally, in the form

$$e' = \frac{2\pi n' N f}{10^8}\sin\alpha\,;$$

whence

$$e_m' = \frac{2\pi n' N f}{10^8},$$

and

$$E' = \frac{\sqrt{2}\,\pi n' N f}{10^8}.$$

This is the value of the electrical pressure developed in a narrow coil, but if the coil is spread over a considerable area, the maximum pressure is less than that given above, since the density of the magnetism which is cut by the conductors falls off from B_m at the centre of the coil to some smaller value, which depends upon the width of the coils. If the coil occupies 180 electrical degrees of circumference, when the middle conductor is in a field of density B_m the outer conductors are in a field of zero density. The maximum pressure developed by the total coil is then

$$\frac{2}{\pi}\times\frac{2\pi n' N f}{10^8}.$$

If the coil spreads over an angle θ, the value of e_m' becomes

$$e_m' = \frac{1}{\theta} \times \frac{2\,\pi n' N f}{10^8} \int_{-\frac{1}{2}\theta}^{\frac{1}{2}\theta} \cos a\,da.$$

The field windings of induction motors are usually arranged uniformly on the field, so that in two-phase motors $\theta = 90°$, and in three-phase motors $\theta = 60°$. The respective values of e_m' become

$$e_{m2}' = \frac{4\sqrt{2}\,n' N f}{10^8}, \quad e_{m3}' = \frac{6\,n' N f}{10^8},$$

and

$$E_2' = 4\,\frac{n' N f}{10^8}, \quad E_3' = \frac{3\sqrt{2}\,n' N f}{10^8}.$$

Hence the electrical pressure set up in a uniformly distributed field winding of a two-phase motor is, other things being equal, about 10 per cent less than if the windings were in narrow coils; and in a three-phase motor the deficit is nearly 5 per cent. The exact ratios are $\pi : 2\sqrt{2}$ and $\pi : 3$. To give the same counter electric pressure in the uniformly distributed field windings of an induction motor arranged for two phases, requires about 6 per cent more turns in the windings than when the same machine is arranged for three phases. If the windings are placed on salient poles, as is sometimes done in two-phase motors, all the lines of force pass through the windings, and the coils therefore act as though they were very narrow, but the increased reluctance of the magnetic circuit caused by this construction more than destroys any advantage pertaining to the form of the winding.

173. Formulas derived from Transformer Formulas.— In the case of a transformer, the following formula (Sect. 111) gives the relation between electrical pressure, frequency, magnetism, and the turns of the coils:

$$E' = \frac{\sqrt{2}\,\pi n' N f}{10^8},$$

provided all the m [illegible] s included within all the turns. This provis [illegible] rue in the ordinary induction motor, since th [illegible] ic density in the air gap may be assumed to v [illegible] sinusoid, and that condition requires that the [illegible] of lines of force passing through the different turns of the coils shall also vary as a sine function. This is illustrated in Fig. 290. Consequently we have for the induction motor,

$$E' = \frac{\sqrt{2}\,\pi n' N f}{10^8 \theta} \int_{-\frac{1}{2}\theta}^{\frac{1}{2}\theta} \cos a\,da,$$

where $\frac{\theta}{2}$ is the value of a corresponding to the sine ordinate which is proportional to the number of lines of force passing through the extreme turns, when the total magnetism, N, passes through the centre turns of the coil.

For uniformly distributed windings in two phases, $\theta = 90^\circ$, and in three phases, $\theta = 60^\circ$, while for coils on salient poles $\theta = 0^\circ$. The values of E' for the field winding of the induction motor become

$$E_2' = \frac{4\, n' N f}{10^8} \quad \text{and} \quad E_3' = \frac{3\sqrt{2}\, n' N f}{10^8},$$

and the formulas thus developed from the fundamental theorems of the transformer are exactly the same as iose developed from the conception of the rotary field. or various reasons it is more convenient to study iduction motors from the transformer standpoint, and e may consider them as *transformers with a relative*

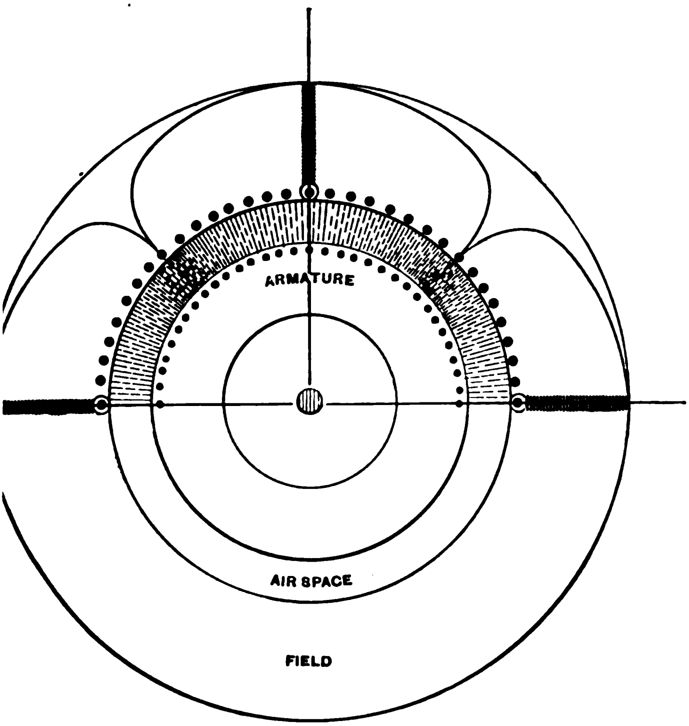

Fig. 290

otion between the primary and secondary windings. n this case, *the field winding is always the primary rcuit and the armature winding the secondary circuit.*

174. Exciting Current. — The exciting current for an iduction motor may be calculated for each circuit in exactly the same manner as that for a transformer. It is composed of two components in quadrature:

(1) The active current, which is equal to the sum of the no-load losses in the circuit, in watts, divided by the volts per circuit. The number of circuits is equal to the number of phases. The total losses entering into the exciting o-load losses) are the core losses in the ure; the C^2R loss in the field, due to ent; a small C^2R loss in the armature ature current required to run the arm tion and core losses (usually negligib on loss. The watts represented in the exciting current per electrical circuit are equal to the total no-load losses divided by the number of phases.

(2) The wattless magnetizing current, which is in quadrature with the active component of the exciting current, is calculated, as in the case of transformers, from the formula $\sqrt{2}\,nC_\mu = \frac{PN}{1.25}$, where P is the reluctance of the magnetic circuit. By Section 160, if n'_p is number of turns per phase which link each magnetic circuit, the actual magnetizing current per circuit is

$$C_\mu' = \frac{P \times N}{n'_p \times \sqrt{2} \times 1.25} \times \frac{2}{m},$$

which for a two-phase machine becomes

$$C_{2\mu}' = \frac{PN}{1.75\, n'_p},$$

and for a three-phase machine,

$$C_{3\mu}' = \frac{PN}{2.65\, n'_p}.$$

Since mechanical clearance between the armature and fields is an essential feature of a motor, the reluctance of the motor magnetic circuit is much greater than that of a transformer of corresponding capacity. This makes the magnetizing current greater, increases the exciting current, and reduces the no-load power factor.

The total exciting current is equal to the square root of the sum of the squares of its two components, or $C_1 = \sqrt{C_{2\omega} + C_{2\mu}}$.

175. Field Ampere-Turns. — The ampere-turns on each magnetic circuit of an induction motor are the resultant of the ampere-turns due to all the phases (Sect. 160). It may readily be shown that the resultant of any number of equal sine functions with a uniform phase difference is a circular function equal to $\frac{m}{2}x$, where x is the amplitude of the components, and m the number of components. It is therefore evident, from the deductions of Section 160, that the resultant ampere-turns in the magnetic circuit of an induction motor is $\frac{m}{2}(n'_p C' \sqrt{2})$, where n'_p is the number of turns belonging to each phase which link each magnetic circuit, and C' is the current in each phase. Consequently, if y ampere-turns are required in the magnetic circuit, the winding in each phase must furnish $\frac{2}{m}$ times the whole. For two-phase motors, $\frac{2}{m} = 1$, and for three-phase motors, $\frac{2}{m} = \frac{2}{3}$.

176. Slip and Armature Pressure. — As stated in Section 165, if the field rotation is V, the armature rotation must be somewhat less, or V', and therefore the slip is

$V - V'' = v$. It is evident that v is proportional to the frequency with which the total useful magnetism per pole, N_a, cuts the armature conductors, and it will vary from a value $v = V$ to $v = 0$ as the motor armature changes from a condition of rest to a speed of synchronism. Slip is frequently named in per cent of motor speed. In ordinary practice, the slip varies, at full load, from 2 per cent to 10 per cent of V, having the smaller value in large machines. The maximum pressure induced in any conductor on the armature is

$$e_m'' = \frac{2\pi N_a v_1}{10^8} = \frac{2\pi N_a p v}{10^8 \times 60},$$

where N_a is the armature magnetism per pole; and the effective pressure per conductor is

$$E_e'' = \frac{\sqrt{2}\,\pi N_a p v}{10^8 \times 60},$$

since the pressure curves in the conductor must be sinusoidal if the magnetism has a sinusoidal distribution in the air space, as has been assumed. The effective current flowing in a conductor of a squirrel-cage armature will be

$$C_e'' = \frac{\sqrt{2}\,\pi N_a p v}{10^8 \times 60 \times I_e''} = \frac{\sqrt{2}\,\pi N_a p v}{10^8 \times 60 \times R_e''} \cos\phi_a,$$

where I_e'' is the impedance of the armature conductor, ϕ_a is the angle of lag of the current, and R_e'' the resistance of the conductor. The C^2R loss in the armature conductors will be $W_a = S''C_e''^2R_e''$, where S'' is the number of armature conductors. The torque is proportional to the current times the magnetic field which is nearly constant); and, if we neglect the ect of magnetic leakage, it is evident that the armae must run at a slip which sets up an electric pressure the armature conductors which is equal to the drop of essure in the conductors caused by the current dended to give the torque. So that the slip is directly oportional to the armature current for a fixed armae resistance, and for a fixed armature current is ectly proportional to the armature resistance. In tual motors, magnetic leakage is not negligible, and slip is increased, since the magnetic leakage in a nsformer is proportional to the secondary amperens. The slip is also increased by the drop of essure in the field winding, which increases with load, and causes a corresponding decrease in the lue of N_a.

The fact then stands that the increase of slip between -load and full-load must be sufficient to increase the essure in the armature conductors by an amount equal the increased loss of pressure in the conductors due the increased current and the decreased armature agnetism. If magnetic leakage increases with the mature current, the total slip becomes proportional to $''R_c''+AN_l+\frac{S''}{S'}C_c'R_c'$, where A is a constant, N_l the mber of leakage lines of force passing through the mature coils, S', S'' the number of field and armature nductors, $C_c'R_c'$ the drop of pressure per field conctor.

177. Design of Induction Motors. — The general principles of transformer design may be so applied to the duction motor that its construction becomes, in many respects, the same as that of the transformer. If it is desired to design a rotary-field motor to supply W' horse power, or $W = W' \times 746$ watts, the formula

$$E' = K\frac{\sqrt{2}\,\pi n' N f}{10^8}$$

will give the product of the field turns into the magnetism, $n'N$; E' and f being given by the conditions of the problem, and p the number of pairs of poles (which is dependent upon armature speed and frequency) being chosen from the table in Section 170. K is a constant which is equal to $\frac{2\sqrt{2}}{\pi} = .90$ for a two-phase machine, and $\frac{3}{\pi} = .95$ for a three-phase machine (Sect. 172). It must be remembered that E' is the *primary pressure per coil* in each phase, and its relation to the line pressure per phase depends upon the connections of the coils.

The safe circumferential speed for induction motors is even higher than that for alternators, since the conductors are nearly always embedded in both field and armature cores, but the usual periphery velocities are between 4000 and 6000 feet per minute. With a given frequency and number of revolutions per minute, the armature diameter in feet will be

$$D = \frac{U}{\pi V'},$$

where U is circumferential speed in feet per minute. The ratio between n' and N must be determined in a great measure by practice. A reasonably large value of N with a proportionate iron section increases the no-load losses, but increases the full-load efficiency as in the case of a transformer (Sect. 129). It is usually desirable to have the maximum efficiency of a motor occur at about three-fourths load, as motors commonly run on loads which average less than full load. Kolben* states that for ordinary good practice, the number of ampere-turns per centimeter of the field circumference at full load should be from 100 to 150, when the frequency is from 40 to 80 and the induction in the air gap from 2000 to 3000. This should only be used as a guide or check in making a design. The number of field turns per volt appears from the examination of a limited number of machines to be more than double the number of turns per volt used in transformers (p. 532); the range being from $\frac{50}{\sqrt{\text{output}}}$ to $\frac{100}{\sqrt{\text{output}}}$, when the output is in watts.

The magnetic density in the field cores may be about the same as in transformers. Kolben gives these values for different frequencies.

f	B	f	B
40	5500 to 6500	80	4000 to 4500
50	5000 to 6000	100	4000 to 3500
60	4500 to 5000	120	3500 to 3000

Having the total magnetization N, which is obtained from the pressure formula when n' is assumed, and assuming the maximum magnetic density B_m, the cross-section of the iron may be found. The magnetic density between the core slots may be allowed to become as great as two or two and a half times the density in the core, but every effort to keep it small in value should be made.

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* *Electrical World*, Vol. 22, p. 284; *London Electrician*, Vol. 31, p. 591.

There must be n' turns in the coils of each phase, since the counter pressure E' must be generated in the coils of each phase. The length of field or armature will be dependent upon the magnetic density in the air space, which may be made quite large. The limit being determined by the reluctance permissible in the magnetic circuit and hence by the magnetizing force required to drive the magnetism through the circuit. The air space of the motor may be very small, so that it may be permissible to run the maximum magnetic density somewhat higher than in the case of dynamos or alternators, though the practice for these machines may be safely followed. From 2000 to 6000 lines would be a safe range. If the fields are wound through holes, as in Fig. 287, the length of the field will be

$$l = \frac{\pi}{2} \times \frac{2pN}{\pi D B_m} = \frac{pN}{DB_m},$$

where p is the number of pairs of poles, B_m the maximum air-space induction, πD the circumference of the polar surface, and N is the magnetism emanating from a pole. The paths of leakage are somewhat as shown in Fig. 291, and the coefficient of leakage may be taken between 1.05 and 1.25 when the motor is running at full load. At starting, the leakage will be increased on account of the strong armature reaction ending to force the field magnetism across the pole ips. The magnetizing current for a two-phase machine nay be found from the formula (Sect. 174)

$$C_{2\mu}' = \frac{PN}{1.75\,n_p'} = .56\frac{PN}{n_p'},$$

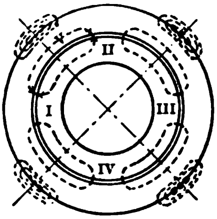

Fig. 291

here P is the reluctance of the nagnetic circuit. For three-phases

$$C_{3\mu}' = \frac{PN}{2.65\,n_p'} = .38\frac{PN}{n_p'}.$$

he working current in each phase of the two-phase inding is

$$C_{2w}' = \frac{\frac{1}{2}W}{E'} \div \text{per cent efficiency}.$$

he efficiency of the motor is assumed during the trial alculation. In the three-phase winding

$$C_{3w}' = \frac{\frac{1}{3}W}{E'} \div \text{per cent efficiency}.$$

he total current in a two-phase machine is

$$C_2' = \sqrt{C_{2w}'^2 + C_{2\mu}'^2},$$

nd for the three-phase,

$$C_3' = \sqrt{C_{3w}'^2 + C_{3\mu}'^2}.$$

aving obtained the field current, the size of the field onductors may be obtained, allowing from 900 to 1200 ircular mils per ampere.

As in transformers, the radiating surface of a stationry field core should not be less than five to seven square inches per watt radiated. If the field rotates, this may be greatly reduced. The usual way to wind the coils is to divide them up into sections and place them in slots or holes (Fig. 292). Slots seem to be preferable if the teeth are close together, as there is then less field leakage in the paths indicated by the dotted lines of Fig. 292*a*.

The armature may be wound in any of the ways explained in Section 169. The wires are usually embedded in the surface of the core, as is the case with the field windings (see Figs. 287 and 288). By this means the air space may be made of exceedingly small depth, and the magnetizing current and magnetic leakage are thus reduced, which is very desirable. The diameter of the armature has already been determined; its speed will be $V' = V - v$. The slip, v, should be made from 2 to 10 per cent, the larger value being for a machine of

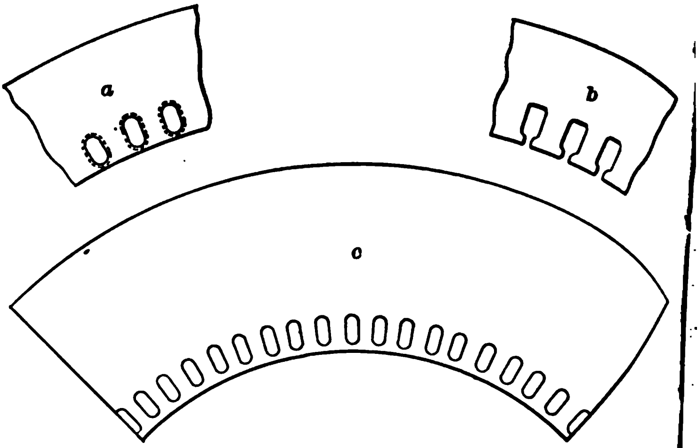

Fig. 292

about 1 H.P. and the lower for 100 H.P;[*] in other words, the regulation of induction motors may be made equal to that of continuous-current motors. The copper loss in the armature bars L_c'' is

$$L_c'' = C_c''^2 R_c'' S'',$$

$$C_c'' R_c'' = K\frac{\sqrt{2}\,\pi N_a v p}{10^8 \times 60}\cos\phi'' \text{ (Sect. 176)},$$

$$R_c'' = K\frac{\sqrt{2}\,\pi N_a v p}{10^8 \times 60 \times C_c''}\cos\phi''.$$

In a squirrel-cage armature $K = 1$, but in coil armatures its value depends upon $\frac{1}{\theta}\int_{-\frac{1}{2}\theta}^{\frac{1}{2}\theta}\cos a\,da$ (Sect. 172). The value of C_c'' is given with ample accuracy from the formula $W = \frac{S''}{S'}E'S''C_c''\cos\phi''$, where S' and S'' are respectively the number of embedded conductors of field and armature. If both field and armature are either drum or ring wound, it is evident that $\frac{S''}{S'} = \frac{n''}{n'}$, but if they have different types of windings the equality does not exist. In every case $\frac{S''}{S'} = \frac{E''}{E'}$. The value of $\cos\phi''$ may be taken as 0.90 for a reasonably close approximation, and the value of the resistance of each armature conductor, including its share of the resistance of the end connections, which corresponds to a fixed value of the slip, is then approximately determined from the formula. The value of N_a used in this computation must correspond to full load; it is $N\frac{E_t' - C'R'}{zE_t'}$

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[*] Kapp's *Electrical Transmission of Energy*, 4th ed., p. 323; Steinmetz, *Trans. Am. Inst. E. E.*, Vol. 11, p. 37.

where N is the field magnetization at no load (that is, assuming no CR drop in the primary), E_i' is the impressed circuit pressure, C' is the primary current at full load, and z is the leakage coefficient.

If the armature is not arranged to allow the insertion of a starting resistance, and if a maximum starting torque is desired, R_e'' mu[illegible] made of such a value as to make $R_e'' = 2\pi f L_e''$, where L_e'' is the self-inductance of an armature bar. Such a value for R_e'' gives an enormously large slip at full load and is unsatisfactory except for special purposes, so that R_e'' is usually smaller.

The density of current in the armature conductors, if the armature rotates, may be made quite large, an allowance of 300 circular mils per ampere being sufficient, since the core losses are insignificant. If the armature is stationary, the current density should not exceed one-third that value. The radiating surface per watt lost in a rotating armature should be the same as that in an alternator or continuous-current dynamo, taking all losses into account. In the same way the radiating surface of a stationary armature should be the same as is allowed for dynamo fields if the same rise of temperatures is admitted.

The foucault current and hysteresis losses may be determined exactly as in the case of a transformer, using v_1 as the frequency of magnetic cycles in the armature, and f as the frequency in the field.

The loss in primary pressure due to C^2R losses in the fields, and foucault currents and hysteresis, will increase proportionally the input required to give a desired output, and proper correction must be made in the design.

The efficiency of a machine is

$$\eta = \frac{W}{W+L}$$

and $L = H_f + H_a + Z_f + Z_a + C'^2 R_f + C''^2 R_a + F$, where W is the output, L the total losses, H hysteresis losses, Z foucault current losses, and F friction losses, all given in watts or horse power. The maximum efficiency evidently occurs, as in continuous-current machines and transformers, at that load which causes the variable copper losses to equal the constant core and friction losses.

The power factor of the machine when running without load is

$$\cos\phi' = \frac{C_\omega'}{\sqrt{C_\omega'^2 + C_\mu'^2}} = \frac{C_\omega'}{C_1'},$$

as shown in Section 174. When the machine is loaded, the power factor is partially dependent upon the lag of current in the armature ($\cos\phi''$), and the self-inductances of both armature and field windings must be calculated before the power factor can be determined. The self-inductances of the windings are due to the leakage lines of force, and the values may be determined from the reluctances of the leakage paths and the arrangements of the windings.* In general, the power factor is approximately equal to

$$\cos\left(\cos^{-1}\frac{R_a}{I_a} + \cos^{-1}\frac{R_f}{I_f} + \cos^{-1}\frac{C_\mu'}{C'}\right).$$

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* *London Electrician*, Vol. 36, p. 578.

The torque of the armature when the output, W, is given in watts is equal to

$$P = \frac{60 \times W \times 10^7}{2\pi V'} \text{ in dyne-centimeters,}$$

$$= \frac{W \times 10^7}{2\pi V' \, 16.3} \text{ in gramme-centimeters,}$$

$$= \frac{W \times 10^7}{2\pi V' \, 226{,}000} \text{ in pound-feet.}$$

177 a. Output Proportional to Square of Primary Pressure. — The output of the motor, plus the armature-core losses and friction, is equal to the product of the number of armature conductors and the effective current in each conductor, multiplied by the product of the pressure which would be developed in each conductor if the armature were driven at its speed in an equal stationary field and the cosine of the angle of lag of the armature current,

or
$$W = S''C_c''\frac{V'}{v}E_c'' \cos\phi'',$$

but
$$C_c'' = \frac{E_c''}{\sqrt{R_c''^2 + 4\pi^2 v_1^2 L_c''^2}},$$

and
$$\cos\phi'' = \frac{R_c''}{\sqrt{R_c''^2 + 4\pi^2 v_1^2 L_c''^2}},$$

and therefore
$$W = \frac{S''E_c''^2 R_c'' \frac{V'}{v}}{R_c''^2 + 4\pi^2 v_1^2 L_c''^2}.$$

Also
$$E_c''^2 = E_c'^2 \frac{v^2}{V'^2} = \frac{1}{S'^2}E'^2\frac{v^2}{V'^2},$$

where E_c' is the induced field pressure per conductor, E'

he total field pressure, and S' the number of field conluctors,

ınd $$W=\frac{\frac{S''}{S'^2}E'^2\frac{V'v}{V^2}R_c''}{R_c''^2+4\pi^2v_1^2L_c''^2}=\frac{\left(\frac{S''}{S'}\right)^2E'^2\frac{V'v}{V^2}R_a}{S''(R_a^2+4\pi^2v_1^2L_a^2)}.$$

This formula is not one which can be made of service n the design of a motor (in fact, it is not needed for uch a purpose), but it plainly shows the effect on the utput of a motor, which is caused by varying any ne of its constructive details while the others remain nchanged. A very important deduction from the forıula is: that *the torque and output of an induction motor ary as the square of the primary pressure,* so that a nachine which will carry an overload of 50 per cent on s normal pressure will barely run at full load if the ressure is reduced 20 per cent. The formula also hows that the slip is inversely dependent on the prinary pressure.

178. Electromagnetic Repulsion. — If a coil of wire is ıeld in an alternating magnetic field in such a way that he lines of force pass through its turns, an alternating ressure is set up in it which has 90° difference of phase rom the alternating magnetism. This in turn causes a urrent in the coil, and the coil experiences a force at ach instant tending to move it in the magnetic field, which is proportional in magnitude and direction to the roduct of the corresponding instantaneous values of urrent and magnetism, paying due attention to their elative signs; and the force for a period is equal to he average of the instantaneous torques during the eriod. If the coil could have no self-inductance, and the phase of the current could therefore be in quadrature with that of the magnetism, the average forces during alternate quarter periods would be equal, but in opposite directions (compare Fig. 49), and the average force during a whole period would be zero, so that the coil would have no tendency to move; but in all practical cases a coil, or even a flat disc, must have some self-induction, so that the current lags behind the impressed pressure, and the current phase is therefore more than 90° behind the phase of the magnetism. In this case the instantaneous values of the force, when plotted in a curve, give a figure similar to Fig. 48, but turned upside down, since the current lags behind the magnetism more than 90°. The ordinates of the large loop represent a negative or repulsive force, and the ordinates of the small loops a positive or attractive force, and the summation of the instantaneous forces during a period is seen to have a finite negative value. This shows that the coil experiences a repulsive force which tends to move it out of the magnetic field. If the coil is pivoted, the force tends to turn it into such a position that the lines of force of the field do not thread through its turns. The conditions here set forth were first fully explained and illustrated in a remarkable lecture by Professor Elihu Thomson,* and a similar lecture by Professor Fleming.†

179. Single-Phase Induction Motors. — If the fields of an induction motor are wound with one set of coils so that the field poles are set up by a single alternating current flowing in the coils, the poles will be stationary but alternating, and the effects of electromagnetic repulsion just described may be utilized for the purpose of causing the armature to rotate. If the armature is wound with uniformly spaced short-circuited coils or conductors, the repulsive effects in the different coils will balance each other when the armature stands still; but if the coils have their independent ends separately connected to the opposite bars of a commutator having as many bars as there are sets of conductors in the armature, brushes may be so arranged as to short-circuit each coil when it is in a position to give a force in one direction. This arrangement was suggested by Professor Thomson [*] and is illustrated in Fig. 293. The motor is self-starting, and runs by virtue of the repulsion

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* Novel Phenomena of Alternating Currents, *Trans. Amer. Inst. E. E.*, Vol. 4, p. 160.

† Electromagnetic Repulsion, *London Electrician*, Vol. 26, p. 567.

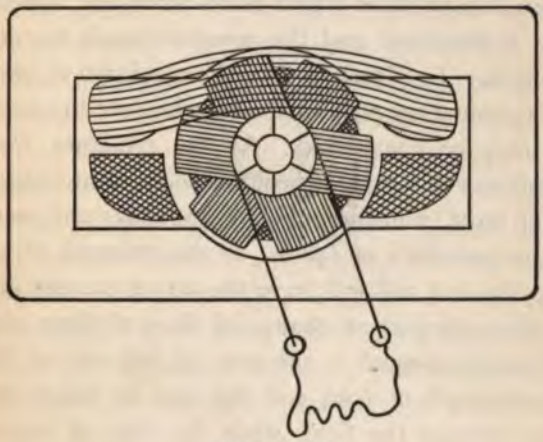

Fig. 293

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[*] *Trans. Amer. Inst. E. E.*, Vol. 4, p. 160.

between the magnetic field and the coils which, as they come into the active position, are short-circuited by the brush connections. Such a motor is bulky, inefficient, and expensive, since only a portion of the armature can be made continuously effective; but if a uniformly wound short-circuited armature (such as is used for polyphase induction motors) is started to revolving in a single-phase a[illegible]nating magnetic field, the balance of repulsions which exists when the armature is at rest is disturbe[illegible]mature tends to continue its motion. To [illegible] the condition of two coils in complementary p[illegible]th reference to one of the poles may be consi[illegible]. As the armature revolves, one coil moves toward a position where it includes more lines of force from the pole, and the other coil moves so as to exclude lines of force. If the strength of pole is rising, the first coil will have the larger current induced in it, since the rate of change of lines of force through the first coil is equal to the sum of the rate of change in the strength of field and the rate at which the coil moves through the field, while the rate of change of lines of force through the second coil is the difference of these two rates. Thanks to the lag in the coil circuits, the currents in both coils are in such a direction as to result in an attractive force on the poles, but a much stronger force is experienced by the first coil than the second. When the field is falling, the magnetic condition of the second coil is changing most rapidly, but the direction of the induced currents in the coils is reversed with respect to the direction of the elds, and the coils experience a repulsive force with reference to the pole. The effect during one complete period of the magnetism therefore tends to cause the armature to rotate in the same direction in which it was started. The torque is a maximum when the positive product of current and magnetism is a maximum, which is when the current lags behind the induced pressure by an angle between 45° and 90°. The torque at any speed (slip, $v = V - V'$) is equal to the torque which would be given by a polyphase induction motor of similar construction at that speed, minus the torque which the polyphase induction motor would give in a field of double the frequency, and with a slip equal to $V + V' = 2V - v$; but with the ordinary ratio of the resistance and inductance in the armature winding, the torque due to the latter slip is negligible at such full-load slips as are satisfactory in practice. In this case, V need not be looked upon as a speed of rotation of a magnetic field, but as the speed of the armature which keeps each conductor at the same position with reference to a pole for any fixed instant in the period of the magnetism. Hence $V = \frac{60f}{p}$, exactly as in rotary-field machines. When the armature is stationary, $v = V$, and the two torques are equal and opposite. A single-phase induction motor may therefore be designed in exactly the same manner as a polyphaser in respect to its operation after it has reached its normal speed, but it requires special treatment in the design for the purpose of making it self-starting.

180. Resolution of Alternating Field. — The single-phase alternating field may be treated in a different manner to get exactly the same result. An alternating field stationary in position may be resolved into two rotary fields, revolving in opposite directions, having the same frequency as the stationary field, and of one-half its magnitude or strength.* This is exactly similar to the principle of mechanics by which a simple harmonic motion may be resolved into two uniform opposite circular motions of one-half the amplitude.

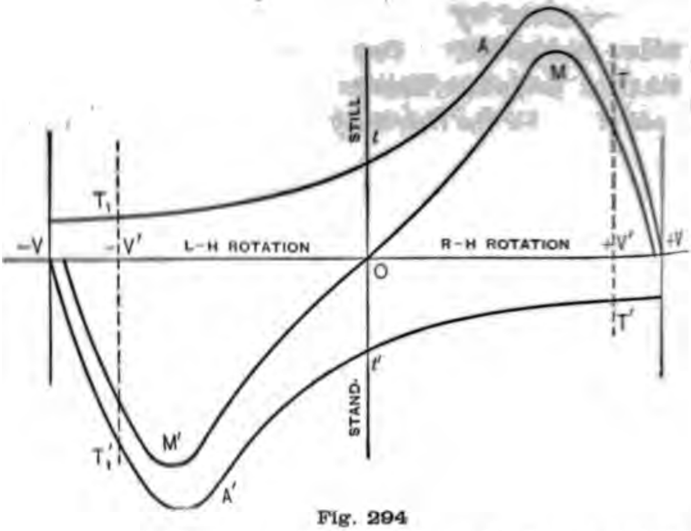

Fig. 294

The torque diagrams of each of these fields acting alone are shown in Fig. 294, where *O* is the point of armature rest, and armature speed is counted from that point along the horizontal axis. The curves *A* and *A'* are the torque curves that would be given by either field acting alone, the torque due to one being in one direction, and that of the other in the opposite direction. It is evident that when the armature is at rest it has no tendency to revolve, as the slip of the armature with respect to the two fields is equal, and torques Ot and Ot' created by the two fields are equal and opposite; but if the armature is started in one direction, for instance toward the right, the slip with respect to field A decreases, the torque caused by it increases and tends to continue the rotation, while the slip with respect to field A' increases, and the torque caused by A' decreases. When the armature speed becomes V', the torque caused by A is T, which is due to a slip $V - V' = v$; while the torque caused by A' is T', which is due to a slip in relative speed between armature and field of

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* Ferraris, A Method for the Treatment of Rotating or Alternating Vectors, *London Electrician*, Vol. 33, p. 110.

$$V + V' = 2V - v.$$

From the relations of torque to slip, which have already been discussed (Sects. 167 and 168), it is evident that the torque caused by A' decreases as the relative speed increases above V. If the differences between the corresponding ordinates of the curves of torque due to A and A' are plotted in a curve, the actual torque curve, M, is given. The ordinates of this will give the actual motor torque with respect to slip. From this curve it is seen that the motor will work at no load at an almost synchronous speed, and may then be loaded until the speed has dropped to a point where the torque is at a maximum. If the load exceeds this, the motor will stop.

If the armature should be started toward the left, instead of the right as here assumed, the conditions would be reversed and the motor would operate under the torque line M'. The ratio $\frac{L}{R}$ should be large in a single-phase motor, in order that the curve A' may be close to the X axis at ordinary running speeds, but the ordinary values of resistance and inductance which are required in l economical design make the effect of A' speed of normal full load that the action be considered.

181. Form ase Induction Motors.—From these is seen that a single-phase motor be ed in exactly the same manner as a polyphaser, that for equal output the resultant ampere-turns upon the field must be equal to the number on a polyphase motor.* The field winding may be arranged, as in polyphase motors, covering the entire polar surface, as is shown in Fig. 295, for a four-pole machine; but the differential action in this case reduces the effectiveness of the winding in the proportion of $1 : \frac{2}{\pi}$, as has already been shown in Section 172. Consequently, the equation from which the field windings are determined becomes

$$E' = K\frac{\sqrt{2}\,\pi n' N f}{10^8} = \frac{2}{\pi} \times \frac{\sqrt{2}\,\pi n' N f}{10^8} = \frac{2\sqrt{2}\,n' N f}{10^8}.$$

The value of K may be increased, and material saved, by leaving space between the coils as in Fig. 296, which shows the windings for a two-pole field.

Not only may the armatures of single-phase induc-n motors be designed in exactly the same way as are ose of polyphasers, but an armature which gives the

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* Kolben, Design of Alternating-Current Motors, *Electrical World*, Vol. 22, p. 284; *London Electrician*, Vol. 31, p. 590.

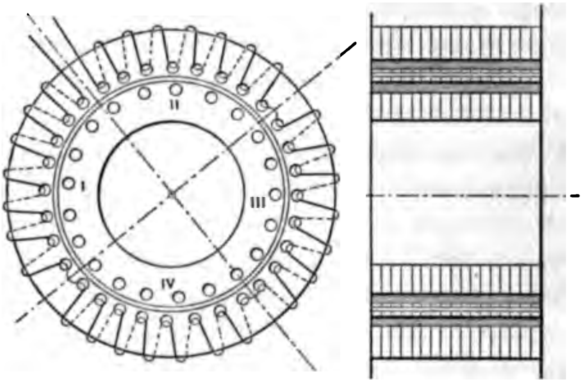

Fig. 295

st results when running in a polyphase field is likely be best for a single-phase machine, other things being ual.

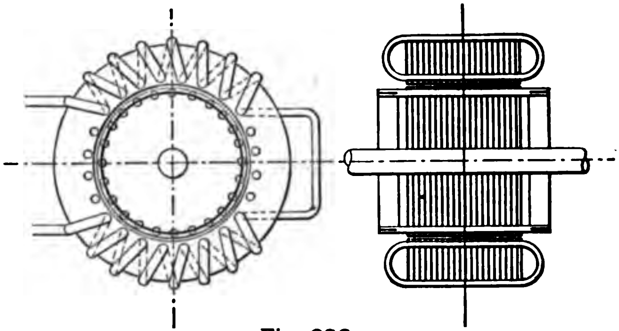

Fig. 296

The efficiency of single-phasers should be slightly ss than that of polyphasers, since the armature-core sses are proportional to the frequency of the main ld instead of to the slip; their slip for a given load and similar design is slightly greater, and their maximum torque is slightly less than that of polyphasers, as is shown by Fig. 294; but these differences in well-designed machines should not be great. The weight of single-phasers is larger than that of equal polyphasers, because the value of K is smaller.

182. Starting Single-Pha nduction Motors. — Since single-phase induction motors are not *per se* self-starting, special starting devices must be included in the design and construction. As a rule is takes the form of what is called a **Phase Splitter.** The field is wound with two coils similar to the windings of a two-phaser, and at starting these are connected in parallel to the circuit; one directly, and the other through a dead resistance or capacity. This throws the current in the two coils into a difference of phase, which may be accentuated by winding one coil so that it has greater self-inductance than the other; and the machine then starts as a two-phaser. After the machine is running, the coils are connected directly to the circuit, or one coil is cut out, and the motor operates as a single-phaser. The operation of "phase splitting," as applicable to such motors, cannot give a large difference of phase between the currents in the two motor circuits with a reasonably large power factor, and consequently single-phase induction motors must have either a very small starting torque or an unreasonably small power factor at starting.

183. Efficiency of Induction Motors and Methods of king Tests. — Polyphase induction motors can be with about the same efficiency as continuous-nt motors, and with somewhat less cost on account of the absence of a commutator and the low insulation required on the armature conductors, but with a counterbalancing extra cost on account of the high grade and expensive sheet iron stampings which are required for the field.

1. *Direct Measurement.* In testing the efficiency of these motors, the output may be measured by a brake or transmission dynamometer, as is explained for testing continuous-current motors in Vol. I., p. 255, but the input must be measured by one of the wattmeter methods of Section 44. The two-wattmeter method is the best, but care must be taken to determine whether the readings are additive or subtractive, since the power factor of a partially loaded induction motor is likely to be quite low and at no load may be only a few per cent. The power factor is determined by taking simultaneous readings of amperemeter, voltmeter, and wattmeter in one circuit, if the machine is balanced; but if the circuits differ, readings for each circuit must be taken. The power factor is then the true watts divided by the apparent watts. This method requires that the motor shall be operated with its full load, and therefore may prove inconvenient, and it does not give any way of separating the losses.

2. *Stray Power Method.* A method similar to that described for testing transformers (Sect. 125, 8 *a*) is often more convenient and satisfactory. By this plan the core losses and friction losses are determined by measuring by wattmeter the power which is absorbed by the motor when running light under normal pressure and frequency. As the power factor under such condi-

tions is likely to be very small, the field current flowing is considerable and the C^2R loss cannot be neglected, but a correction can be made after the test for copper losses is completed. To measure the copper losses, the machine is locked so as to remain stationary, in which case the armature serves the purpose of a short-circuited secondary, and such a reduced impressed pressure is applied as to cause any desired current to flow in the fields. The wattmeter readings give the C^2R losses for the current flowing, and the losses for any other current may be at once calculated. A small core loss is included in this measurement, but should commonly be negligible; an approximate correction may be made on account of it, when necessary, by considering its ratio to the total corrected core losses as the 1.6 power of the pressure applied in the copper loss test is to the 1.6 power of the normal pressure. From these results the total losses and the efficiency at any load may be calculated.

A motor running, without load, or with part load, on an unbalanced circuit, is likely to absorb widely different amounts of power in its coils; one coil may even return power to the circuit, while the others absorb the power required for operation plus that returned. In all such cases, the two-wattmeter method of measuring the power gives the net power absorbed by the machine.

3. *Power Factor.* The power factor at any load may also be calculated from the results of the two loss tests. Thus, from the power factor of the machine running light, which is determined in the core-loss tests, is readily deduced the wattless magnetizing current, since the power factor is equal to cos ϕ' and the magnetizing cur-

rent per coil, C_μ', is equal to C_1' sin ϕ', where C_1' is the current per coil as shown by the amperemeter. The field current at any load is equal to the resultant of the active current corresponding to that load and the wattless current, or

$$C' = \left\{ \left(\frac{W}{mE'} + \frac{L}{mE'} \right)^2 + C'^2_\mu \right\}^{\frac{1}{2}}$$

where C' is the field current per coil, E' is the normal pressure per coil, W is the total output, L is the total losses, m is the number of phases, and C_μ' the wattless magnetizing current per coil. The power factor at any load is

$$\cos \phi' = \frac{W + L}{mC'E'}.$$

4. *Regulation and Torque.* — All desired information in regard to the operation of induction motors (except regulation and starting torque) may therefore be determined by purely electrical measurements, and to a high degree of accuracy. Using commercial amperemeters, voltmeters, and wattmeters which have been properly calibrated, the errors probably affecting the full-load efficiency and power factor need not exceed one per cent. The exact regulation of a machine can be determined only by actual running tests under load, in which the actual slip is measured, but the percentage slip may be taken to be approximately equal to the percentage drop of pressure in the windings. The starting torque can be measured by clamping a lever upon the pulley, and measuring the pull at the end of a fixed length of arm. For machines having an improperly divided starting resistance in the armature, this gives a value which is

higher than the torque against which the motor will start and run up to speed. In such machines, the *standing torque* and *starting torque* are different, but in machines having a properly arranged starting resistance in the armature, the standing torque and starting torque are equal, and are practically equal to the maximum torque which the machine can exert.

The following table gives the efficiency, power factors, and other characteristics of a number of European polyphase motors.*

Designation	0	1	2	3	4
Capacity — H.P.	¾	2	6	24	60
Field speed	1500	1500	1000	1000	750
Speed at full load	1435	1445	960	970	730
Number of phases	3	3	3	3	3
Frequency	50	50	50	50	50
Pressure	190	190	190	190	190
No-load current	2.2	3.6	12	25	48
Full-load current	3.3	7.5	20.5	70	160
Full-load efficiency	68	75	82	91	93
Power factor at full load	.75	.80	.80	.85	.90
Number of poles	4	4	6	6	8
Slip at full load	4.3	3.7	4	3	2.7

Designation	5	6	7	8	9
Capacity — H.P.	100	125	1	5	50
Field speed	600	500	1500	1500	750
Speed at full load	588	488	1375	1395	725
Number of phases	3	3	3	3	3
Frequency	50	50	50	50	50
Pressure	190	190	100	100	100
No-load current	75	90	4.5	15	150
Full-load current	265	330	8	36	280
Full-load efficiency	93	94	75	84	91
Power factor at full load	.91	.91	.70	.70	.82
Number of poles	10	12	4	4	8
Slip at full load	2	2.4	8	7	3.3

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* Thompson's *Polyphase Electric Currents*, p. 203; Rodet et Busquet's *Courants Polyphasés*, p. 89.

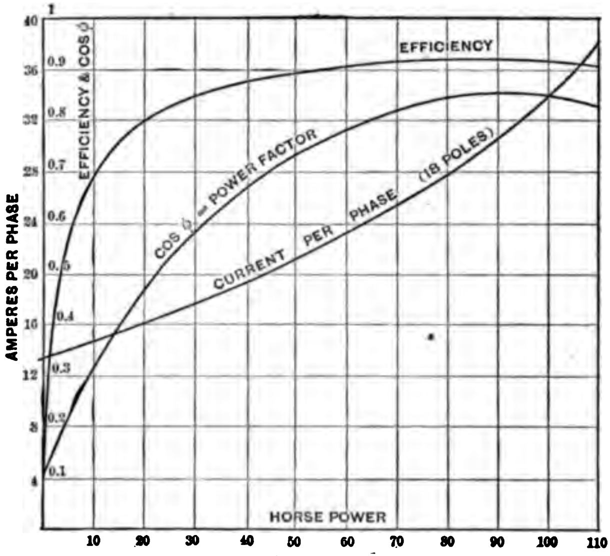

Fig. 297

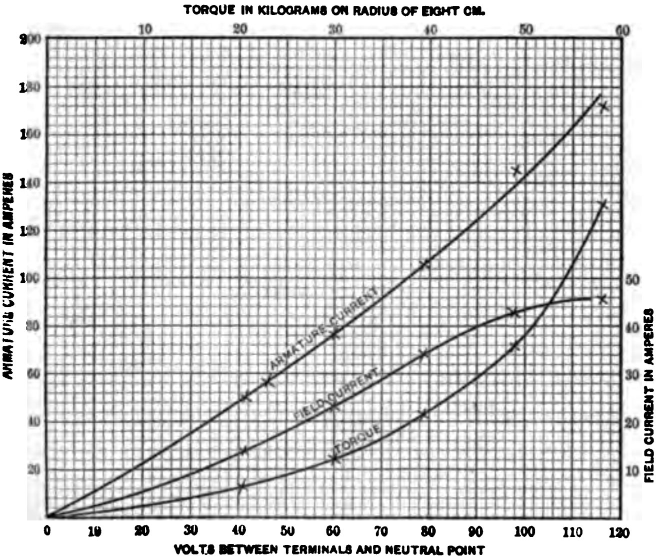

Fig. 298 a

Figure 297 gives the curves of efficiency, power factor, and current of an Oerlikon 100 H.P. three-phase motor of eighteen poles, working under a pressure of 1730 volts at a frequency of 50 and a speed of 320 revolutions per minute. Figure 298 *a, b, c* gives similar curves for a 3 H.P. three-phase Oerlikon motor running at a frequency of 50 and a speed of 1500. A Tesla 50 H.P. two-phase motor running at a frequency of 25 and a speed of 750 revolutions per minute gives an

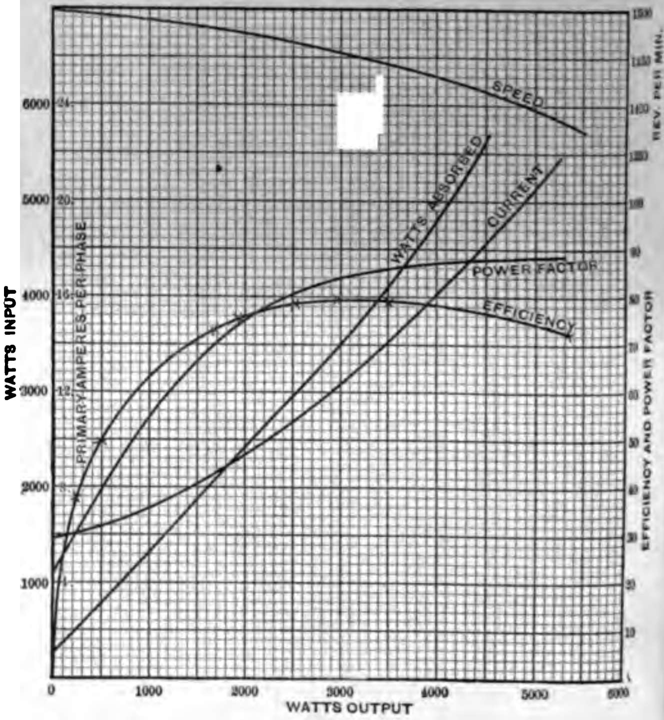

Fig. 298 b

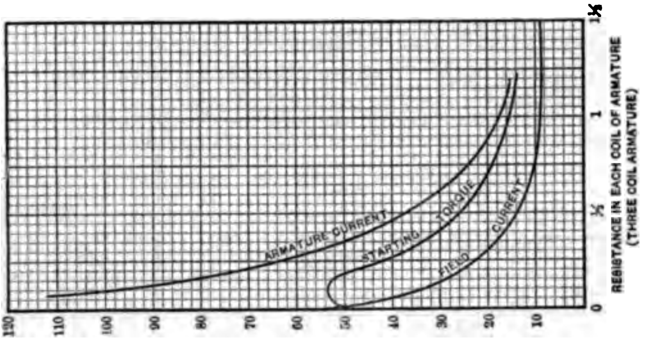

Fig. 298 c

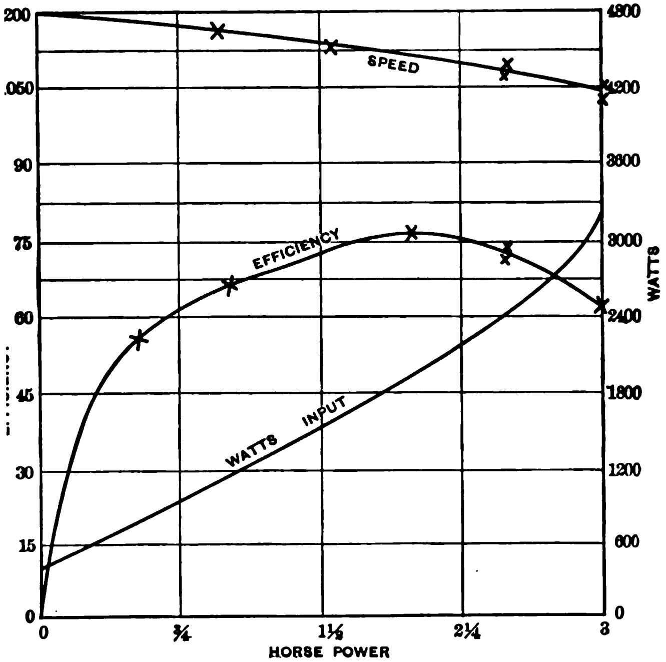

Fig. 299 a

efficiency of 89 per cent and a slip of 2 per cent at full load and a maximum efficiency of 91 per cent at about three-fourths load. Figure 302 gives the curves of efficiency of a single-phase 2 H.P. motor, running at frequencies of 42 and 50. Figures 299 *a* and 299 *b* give the curves of efficiency, current per phase, apparent watts, true watts, and regulation for a 3 H.P. Brown

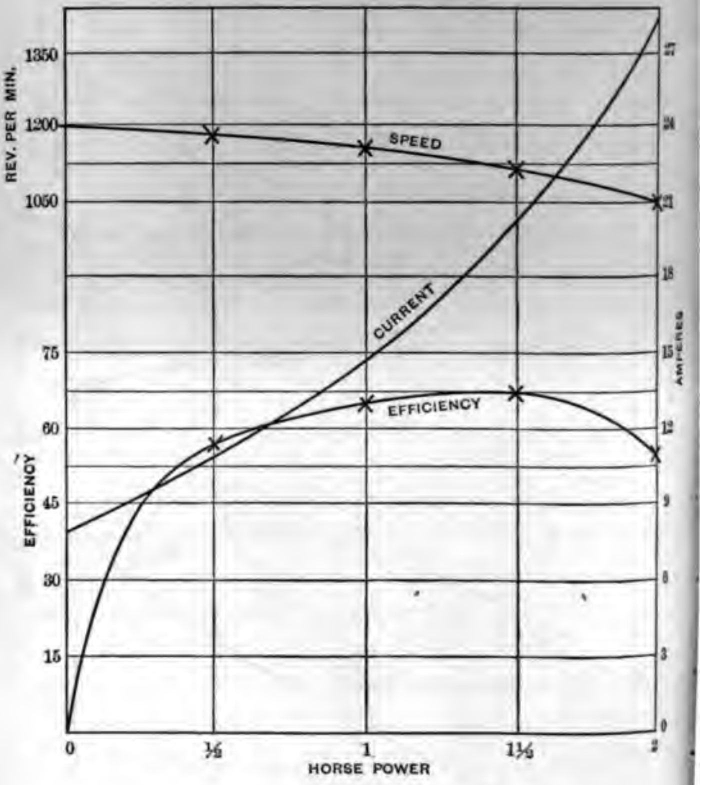

Fig. 299 b

two-phase motor, running at a frequency of 40 and a speed of 1200 when operated on two phases and when operated on one phase.*

In Fig. 300, curve *A* shows the relation between current and starting torque of a three-phase motor when a proper resistance is introduced into the armature circuit. Curve *B* shows the same for the motor without external armature resistance; this would give the relations obtaining if the resistance were placed in the field circuit. Curve *C* gives the same relations for a 15 H.P. motor, which has a full-load torque of 52.5 lbs.

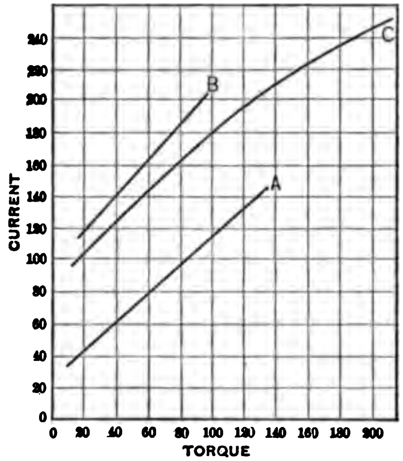

Fig. 300

184. Weight Efficiency. — The following table gives the weights per horse-power of three types of American induction motors.

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* Boucherot, *Bull. Soc. Inter. des Électriciens*, Vol. II., p. 264.

Horse-power.	TYPE NUMBER 1.		TYPE NUMBER 2.		TYPE NUMBER 3.	
	Revolutions per minute.	Pounds per H.P.	Revolutions per minute	Pounds per H.P.	Revolutions per minute.	Pounds per H.P.
1	2666	246	1800	150	1800	275
2	2666	165	—	—	1800	175
3	2000	180	1800	167	—	—
5	2000	164	1200	140	1200	125
7	2000	182	—	—	—	—
7½	—	—	1200	160	—	—
10	1600	125	1200	120	1200	85
15	1600	101	1200	100	900	80
20	1333	105	1200	75	900	100
30	1333	97	1200	67	900	88
40	1333	90	1200	62	—	—
50	1333	84	900	84	720	100

Speeds given are field speeds or "theoretical speeds," and the full-load speeds are less by from 2 to 10 per cent. Frequencies to which type number 1 is adapted are 133 and 66⅔. The other types are adapted to a frequency of 60.

S. P. Thompson gives the following data as representing European induction motors.*

Horse-power.	Weight, pounds per horse-power.	Horse-power.	Weight, pounds per horse-power.
2	120	50	70
6	100	70	66
13	88	100	58

Figure 301 gives the averages for the American motors in the form of a curve (*A*), with the addition of ırve (*B*) for continuous-current machines reproduced from the equations on page 263 of Vol. I. This that the alternating-current machines are the r in the larger sizes, on account of their higher s, but for equal speeds the two classes of machines ry near an equality in respect to weight efficiency.

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* Thompson's *Polyphase Electric Currents*, p. 209.

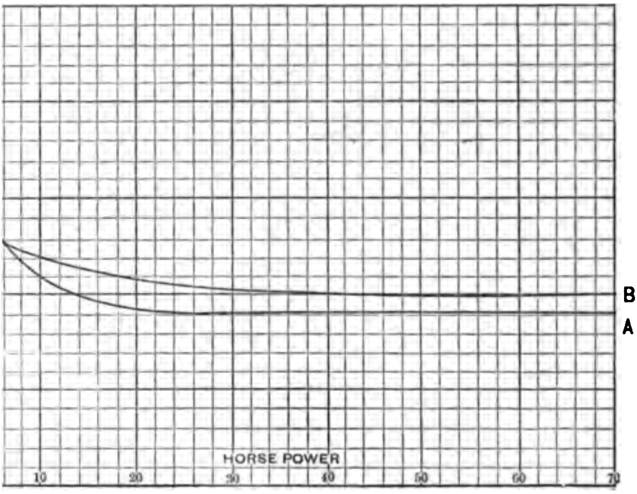

Fig. 301

Effect of Frequency.* An examination of the las relating to the design of induction motors that the frequency of the current for which a ne is designed does not affect its efficiency, slip, factor, or starting torque, but that for a given the number of poles must be directly as the frey. Increasing the number of poles of a given

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:inmetz, *Trans. Amer. Inst. E. E.*, Vol. 11, p. 47 ; Stanley, Effect uency in Induction Motors, *Electrical Engineer*, Vol. 17, p. 125.

machine reduces the cross-section of each pole, but the number of lines of force at each pole is equally reduced so that the magnetizing current is unaltered. Consequently, induction motors of equal merit may be designed for all reasonable frequencies, though magnetic leakage may interfere with the operation when the poles become too numerous (compare Transformers, Sect. 121).

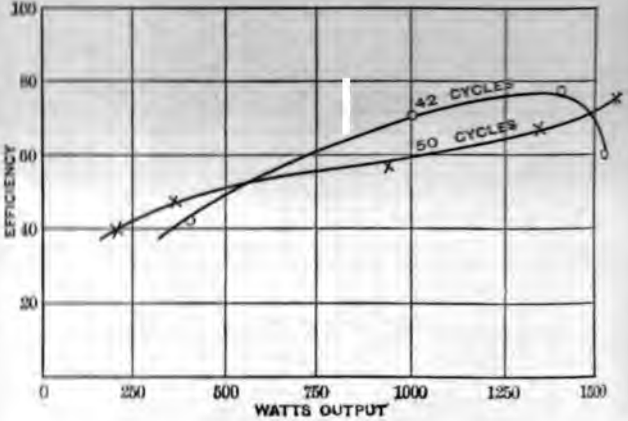

Fig. 302

On the other hand, when a machine which has been designed for a certain frequency is operated at another frequency, the speed is changed in direct proportion to the frequency, its percentage slip is practically unaltered, the starting torque varies inversely with the frequency, and the efficiency and power factor both vary directly with the frequency cause the magnetic density is inversely as the fre- ency, as in transformers. Figure 302 shows the curves of efficiency of an Oerlikon single-phase induction motor for two frequencies; * and Figs. 303 and 304 give the curves of efficiency and power factor for a 50-frequency Allgemeine three-phase motor when operated on four different frequencies.†

The frequencies which are commonly used with induction motors cover a wide range. In Europe, 50 seems to be quite universally adopted for three-phase and single-phase motors; while in this country the General Electric Company prefers 60, which is equally suitable for lighting and power purposes. The Westinghouse Company has built two-phase machines for

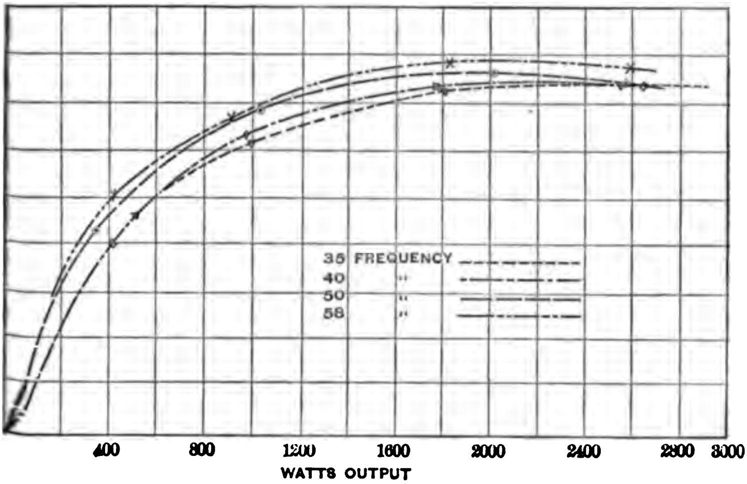

Fig. 303

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* Kolben, Design of Alternating-Current Motors, *Electrical World*, Vol. 22, p. 284; and *London Electrician*, Vol. 31, p. 590.

† Jackson, Tests of a Polyphase Motor, *Electrical Journal*, Vol. 1, p. 101.

60 frequency, but appears to prefer about 25, which is that adopted at Niagara Falls, for plants where power service is of greater importance than the lighting service. The Stanley Company, on the other hand, prefers a frequency of 133 for its two-phasers, though it also builds standard machines for a frequency of 66⅔, both of which are excellent frequencies for lighting circuits. The Fort Wayne Electric Corporation and the Emerson Electric Company build self-starting synchronous single-phasers for frequencies of 60, 125, and 133.

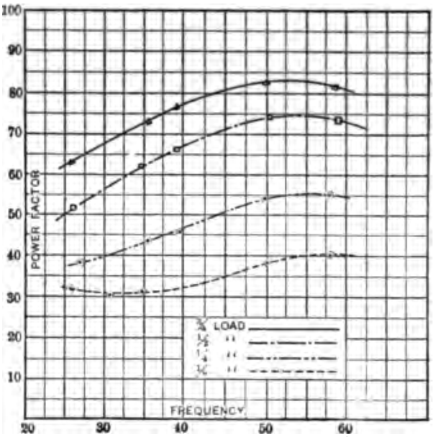

Fig. 304

186. Other Forms of Induction Motors. — The effects of a rotary field, electromagnetic repulsion, or magnetic screening may be utilised in an almost indefinite number of ways, of which only a few will be indicated here.

1. *Motor of Stanley Electric Manufacturing Company.** This is a two-phase machine which may be classified either as a rotary-field motor, or as a sort of double single-phaser; and its design may be worked out upon either hypothesis, though it should properly be worked out upon the rotary-field basis. It consists essentially of two fields set side by side. The field windings are placed upon salient poles, and respectively connected to the two circuits of a two-phase supply system, and corresponding to the two fields are two armature cores on the same shaft, which carry a single set of short-circuited armature coils. The two fields have an equal number of poles, but they are set with the angular position of the poles ninety electrical degrees apart, while the armature conductors are laid in slots straight across both armature cores. A diagrammatic development of a six-pole machine which shows the arrangement very plainly is given in Fig. 305. A view of an armature is shown in Fig. 306. The operation of the machine is easily understood by referring to Fig. 305. When the armature is stationary in the position shown, current is induced in the *a* armature windings by the lower crown of poles belonging to the *A* field. The conductors of the *a* coil lie directly under the upper or *B* field, and a torque tending to move the coil is developed, which, at any instant, is proportional to the product of the instantaneous strength of the *B* field and the current induced in the coil by the *A* field. At the same time, the *B* field induces a current in the *b* coil, which causes

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* *Electrical World*, Vol. 21, p. 326; *Electrical Engineer*, Vol. 17, p. 505.

a torque with the *A* field. The motor is therefore self-starting. After the machine has rotated through an angle corresponding to one-fourth the polar pitch, both coils are inductively acted upon to an equal extent by both poles, and opposite torques are produced by the two fields. A further rotation places the *b* coil in inductive relation with the *A* field, and the *a* coil with

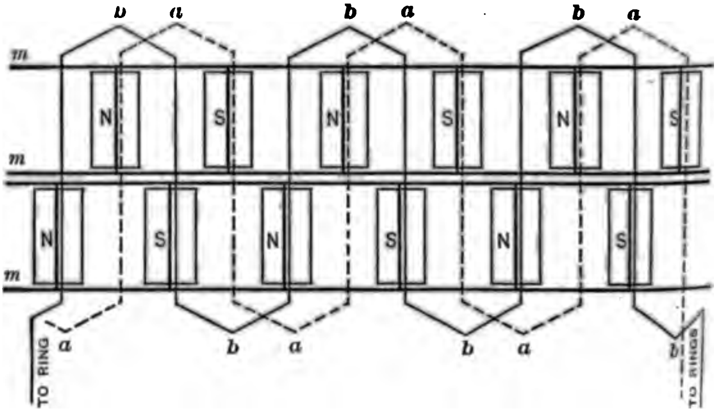

Fig. 305

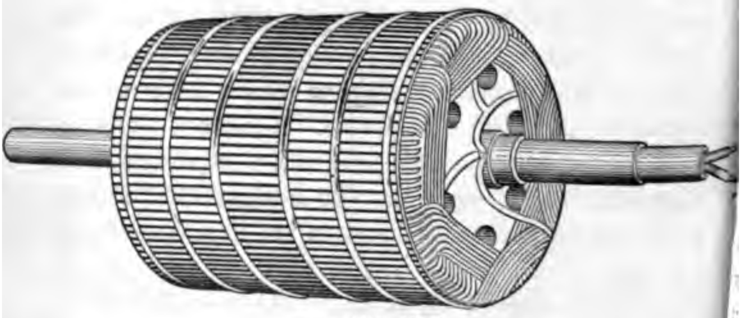

Fig. 306

the B field, and the torque again becomes positive. It is possible, with such a winding, to have dead points, though they may be avoided by a proper disposition of the armature winding. The Stanley motors are usually equipped with a well-proportioned external starting resistance which is introduced into the armature circuit by means of collector rings, and the motors have a fairly large starting torque. A set of short-circuited windings, $m\,m\,m$, is placed in the pole faces to decrease the apparent self-inductance of the armature windings, and condensers are used in parallel with the field circuits to supply the wattless magnetizing current, and thus increase the power factor.

Capacity of Condenser required to supply Wattless Current. As the current of a condenser is equal to $2\pi fsE$, a high frequency and a high pressure both serve to reduce the capacity of a condenser which is required to give any desired current, and in order that the condensers which are required for the Stanley Company's motors may be of reasonable capacity, the motors are designed for 500 volts pressure, and the use of high frequencies is recommended by the company. In illustration of this, supposing that a 500 volt motor for 120 frequency requires 20 microfarads to exactly supply its wattless current, an exactly similar machine designed for a frequency of 60 would require from 40 to 50 microfarads, while if the pressure is also reduced to 250 volts the capacity required is increased to from 160 to 250 microfarads. Figure 307 shows the regulation, power factor, and efficiency as a function of load for a 2 H.P. Stanley motor, and Fig. 308 shows the same for a 6 H.P. machine. The armature core losses in Stanley machines are similar to those of single-phasers, and are, therefore, greater than those of plain rotary-field machines. A properly built machine of this type gives a true rotary field in its effect on the armature windings, as was early pointed out by Sahulka,* but the effect on the armature core losses is similar to that found in a single-phase machine. The number of pairs of poles in the rotating field is equal to the number of salient poles on one ring.

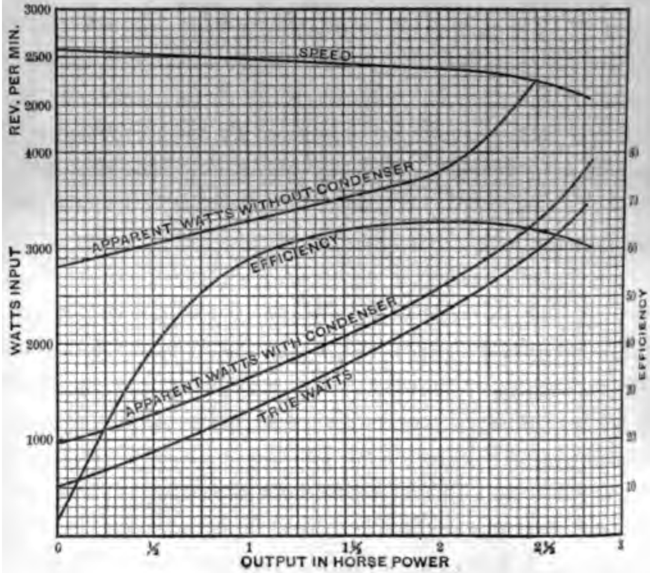

Fig. 307

2. *Shallenberger Meter.* The running parts of the

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* Sahulka's *Ueber Wechselstrom-Motoren mit magnetischem Drehfelde.*

lectrical meter manufactured by the Westinghouse ompany consist essentially of a single-phase induction otor. The armature consists of an iron disc, across ne diameter of which is wound a stationary coil that arries the main current. A short-circuited coil, consisting of heavy copper strips, lies within and at a slight ngle with the main coil, and the lagging induced cur-

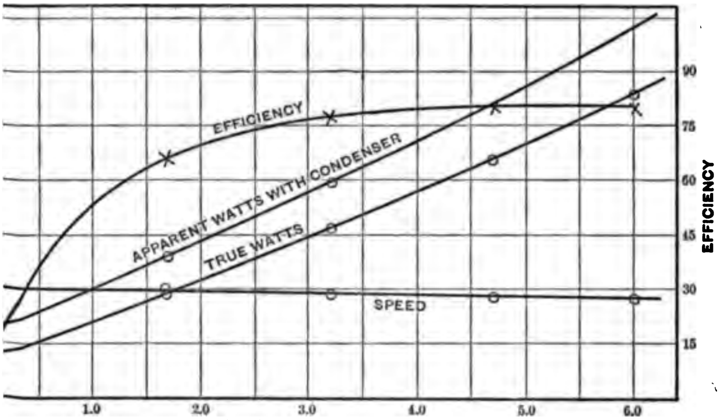

Fig. 308

nt in this coil sets up a magnetic field which joins ith the magnetism of the main coil to set up an irregularly rotating field with a frequency equal to that of e main current. This causes the iron armature to tate. The strength of the resultant field and the rque on the armature are proportional to the main cur-nt. A proper retarding force is used to cause the rmature to rotate at a speed directly proportional to e main current as long as the frequency is constant.

The speed of the armature depends on the frequency of the main current.

3. *Scheeffer Meter.* The running parts of a recording wattmeter, manufactured by the Diamond Electric Company, consist of a split-phase induction motor. The armature is an iron cylinder, which is embraced by a three-legged magnet made of iron stampings, upon one leg of which is wound a coil carrying the main current, and upon another leg is wound a shunt or pressure coil. The magnetism set up by these two coils, in which the current has different phases, sets up an irregular rotary field, the strength of which depends upon the product of the main current and pressure, by means of which the armature is driven. By a proper retarding force the armature may be caused to run at a speed which is directly proportional to the watts in the circuit. It is possible to adjust the magnetic density in the cores, by adjusting the resistance of the shunt coil, so that the speed of the armature is practically independent of the frequency within the ordinary commercial limits.

4. *Ferranti Meter.* The armature in this is an iron disc which is embraced by two elongated pole pieces, and these are surrounded at equal intervals by short-circuited copper bands. The bands exert what may be called a shielding effect on the magnetism, and cause magnetic poles to apparently creep along the pole-pieces. These cause the revolution of the armature. The speed of the poles and the armature torque depend upon the strength of the magnetism set up in the main coil.

Thomson's recording wattmeter consists of a small tor with a Gramme armature, and without iron in lds or armature cores, so that it works equally well alternating or continuous currents. It is not an duction motor, but seems to require notice amongst other meters already described. It is well known at a small series motor, with either Gramme or Sie- ns armature, will run on alternating circuits exactly it runs on continuous-current circuits, but its power ctor is so minute as to preclude its commercial value. the Thomson meter, the main current passes through e field coils, and the armature is connected directly ross the leads through a large non-inductive resist- ce.

The only other type of induction motor to which ntion can be given is that developed by Mr. C. P. inmetz, together with a special arrangement of alter- r windings and transmission lines which is called **Monocyclic System.**

7. **Monocyclic System.*** — A diagram of a "mono- " alternator armature is shown in Fig. 309, and ame is developed in Fig. 310*a*. The winding is alternator consists of an ordinary coil winding ts or grooves, which may be called the main g or coil, and an auxiliary or "teaser" winding in smaller slots half-way between the main The electrical pressure developed in the auxil- ding has, on account of the position of the hase difference of ninety degrees from that d in the main winding, exactly as would

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* *Electrical World*, Vol. 25, pp. 182 and 302.

be the case in a two-phase alternator, but one end of the auxiliary coil is connected to the middle of the main coil. The free end of the auxiliary coil and the ends of the main coil are connected to separate collector rings. If the number of turns in the auxiliary coil bore the relation to the number in the main coil of $\frac{\sqrt{3}}{2} : 1$, the pressure measured between the collector rings taken in pairs would be equal for each pair, and the machine would be a balanced three-phase generator giving three equal pressures at 120° difference of phase (Fig. 310 *b*). But in the monocyclic generator the aux-

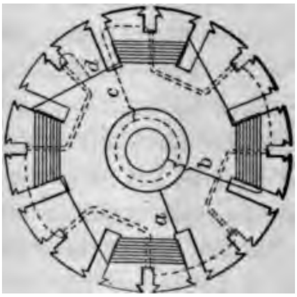

Fig. 309

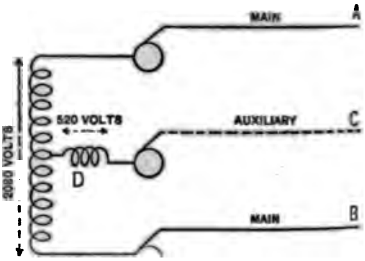

Fig. 310 a

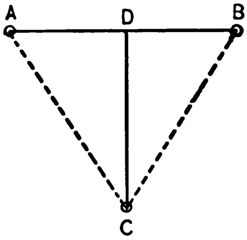

Fig. 310 b

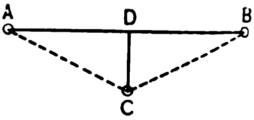

Fig. 310 c

iliary coil has only one-fourth as many turns as the main coil, and therefore the three phases developed by the machine are not 120° apart, but have the angular relations shown in Fig. 310*c*, in which *AB* is the pressure measured between the main terminals, *AC* and *BC* pressures measured between the auxiliary terminal and the main terminals, and *CD* the pressure developed in the auxiliary coil. The angle *ACD* is nearly 60° and *AC* is nearly .56 of *AB*.

If two transformers are connected in circuit with a monocyclic generator, it is possible to get a three-phase

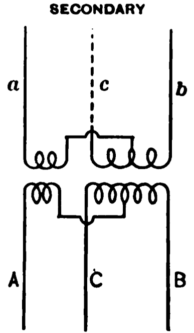

Fig. 311 a

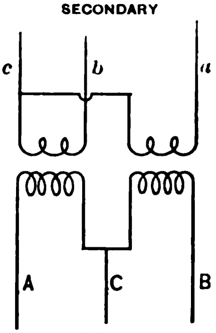

Fig. 311 b

secondary circuit with 120° difference of phase by the arrangement shown in Fig. 311 *a*, provided the ratios of the number of turns *cb* to *ab* and *CB* to *AB* are properly proportioned. Two ordinary transformers with the same ratio of transformation may be used as shown in Fig. 311 *b*, one of the secondaries being reversed, though the pressures in the three circuits are then not exactly equal. The way in which the pressures come out in this case is illustrated in Fig. 311 *c*, where AC is the pressure measured from a to c, BC is the pressure measured from b to c, $B'C$ shows the phase of the pressure BC without a reversed transformer, and AB is the resultant pressure measured between a and b. The last is equal to twice the pressure developed in the teaser coil, or CD in Fig. 310 *c*. This may be treated as a regular three-phase system in bad balance, but the system was designed to be operated with a special two-coil induction motor which is shown diagrammatically in Fig. 312. The field of this motor is wound with two coils, one of which, *m*, is connected to the main circuit, and the other,

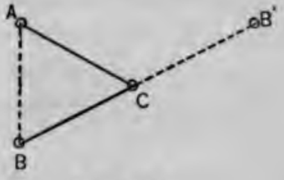

Fig. 311 c

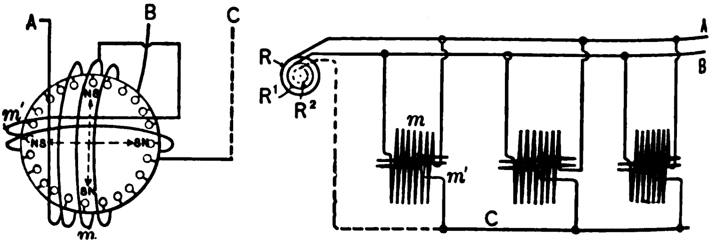

Fig. 312

m', which has fewer turns, is connected to the auxiliary conductor. The motor acts in starting as an unbalanced three-phaser, but after getting under way takes most of power from the main circuit. This arrangement was intended to avoid the unbalancing which is likely to occur in polyphase systems where lighting and power are used together. The monocyclic system is essentially a single-phase system; all the lighting apparatus is operated from the main circuit, and while the motors are, broadly speaking, polyphasers, and may be ordinary three-phasers, they operate more after the manner of single-phasers which are started by splitting the phase, than as balanced three-phasers. Quite a large number of monocyclic alternators have been put in service in this country as ordinary single-phase lighting generators [*] but comparatively few have been put into operation for use in a combined light and power service. The motors in use, where the combined service is furnished, are standard three-phasers.

The accomplished designer of the monocyclic system has made many plans in which it is proposed to utilize in remarkably varied ways the flexibility of alternating-current machinery of the induction types. One of these plans is shown diagrammatically in Fig. 313, and is introduced here for the purpose of illustrating more fully the varied purposes to which alternating-current apparatus lends itself.[†] *G*, in the figure, is an ordinary single-phase generator, with its field excited by exciter, *E*, and its armature, *A*, connected, through the collector rings, *rr*, to feeders, to which lighting circuits may be directly connected through transformers, as at *a*. At *b* is a circuit containing several monocyclic motors, which, after one is started, furnish each other the current required for the auxiliary winding; while at *M* is a synchronous motor wound like a monocyclic generator, but which operates as a single-phase synchronous motor

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[*] Jackson and Fortenbaugh, Some Observations on a 300 K. W. Monocyclic Alternator, *Trans. Amer. Inst. E. E.*, Vol. 12, p. 350.

[†] See also Emmett, Existing Commercial Applications of Electrical Power from Niagara Falls, *Trans. Amer. Inst. E. E.*, Vol. 12, p. 482.

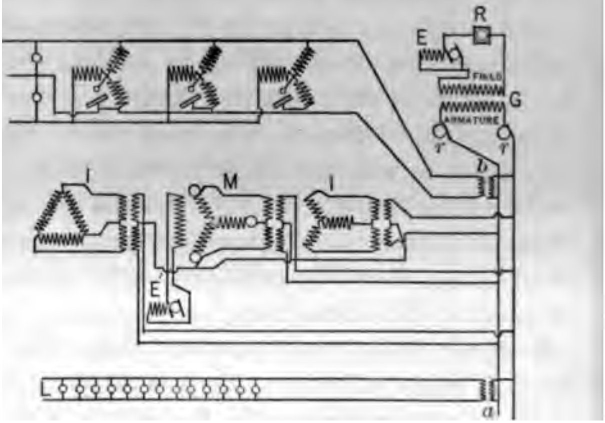
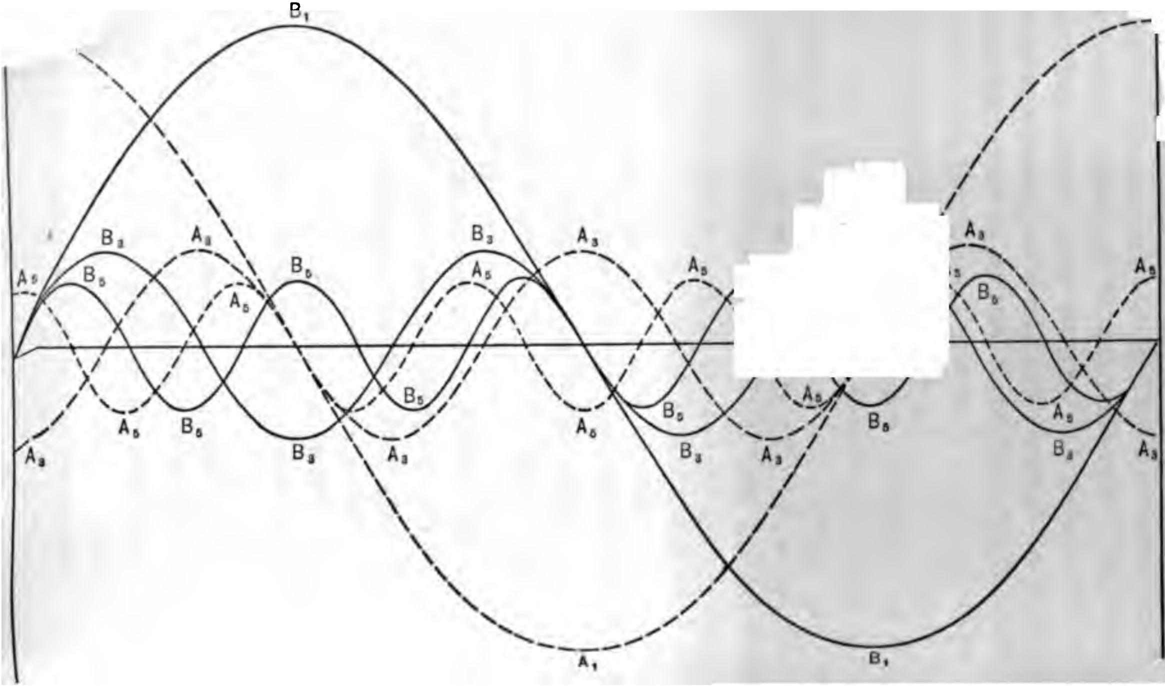


Fig. 313

with its main coil connected to the generator circuit. Its auxiliary coil serves to furnish the additional current needed to operate plain three-phase induction motors, *I*, *I*, which are on the circuit.

188. Effect of the Form of Curves of Pressure. — The effect of distorted curves upon the operation of induction motors depends upon the number of phases. The harmonics of three and five times the fundamental frequency are the only ones which need be considered; and indeed, that of three times the frequency is the only one which has an appreciable influence (Sect. 30, and Appendix A). In single-phase motors the harmonics must affect the magnetic field exactly as they affect that of a transformer, so that peaked pressure curves should cause a decrease in core losses, and the operation of the motor should not be otherwise greatly influenced. In polyphase motors, however, the harmonics may set up a rotating field of their own, which is superposed upon the regular field, and may interfere with the operation of the machine. The harmonics with three times the fundamental frequency belonging to the two circuits of a two-phase system, have a phase difference of 90° (Fig. 314), and these set up a superposed rotating field in the induction motor which has a field velocity of three times that of the main field. The figure shows that the harmonics of triple frequency, belonging to the two phases, are reversed in relative position compared with the fundamental waves. The field due to these harmonics rotates in the reverse direction from that of the main field, and therefore tends directly to decrease the torque of the motor and to increase the slip. The field due to the harmonics of five times the frequency rotates in the same direction as the main field, and its only disadvantageous effect is in causing eddy currents which may slightly decrease the efficiency of the motor. The frequency of the harmonic curves is indicated in the figure by subscripts.

In three-phase circuits the harmonics of triple frequency belonging to the different currents are directly superposed in phase as is shown in Fig. 315, and therefore the superposed field which they cause in three-phase induction motors is a stationary one whose


B1
B3
A3
B3
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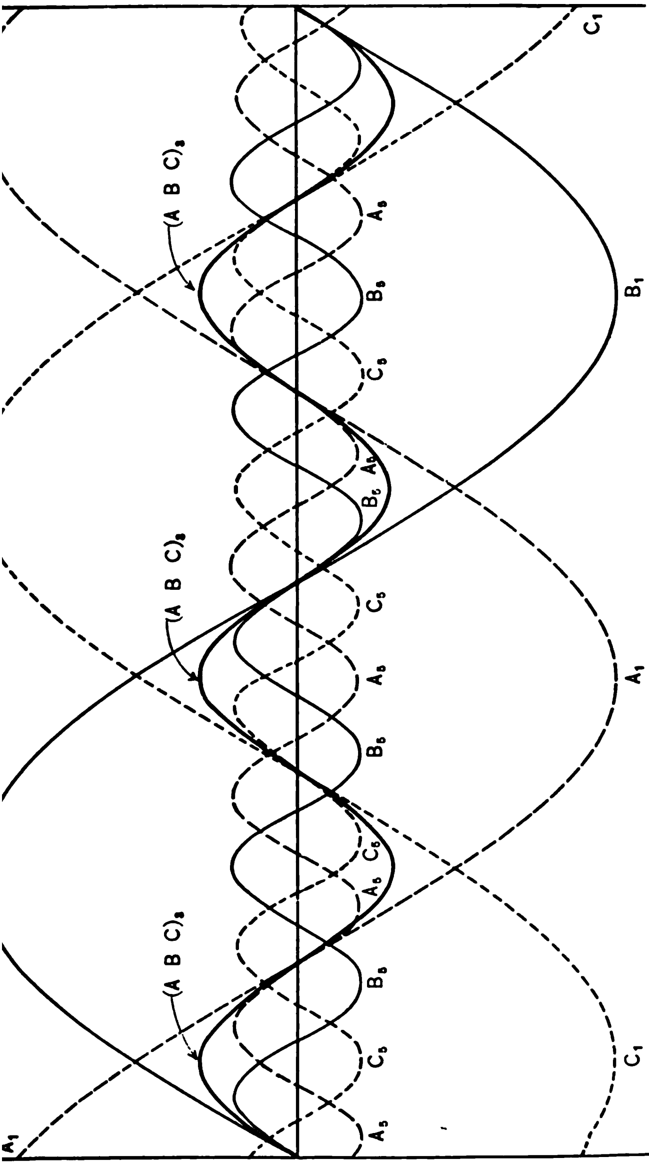

Fig. 315

influence is only to decrease the efficiency by setting up extra core losses. The figure also shows that the harmonics of five frequencies have 120° difference of phase and are in reversed order, so that they set up a reverse rotating field, and if they are in much strength may affect the torque.

188 a. Reversing Polyphase Motors. — Polyphase motors may be reversed by reversing the direction of rotation of the field.

In two-phase motors with independent circuits, reversing the terminal connections of either circuit will effect the reversal of rotation, but reversing the terminals of both circuits will not alter the direction of rotation. Two-phase motors with three-wire connections cannot be reversed by any change of the external connections.

The direction of rotation of three-phase motors may be reversed by interchanging the connections of any pair of leads.

CHAPTER XV.

POLYPHASE TRANSFORMERS.

189. Stationary Transformers for Polyphase Circuits. —The transformation of pressure in polyphase circuits ay be compassed by using single-phase transformers n groups. A two-phase circuit then requires two transormers at each point of transformation, and a threehase circuit requires either two or three transformers. The individual transformers must each have a capacity qual to the power required to be transformed in each hase divided by the power factor of the secondary ciruit. As the power factor of an incandescent lamp ircuit is practically 100, and as circuits supplying motors are likely to have a full-load power factor at est as small as 80, it is evident that transformers which upply currents to motors must be of greater capacity han those which supply an equal power to incanescent lamps. This rule applies equally to single-phase nd polyphase circuits and is important to bear in mind t the present time when alternating-current motors are oming into use. Figure 316 shows the connections f a three-phase circuit with two and with three transrmers.

A saving in the amount of material used, and therere also in the economy of operation, may be effected by combining the magnetic circuits of the individual transformers in the several phases, exactly as polyphase electric circuits are combined into common wires (Chap. XIII.). Figure 317 represents a two-phase transformer with a combined magnetic circuit. Since the phases of the magnetism in the two halves of the transformer are 90° apart, the resultant magnetism in the middle tongue

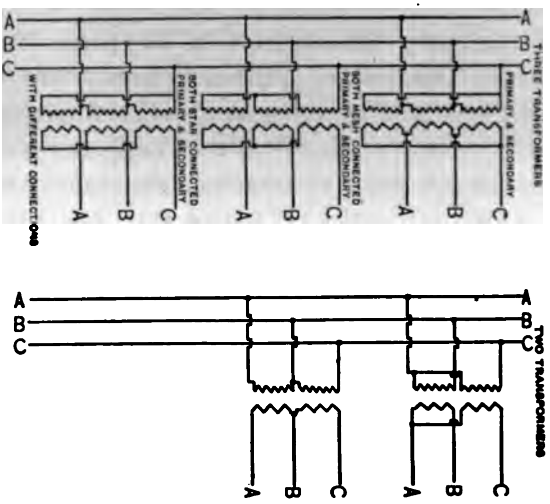
A
B
C
WITH DIFFERENT CONNECTIONS
BOTH STAR CONNECTED PRIMARY & SECONDARY
BOTH MESH CONNECTED PRIMARY & SECONDARY
THREE TRANSFORMERS PRIMARY & SECONDARY
TWO TRANSFORMERS

Fig. 316

is $\sqrt{2}$ times as great as that in the cores under the windings, so that this central tongue must have $\sqrt{2}$ times as great a cross-section as the remainder of the magnetic circuit. There is a saving of iron in the combined transformer, as compared with two independent transformers, which is equal to $\sqrt{2}$ times the weight of the central tongue. This is of little moment in small transformers, but may make quite a difference in the ost and efficiency of transformers of very large capac-ty. The same sort of combination may be effected in

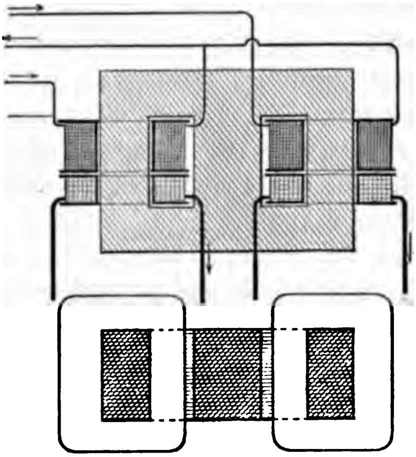

Fig. 317

hree-phase transformers, and the magnetic circuit may e coupled in either the star or the mesh arrangement.

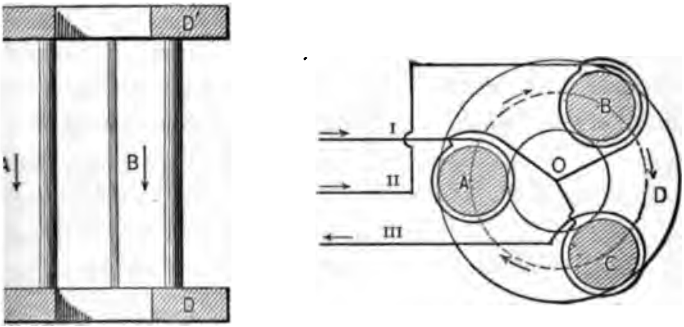

Fig. 318

igure 318 shows a three-phase transformer used by iemens and Halske, and others, in which the magnetism in the yokes, DD', which join the cores A, B, and C, is $\sqrt{3}$ times as great as that in the cores, and the construction allows a considerable economy in comparison with separate transformers in the three phases; while, if the windings for the three phases are placed on the three sides of a triangle or are arranged in consecutive order on a ring, the core must be $\sqrt{3}$ times as great as would be required for one phase alone and the saving of iron is in the relation of $3 : \sqrt{3}$.

After giving due regard to the resultant magnetism in the cores, the principles and practice in transformer design, construction, and testing, which have already been fully developed (Chaps. X., XI., and XII.), are directly applicable to polyphase transformers.

190. Transformation of Phases. — Arrangements for transforming one polyphase system into another system with a different number of phases may be readily developed from the principles which have been fully set forth in the chapters on single-phase transformers and induction motors. Quite a number of commercial devices for this purpose have been proposed. Mr. C. F. Scott* has patented a method for transforming two-phases into three-phases which has been in some commercial service. It is arranged as follows: in Fig. 319*a*, the primaries of the transformers M and M' are connected to a two-phase source. The secondary of M is attached to the middle of the secondary of M', as shown at O. The secondary of M has $\frac{\sqrt{3}}{2}$ times the turns of that of

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* Polyphase Transmission, *Electrical World*, Vol. 23, p. 358; *Lond. Electrician*, Vol. 32, p. 640.

M'. Then in Fig. 319 *b* the line *OB* represents the pressure between the points *O* and *B* in the former figure, *OC* that between *O* and *C*, and *OA* that between *O* and *A*. *OA* must be at right angles to *OB* or *OC*, as the two-phases of the primaries are 90° apart. Thus it is seen that between the points *A*, *B*, and *C* three equal pressures are set up at 120° apart. By reversing the apparatus, three-phases may be transformed into two-phases. Other arrangements for effecting the same result may be readily suggested, such as that shown in Fig. 320, where *aa'*, *bb'*, *cc'* represent the three coils of a three-phase winding uniformly placed upon a ring core, and *AA'*, *BB'* a uniform two-phase winding. If one of the windings is connected to an appropriate polyphase circuit it causes a rotary field to be set up in the core which sets up a polyphase current in the circuit of the other set of coils. In this case the number of phases in the secondary circuit is independent of the number of primary phases and depends only upon the number and arrangement of the secondary coils. The magnetic circuit should be completed by filling the central space with iron stampings.

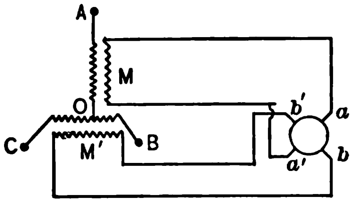

Fig. 319 a

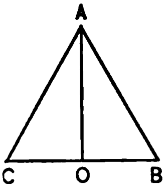

Fig. 319 b

The transformation of single-phase into polyphase

currents by means of stationary transformers may be accomplished by phase-splitting devices,* but no satisfactory commercial method has been developed which does not include moving parts in the transformer.

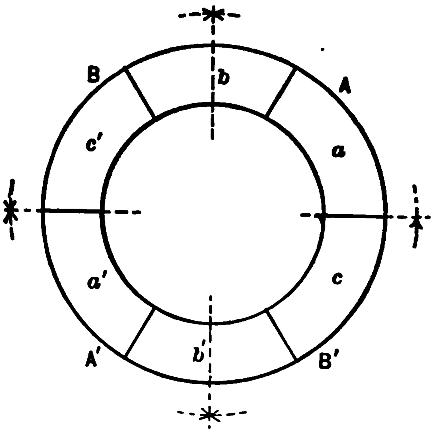

Fig. 320

191. Rotary Transformers. — The possibility of converting a continuous-current dynamo into a single-phase alternator was referred to in Section 5 and later sections, and a machine so constructed with a continuous-current commutator and alternating-current collector rings may be used to convert a continuous current which is fed into its commutator end, and by which it is driven, into an alternating current which is taken from the collector rings. Or, the transformation may be from alternat-

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* Bradley, Phasing Transformers, *Trans. Amer. Inst. E. E.*, Vol. 12, p. 505; Steinmetz, Some Features of Alternating-Current Systems, *Trans. Amer. Inst. E. E.*, Vol. 12, p. 329.

continuous currents, if the armature is prop-
synchronized so that it runs as a synchronous

is possible in the same manner to make a two-
r to be used ***with separate circuits*** out of any
uous-current machine with Gramme or Siemens
ure, by arranging four collector rings on the shaft
onnecting them to the armature windings at points
are 90 electrical degrees apart. It is also possi-
make a three-phaser out of a continuous-current

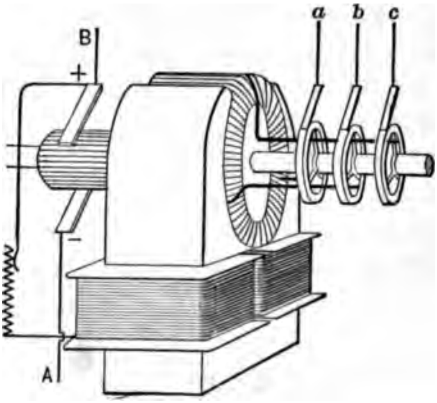

Fig. 321

ne by arranging three collector rings on the
and connecting them to the armature winding,
electrical degrees apart. Such machines may
d to transform continuous currents into polyphase
ts or *vice versa*. (See Fig. 321.) In the case of
asers, it is evident that the maximum value of
ternating pressure is equal to the value of the

continuous pressure, and hence the ratio of transformation is theoretically $1:\sqrt{2}$. In the case of threephasers a little consideration will show that the ratio of transformation is $1:\sqrt{3}$. These theoretical values are found to hold very closely in commercial machines. They are independent of the speed of the machines and of the strength of the fields, provided armature reactions are small.

Machines so constructed are called **Rotary Transformers.** They will run in synchronism when fed with alternating currents, and their speed therefore depends upon the number of poles in the field and the frequency of the currents. Polyphase rotary transformers are generally self-starting from the alternating-current end by the effect of foucault currents set up in the pole pieces by the rotary field which exists in the armature when it is not in synchronism. The starting torque may be increased, as in polyphase synchronous motors, by embedding copper "induction bars" across the pole faces. After a rotary transformer fed by an alternating current is in synchronism, its fields may be magnetized by the continuous current produced by itself and collected from its commutator.

In connecting the armature windings of rotary transformers to the collector rings, the relative angles corresponding to the current phases must be carefully distinguished (compare Sect. 102 *a*). One complete revolution of an armature in a two-pole field corresponds to one complete period of the alternating current, and therefore 360 mechanical degrees corresponds to 360 electrical degrees, but in multipolar machines a rotation of the armature equal to twice the angular pitch of the poles corresponds to one complete period, so that, in general, the relation of electrical degrees to mechanical degrees is $p : 1$, where p is the number of pairs of poles. Two-pole rotary transformers evidently utilize the whole of the armature winding with each collector ring connected to a single point, and the same is true of multipolar machines with series path windings (Vol. I., p. 276). If single connections to the collector rings are used in multipolar machines with multiple path windings, a portion only of the armature, corresponding to 360 electrical degrees, is occupied in the delivery of alternating currents, and the armature capacity is therefore not fully utilized. To fully utilize the armature in this case, each collector ring must be connected to the winding at as many points as there are pairs of poles, the points being 360 electrical degrees apart.

The capacity of a rotary transformer of this type is greater than the same machine used either as an alternator or as a continuous-current generator, and the excess capacity increases with the number of phases. This is due to the fact that the transformed current does not traverse all of the armature conductors, but takes the path from the continuous-current brushes to the alternating-current brushes in which it meets the least opposition, and the heating and armature reactions for a given output are reduced.* The ratio of transformation of the machine when operated to transform alternating currents into continuous currents may be increased by unbalancing the polyphase circuit by the introduction of unequal inductances.

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* Mershon, Output of Polyphase Generators, *Electrical World*, Vol. 25, p. 58.

Rotary transformers are also constructed with two independent armature windings, or by rigidly connecting independent machines together.

APPENDICES.

A. THE APPLICATION OF FOURIER'S THEOREM TO ALTERNATING-CURRENT CURVES.

B. THE CHARACTERISTIC FEATURES OF ALTERNATING-CURRENT CURVES.

C. OSCILLATORY DISCHARGES.

D. ELECTRICAL RESONANCE.

APPENDIX A.

THE APPLICATION OF FOURIER'S SERIES TO ALTERNATING-CURRENT CURVES.

It has been stated in Section 30 of the text that alternating-current curves may be represented by a special form of Fourier's series,

$$e \text{ (or } c) = a_1 \sin \alpha + a_3 \sin 3\alpha + a_5 \sin 5\alpha + \text{etc.}$$
$$+ b_1 \cos \alpha + b_3 \cos 3\alpha + b_5 \cos 5\alpha + \text{etc.},$$

but it is a matter of some labor to determine the constants a and b which apply to any particular curve. This may be done in the following manner, first assuming that an alternating-current curve has been experimentally determined and plotted in the usual manner to rectangular co-ordinates and it is desired to find the constants to be inserted in the Fourier series in order to give the equations of the curve. Divide the base of one loop of the curve into $n+1$ equal divisions, then there will be n points between $\alpha = 0$ and $\alpha = 180°$, which will correspond to $\Delta\alpha = \left(\frac{180}{n+1}\right)^\circ$, $2\,\Delta\alpha = \left(\frac{2 \times 180}{n+1}\right)^\circ$, etc., and the abscissa of any of the points may be represented in general by $k\,\Delta\alpha = \left(\frac{k\,180}{n+1}\right)^\circ$. Corresponding to each abscissa there will be an ordinate which represents a value of e (or c), which may be called e_k (or c_k). Substituting in the original equation gives

$$e_k \text{ (or } c_k)$$
$$= a_1 \sin\left(\frac{k\,180}{n+1}\right)^\circ + a_3 \sin 3\left(\frac{k\,180}{n+1}\right)^\circ + a_5 \sin 5\left(\frac{k\,180}{n+1}\right)^\circ + \text{etc.}$$
$$+ b_1 \cos\left(\frac{k\,180}{n+1}\right)^\circ + b_3 \cos 3\left(\frac{k\,180}{n+1}\right)^\circ + b_5 \cos 5\left(\frac{k\,180}{n+1}\right)^\circ + \text{etc.}$$

By giving k successive numerical values from unity to n, there are found n equations of the first degree from which the values of a_1, a_3, a_5, etc., b_1, b_3, b_5, etc. to n terms, may be determined by the usual algebraic methods. Putting m as a general subscript for a or b, then

$$a_m = \frac{2}{n+1}\left[e_1 \sin\left(m\frac{180}{n+1}\right)^\circ + e_2 \sin\left(2\,m\frac{180}{n+1}\right)^\circ + \cdots \right.$$
$$\left. + e_n \sin\left(nm\frac{180}{n+1}\right)^\circ\right],$$
$$b_m = \frac{2}{n+1}\left[e_1 \cos\left(m\frac{180}{n+1}\right)^\circ + e_2 \cos\left(2\,m\frac{180}{n+1}\right)^\circ + \cdots \right.$$
$$\left. + e_n \cos\left(nm\frac{180}{n+1}\right)^\circ\right].$$

These expressions may be written for convenience

$$a_m = \frac{2}{n+1}\sum_{k=1}^{k=n} e \sin\left(km\frac{180}{n+1}\right)^\circ, \quad b_m = \frac{2}{n+1}\sum_{k=1}^{k=n} e \cos\left(km\frac{180}{n+1}\right)^\circ.$$

The pressure wave of an alternator is represented by the heavy line of Fig. *A*, and the constants of Fourier's series for this curve have been determined up to the seventh harmonic, giving the values

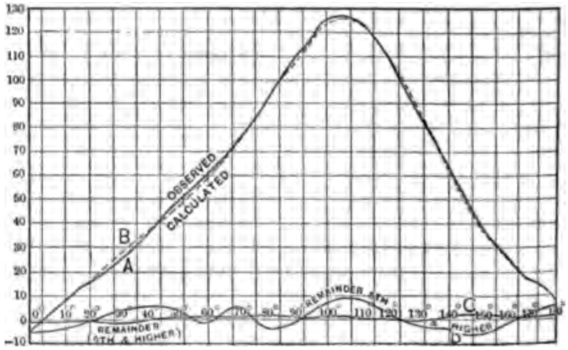

Fig. A

$$\begin{aligned} a_1 &= +98.6, & b_1 &= -14.7, \\ a_3 &= -13.3, & b_3 &= +18.2, \\ a_5 &= -\ \ 1.6, & b_5 &= -\ \ 4.8, \\ a_7 &= +\ \ .25, & b_7 &= +\ \ 1.2. \end{aligned}$$

The equation for the curves as determined by this means is

$$\begin{aligned} e = 98.6 \sin\alpha - 13.3 \sin 3\alpha - 1.6 \sin 5\alpha + .25 \sin 7\alpha \\ - 14.7 \cos\alpha + 18.2 \cos 3\alpha - 4.8 \cos 5\alpha + 1.2 \cos 7\alpha. \end{aligned}$$

Substituting various values of α in this equation, the corresponding values of e are given, and the corresponding curve, which is dotted, has been plotted in the figure. It will be noticed that the calculated curve crosses the original in seven points, and very closely approximates to its exact form. If a larger number of constants had been determined, the calculated curve would have crossed the original curve a proportionally larger number of times, and the approximation would have been still closer. The number of times the calculated curve crosses the original curve is equal to n, and consequently the calculated curve cannot exactly coincide with the original curve unless $n = \infty$. The series used is rapidly convergent, and in this particular curve the effect of the fifth and seventh harmonics is quite small, and the curve is sufficiently well represented for practical purposes by the fundamental and third harmonics, in which case the equation is

$$\begin{aligned} e = 98.6 \sin\alpha - 13.3 \sin 3\alpha \\ - 14.7 \cos\alpha + 18.2 \cos 3\alpha. \end{aligned}$$

The corresponding sine and cosine terms of the series

$$\left.\begin{matrix} a_1 \sin\alpha \\ + b_1 \cos\alpha \end{matrix}\right\} + \left\{\begin{matrix} a_3 \sin 3\alpha \\ + b_3 \cos 3\alpha \end{matrix}\right\} + \left\{\begin{matrix} a_5 \sin 5\alpha \\ + b_5 \cos 5\alpha \end{matrix}\right\} + \left\{\begin{matrix} + \text{etc.,} \\ + \text{etc.} \end{matrix}\right.$$

may be conceived as representing the rectangular sine components of the terms of a single sine or cosine curve. This is illustrated in Fig. *B*, from which it is evident that

$$a_m \sin m\alpha \pm b_m \cos m\alpha = c_m \sin (m\alpha + \theta_m)$$

or $$a_m \sin m\alpha \pm b_m \cos m\alpha = c_m \cos (m\alpha - \theta_m').$$

Where $c_m = \sqrt{a_m^2 + b_m^2}$, and $\tan \theta_m = \frac{b_m}{a_m}$ or $\tan \theta_m' = \frac{a_m}{b_m}$.

Substitution gives

$$e = c_1 \sin (\alpha + \theta_1) + c_3 \sin (3\alpha + \theta_3) + c_5 \sin (5\alpha + \theta_5) + \text{etc.}$$
$$= c_1 \cos (\alpha - \theta_1') + c_3 \cos (3\alpha - \theta_3') + c_5 \cos (5\alpha - \theta_5') + \text{etc.}$$

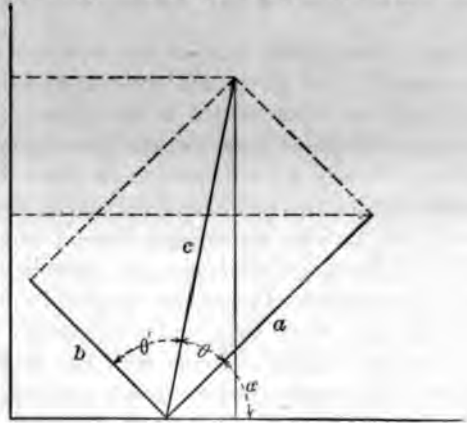

Fig. B

The equation given previously, when reduced to this form (using θ), has the following constants:

$c_1 = 99.7,$ $\theta_1 = -\ 8^\circ\ 29',$
$c_3 = 22.5,$ $\theta_3 = -\ 53^\circ\ 50',$
$c_5 = 5.1,$ $\theta_5 = +\ 71^\circ\ 34',$
$c_7 = 1.2,$ $\theta_7 = +\ 78^\circ\ 34',$

and the equation is

$$e = 99.7 \sin (\alpha - 8^\circ\ 29') - 22.5 \sin (3\alpha - 53^\circ\ 50')$$
$$- 5.1 \sin (5\alpha + 71^\circ\ 34') + 1.2 \sin (7\alpha + 78^\circ\ 34'),$$

and its value to a considerable degree of approximation is

$$e = 99.7 \sin (\alpha - 8^\circ\ 29') - 22.5 \sin (3\alpha - 53^\circ\ 50').$$

Following are examples of the calculation of the constants of these equations:

$$n = 7, \qquad n + 1 = 8, \qquad \left(\frac{180}{n+1}\right)^\circ = 22^\circ\tfrac{1}{2}.$$

Values of e from curve:

$$e_1 = 18, \qquad e_3 = 70, \qquad e_5 = 125, \qquad e_7 = 30.$$
$$e_2 = 42, \qquad e_4 = 110, \qquad e_6 = 80,$$

$$a_1 = \tfrac{1}{4}\{18 \sin 22^\circ\tfrac{1}{2} + 42 \sin 45^\circ + 70 \sin 67^\circ\tfrac{1}{2} + \cdots + 30 \sin 157^\circ\tfrac{1}{2}\} = 98.6,$$
$$a_3 = \tfrac{1}{4}\{18 \sin 67^\circ\tfrac{1}{2} + 42 \sin 135^\circ + 70 \sin 202^\circ\tfrac{1}{2} + \cdots + 30 \sin 112^\circ\tfrac{1}{2}\} = -13.3,$$
$$b_3 = \tfrac{1}{4}\{18 \cos 67^\circ\tfrac{1}{2} + 42 \cos 135^\circ + 70 \cos 202^\circ\tfrac{1}{2} + \cdots + 30 \cos 112^\circ\tfrac{1}{2}\} = 18.2.$$

$$e_3 = \sqrt{a_3^2 + b_3^2} = 22.5, \quad \theta_3 = \tan^{-1}\frac{b_3}{a_3} = -53^\circ\,50'.$$

The effective ordinate of an alternating-current curve may be determined by integrating directly from its equation. The effective value squared is

$$E^2 = \frac{1}{\pi}\sum_0^\pi e^2 = \frac{1}{\pi}\int_0^\pi e^2 da$$
$$= \frac{1}{\pi}\int_0^\pi (a_1 \sin a + a_3 \sin 3a + \text{etc.} + b_1 \cos a + b_3 \cos 3a + \text{etc.})^2 da,$$

and since

$$\int_0^\pi \sin ma \sin na da, \qquad \int_0^\pi \sin ma \cos ma da, \qquad \int_0^\pi \cos ma \cos na da$$

are each equal to zero when m and n are unequal integers, there results

$$E^2 = \frac{1}{\pi}\int_0^\pi e^2 da = \frac{a_1^2}{\pi}\int_0^\pi \sin^2 a da + \frac{a_3^2}{\pi}\int_0^\pi \sin^2 3a da$$
$$+ \frac{a_5^2}{\pi}\int_0^\pi \sin^2 5a da + \text{etc.} + \frac{b_1^2}{\pi}\int_0^\pi \cos^2 a da$$
$$+ \frac{b_3^2}{\pi}\int_0^\pi \cos^2 3a da + \frac{b_5^2}{\pi}\int_0^\pi \cos^2 5a da + \text{etc.}$$

But $\int_0^\pi \sin^2 ma\,da$ and $\int_0^\pi \cos^2 ma\,da$ are always equal to $\frac{\pi}{2}$;

hence $E^2 = \frac{a_1^2}{2} + \frac{a_3^2}{2} + \frac{a_5^2}{2} + \text{etc.} + \frac{b_1^2}{2} + \frac{b_3^2}{2} + \frac{b_5^2}{2} + \text{etc.},$

and $$E = \left\{ \sum_{k=1}^{k=\infty} \frac{a^2}{2} + \sum_{k=1}^{k=\infty} \frac{b^2}{2} \right\}^{\frac{1}{2}} = \left\{ \sum_{k=1}^{k=\infty} \frac{c^2}{2} \right\}^{\frac{1}{2}}$$

In the example which has been given, the value of E calculated from the constants up to the seventh is $E = 72.4$.

Taking the first and third constants only, gives $E = 72.3$, which is correct to a close approximation.

The value of E found by plotting the curve to polar co-ordinates is $E = 72\frac{1}{2}$.

The example which has been taken fairly represents the complexity of the average distorted waves. Some alternating-current curves are so greatly distorted that a larger number of terms of the series is required to closely represent them, but for practical purposes three or four terms are always sufficient. In a large number of waves the forms are so simple that two terms of the series give ample approximation for practical purposes. Examples of waves of greater or less complexity than that already operated upon are given in figures C, D, and E of

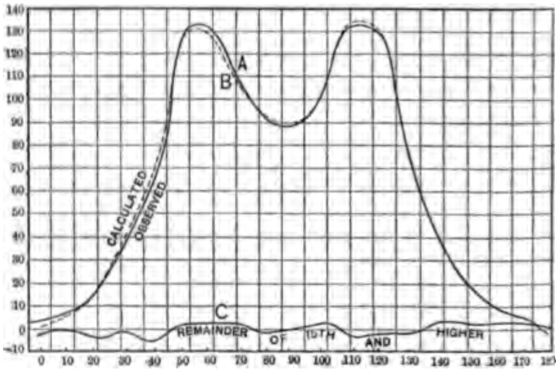

Fig. C

this appendix and various figures of the text. Figure *C* is a curve given by Steinmetz[*] for which the constants up to the 13th are

$$\begin{array}{ll} a_1 = +109.5 & b_1 = +10.5 \\ a_3 = -\ 12.8 & b_3 = -\ 3.25 \\ a_5 = -\ 22.8 & b_5 = -10.6 \\ a_7 = -\ 12.4 & b_7 = +\ 7.87 \\ a_9 = -\ .55 & b_9 = +\ .245 \\ a_{11} = -\ 2.95 & b_{11} = -\ 4.2 \\ a_{13} = +\ .595 & b_{13} = +\ 3.38 \end{array}$$

The ninth and higher constants are practically negligible, so hat the curve may be represented by the formula

$$e = \begin{array}{l} 109.5 \sin \alpha \\ +\ 10.5 \cos \alpha \end{array} + \begin{array}{l} -12.8 \sin 3\alpha \\ -3.25 \cos 3\alpha \end{array} + \begin{array}{l} -22.8 \sin 5\alpha \\ -10.6 \cos 5\alpha \end{array} + \begin{array}{l} -12.4 \sin 7\alpha \\ +7.87 \cos 7\alpha \end{array}$$

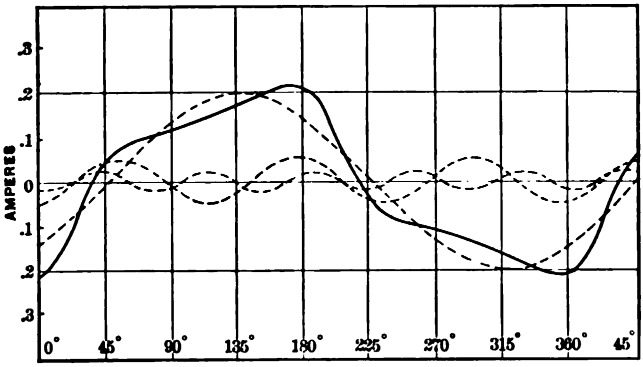

Fig. D

Figure *D* is a curve given by Fleming[†] after the results of tests by Merritt and Ryan. Its equation is

$$e = .196 \sin (\alpha - 48° 55') + .048 \sin (3\alpha - 76° 50') \\ + .016 \sin (5\alpha - 90°)$$

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[*] *Trans. Amer. Inst. E. E.*, Vol. 12, p. 476.

[†] Fleming's *Alternate Current Transformer in Theory and Practice*, Vol. II., p. 454.

ditions brought about by the discharge of a condenser through an inductive circuit are not entered upon in the text, and as they have some incidental interest to the electrical engineer they will be explained here.

If a condenser of capacity s, charged to a difference of potential or electric pressure E, be introduced into an electric circuit, it will at once discharge; that is, it will send a current through the circuit and thus bring the difference of potential of its plates to zero. At any instant the electrical pressure in the circuit will be

$$e = L\frac{dc}{dt} + cR,$$

where L and R are the self-inductance and resistance of the circuit. From the fundamental definition of a condenser,

$$e = \frac{q}{s} \text{ and } c = -\frac{dq}{dt},$$

q representing the quantity of electricity in the condenser at any instant during the discharge, when the electrical pressure is e, and the current c. Substituting these values gives

$$\frac{q}{s} = -L\frac{d^2q}{dt^2} - \frac{dq}{dt}R,$$

or

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{Ls} = 0.$$

In order to find the value of the quantity of electricity in the condenser at any instant, and thus determine the rate at which the condenser discharges, this equation must be solved by integration.* The characteristic equation is

$$x^2 + \frac{Rx}{L} + \frac{1}{Ls} = 0,$$

and the roots of this determine the form of the solution. As this is a quadratic equation, it may have either two real or two aginary roots depending upon circumstances; these roots are

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* Price's *Calculus*, Vol. II., p. 458; Forsyth's *Differential Equations*, p. 86.

$$x_1 = -\frac{R}{2L} + \sqrt{\frac{1}{4}\frac{R^2}{L^2} - \frac{1}{Ls}},$$

$$x_2 = -\frac{R}{2L} - \sqrt{\frac{1}{4}\frac{R^2}{L^2} - \frac{1}{Ls}}$$

is evident that the roots are real when

$$\frac{1}{4}\frac{R^2}{L^2} \geqq \frac{1}{Ls} \text{ or } R^2 > \frac{4L}{s};$$

d that they are imaginary when

$$R^2 < \frac{4L}{s}.$$

In the first case the solution takes the form

$$q = A\epsilon^{x_1 t} + B\epsilon^{x_2 t},$$

d
$$c = -\frac{dq}{dt} = -Ax_1\epsilon^{x_1 t} - Bx_2\epsilon^{x_2 t},$$

ere A and B are constants which must be found by sub- tuting the value of zero for t, in which case $q = Q$, and $= 0$. Whence,

$$Q = A + B, \text{ and } Ax_1 + Bx_2 = 0,$$

m which
$$A = -\frac{Qx_2}{x_1 - x_2}, \text{ and } B = \frac{Qx_1}{x_1 - x_2}.$$

nce

$$q = \frac{Q}{x_1 - x_2}(x_1\epsilon^{x_2 t} - x_2\epsilon^{x_1 t})$$

d
$$c = -\frac{dq}{dt} = \frac{x_1 x_2}{x_1 - x_2}Q(\epsilon^{x_1 t} - \epsilon^{x_2 t})$$

$$= \frac{Q}{2Ls\sqrt{\frac{R^2}{4L^2} - \frac{1}{Ls}}}\left[\epsilon^{\left(-\frac{R}{2L}+\sqrt{\frac{1}{4}\frac{R^2}{L^2}-\frac{1}{Ls}}\right)t} - \epsilon^{\left(-\frac{R}{2L}-\sqrt{\frac{1}{4}\frac{R^2}{L^2}-\frac{1}{Ls}}\right)t}\right]$$

ese equations show that q and c never fall to zero, but grad- ly decrease according to a logarithmic function as t increases.

The time constant of the circuit decreases as $\frac{1}{4}\frac{R^2}{L^2}$ approaches $\frac{1}{Ls}$ in value and is a minimum of $\frac{1}{2}sR$ when they are equal.*

When $R^2 < \frac{4L}{s}$ the roots are imaginary, and if i be taken to indicate the imaginary unit $\sqrt{-1}$,

$$x_1 = -\frac{R}{2L} + i\sqrt{\frac{1}{Ls} - \frac{1}{4}\frac{R^2}{L^2}},$$

$$x_2 = -\frac{R}{2L} - i\sqrt{\frac{1}{Ls} - \frac{1}{4}\frac{R^2}{L^2}}.$$

Inserting these values in the formulas for q and c and reducing to trigonometrical forms, the equations become

$$q = \epsilon^{-\frac{tR}{2L}}\left\{C\cos t\sqrt{\frac{1}{Ls} - \frac{R^2}{4L^2}} + D\sin t\sqrt{\frac{1}{Ls} - \frac{R^2}{4L^2}}\right\}$$

$$c = \frac{Q}{Ls\sqrt{\frac{1}{Ls} - \frac{R^2}{4L^2}}}\epsilon^{-\frac{tR}{2L}}\sin t\sqrt{\frac{1}{Ls} - \frac{R^2}{4L^2}}.$$

From these formulas we see that when the roots of the differential equations are imaginary, q and c are periodic functions which have alternately positive and negative values, so that the discharge is an oscillatory one. In other words, when the condenser is discharged, during the first flow of current a certain amount of energy has been stored in the magnetic field and in the return of this to the circuit the condenser is charged up in the opposite direction. This is repeated over and over again with incredible rapidity but with decreasing intensity, until the total energy of the original charge is dissipated in overcoming the resistance of the circuit. The current passes through one complete period while $t\sqrt{\frac{1}{Ls} - \frac{R^2}{4L^2}}$ passes through all values from 0 to 2π and therefore the period $T = \frac{2\pi}{\sqrt{\frac{1}{Ls} - \frac{R^2}{L^2}}}$, and if s very small compared with L this becomes $T = 2\pi\sqrt{Ls}$. period of oscillation set up in any circuit may therefore be trolled by increasing L. By this means Professor Lodge :eeded in getting periods a considerable fraction of a second ɜngth, but in general the discharge of a condenser may be to be practically instantaneous. If iron cores are used in inductance coils for use with oscillating discharges they t be very finely subdivided, or the excessive foucault 'ents set up in the outer layers of the cores screen the :r parts from any magnetic effects.

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* Lodge, On the Influence of Self-induction on the Rate of Discharge of a Condenser, *Lond. Electrician,* Vol. 21, p. 39.

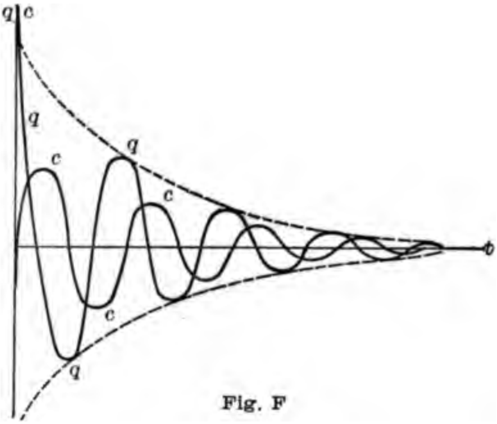

Fig. F

'he formulas for the discharge of a condenser, through an ıctive circuit, apply equally well to the charging current, ch may be logarithmic or oscillating depending upon whether $\gtrless \frac{1}{Ls}$. Figure F shows the dying away of the charge and oscillations of the discharge current in an oscillating circuit. : curve which touches the maximum points of the quantity ʀe is logarithmic, and a similar curve similarly touching current curve would be logarithmic. Figure G shows the ɪwth and dying away of a current due to a transient pressure an oscillating circuit. Figure H shows the curve of disrge and of the discharging current in a non-oscillating circuit.

The oscillating electric circuit may be likened to a pendulum or an oscillating spring (Fig. *I*). Such a spring will have a period of vibration dependent upon the mass (inertia) of its

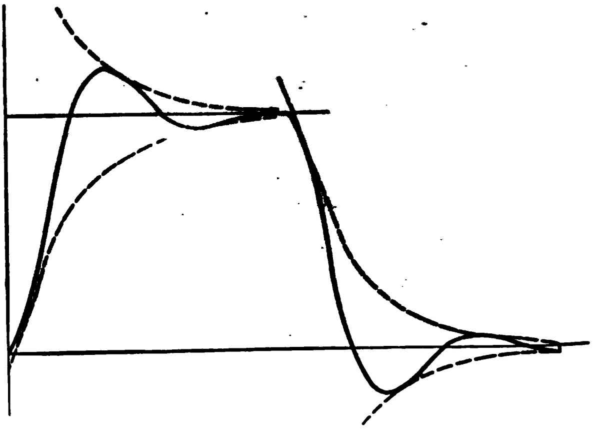

Fig. G

load, its elasticity, and the frictional resistance to its motion. The formula giving its period is exactly similar to that for the period of an oscillating discharge, putting mass for self-inductance, friction for resistance, and the reciprocal of elasticity

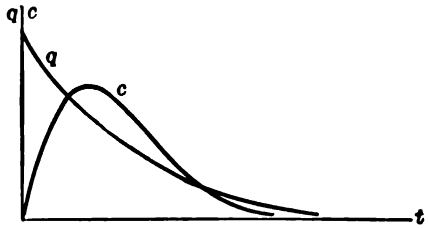

Fig. H

(compressibility) for capacity. When the spring stands at its neutral point it is analogous to the condenser when discharged. Extending or compressing the spring is equivalent to charging condenser. If the resistance to motion is small and the extended spring is released, it will oscillate through decreasing distances with an isochronous period until the energy stored in the spring by its extension is used up in overcoming the frictional resistance to its motion. If the resistance to its motion is increased, its period will be lengthened and the number of oscillations decreased. While if the resistance is made sufficiently great (as for instance, if the spring is immersed in syrup) the motion will be dead beat. This condition is analogous to the electric discharge in a circuit in which $\frac{R^2}{4L^2} > \frac{1}{Ls}$.

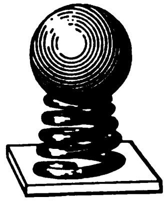

Fig. I

This subject is treated at great length in Fleming's *Alternate Current Transformers*, Vol. I, p. 364, *et seq.;* Bedell and Crehore's *Alternating Currents*, Chaps. 7 and 8; and Gerard's *Leçons sur l'Électricité*, 3d ed., Vol. I, p. 253, *et seq.*

APPENDIX D.

ELECTRICAL RESONANCE.

The deductions of Chapters III. and IV. of the text have shown very clearly that self-inductance and capacity in a circuit may be made to neutralize each other when a sinusoidal alternating pressure is applied to the circuit, and the self-inductance and capacity are constant. In this case the self-inductance and capacity act in opposition, so that at each instant energy is being stored or released in the magnetic field at exactly the same rate as energy is being released or stored in the charge of the condenser. The self-inductance and capacity may therefore be said to supply each other's demands, and the pressure impressed on the circuit may be wholly utilized in doing work on a non-reactive receiver, such as incandescent lamps, and in heating the wires of the circuit.

The actual energy which is transferred back and forth between the self-inductance and capacity may be many times as great as that given to the circuit by the generator, and the pressure at the terminals of the self-inductance and of the condenser must then be proportionally greater than that of the generator.

This condition can exist only when $2\pi fL = \frac{1}{2\pi fs}$[*] or $Ls = \frac{1}{4\pi^2 f^2}$, and when $T = 2\pi\sqrt{Ls}$.[†] From the condition $2\pi fL = \frac{1}{2\pi fs}$ it is seen that $\frac{1}{f} = 2\pi\sqrt{Ls}$, and $\frac{1}{f}$ is therefore equal to T, the natural period of the circuit. The natural period of discharge of the circuit is therefore exactly equal to the period of the impressed pressure, or, as we may say, to the actual rate of the electrical vibrations impressed on the circuit by the generator. This relation between the vibrations of the line and of the generator is similar to that of a vibrating tuning fork or string and its sounding board when they are in resonance, and therefore the term **Electrical Resonance** has, on account of the analogy, been applied to the electrical circuit. *An electrical circuit is said to be in resonance with an impressed pressure when the natural period of the circuit is equal to the period of the impressed pressure.* When this condition exists, the maximum current is caused to flow in the circuit by the application of a given impressed pressure, the value of the current in a resonant circuit from which no external work is supplied being

$$C = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fs}\right)^2}} = \frac{E}{R}, \text{ since } 2\pi fL = \frac{1}{2\pi fs}.$$

If the self-inductance and capacity are in series in the circuit, it is evident that when the circuit and applied pressure are in resonance the pressure between the terminals of the capacity, $= \left(\frac{C}{2\pi fs}\right)$,[‡] is a maximum, since the circuit current is a maximum. If either the frequency, the self-inductance, or the capacity is changed in value, the value of the current falls, and the condenser pressure falls, unless the other elements are changed in value in such a way as to continue the condition of resonance. A condenser in a resonant circuit may be used as a transformer of pressure by connecting non-reactive apparatus across its terminals, as has been suggested by Blakesley,* Loppé et Bouquet,† Pupin,‡ and others.

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[*] *Text*, Chapters III. and IV.

[†] Appendix C.

[‡] *Text*, Section 33.

If the self-inductance and capacity are in parallel in the circuit, the pressure at their terminals cannot be greater than that impressed upon the circuit minus the loss of pressure in the lead wires, but when the circuit is resonant, the circuit current furnished by the generator is at a maximum which is equal to $\frac{E}{R}$, while the current transferred between the inductance and capacity is also a maximum which may be a great many times as great as the maximum value of the generator current.

Resonant circuits in the hands of renowned experimenters such as Hertz, Lodge, and others have produced remarkable results, which have led to great advances in our knowledge of electricity, while mathematical analysis of such circuits has led to further discoveries. These results have caused some to expect remarkable effects to be gained from the use of resonant circuits (or *tuned circuits*, as they are sometimes called) for the purposes of the electrical transmission of power. Circuits which are installed for the transmission of energy over considerable distances (whether the wires are overhead or underground) always contain capacity and self-inductance § distributed along their length. It would be possible in such lines to adjust the capacity and self-inductance so as to give resonance, and the results to be gained from so doing may be examined through analogy.

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* Blakesley's *Alternating Current of Electricity*, 2d ed., p. 53.

† Loppé et Bouquet's *Courants Alternatifs Industriels*, p. 77.

‡ Pupin, *Trans. Amer. Inst. E. E.*, Vol. 10., p. 382.

§ *Text*, Section 47.

A mechanical analogue of a resonant circuit is shown in Fig. *J*. This consists of a tube fitted with two plungers and filled with a perfectly elastic fluid. The properties of this fluid may be used to represent electrical quantities according to the analogies; fluid velocity — electric current; fluid pressure — electric pressure; inertia — self-inductance; compressibility* — capacity; frictional resistance — electrical resistance. Now suppose the fluid to be without inertia and perfectly incompressible; then if plunger *A* be moved toward *D*, a uniform current will be instantly set up in the whole tube, the velocity of which is equal (in proper units) to the pressure applied to the plunger divided by the frictional resistance. If plunger *A* is caused to move up and down harmonically, the other plunger will have an exactly equal synchronous harmonic motion. This is exactly analogous to the state of an electric circuit without inductance or capacity. Figure *K* shows diagrammatically the state of the circuit, where the distance of the broken line from the heavy line is equal to the current at each point, and the light line shows the gradual fall of pressure between *A* and *A'*, caused by the resistance, and the sudden fall of pressure at *A'*, caused by the external work done by plunger *A'*.

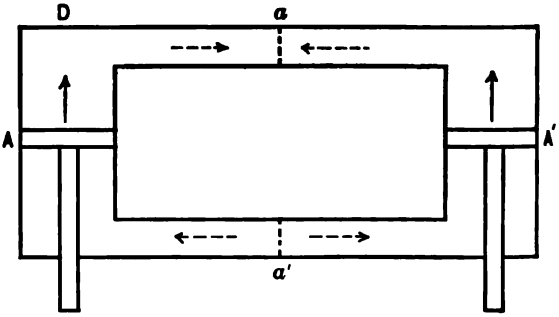

Fig. J

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* Compressibility of a fluid is the ratio of compression (change of volume) to the pressure producing it, and electrical capacity is the ratio of the charge (change of quantity) to the electrical pressure producing it.

If the fluid be compressible but have no inertia, it is evident that the motion of the plunger at *A'* will be less than that at *A*, which is analogous to the decadence of current as it flows along a circuit having capacity, due to the quantity of electricity entering into the static charge. The movements of the plungers are isochronous but not in synchronism. In this case the motion of the plunger *A* will exert its maximum pressure when the fluid is most compressed, or at the end of its stroke where its velocity is least. Hence the velocity (current), which is greatest at the middle of the stroke, leads the pressure by 90° of phase. The electric circuit corresponding to this is shown in Fig. *L*.

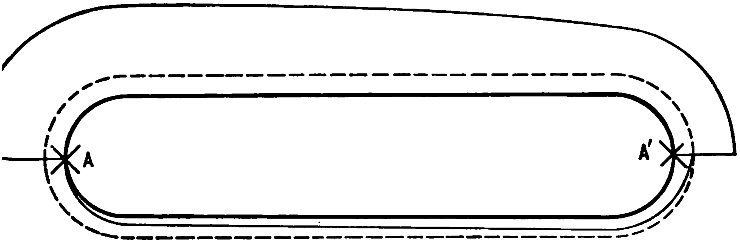

Fig. K

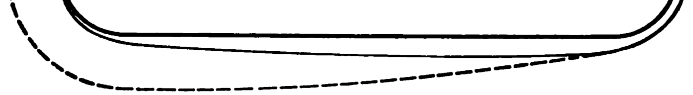

Fig. L

If the fluid has inertia but is incompressible, the velocity at *A* and *A'* will be equal, or the current through the circuit will be uniform, but the pressure exerted upon piston *A* must be greatest where the acceleration is greatest, which is at the beginning of the stroke where the velocity is least. Consequently the current lags behind the pressure by 90°. This is analogous to the electric circuit with self-inductance only.

If the fluid has both inertia and compressibility, the column of fluid in the tube will then take upon itself the properties of all material elastic bodies, and will have a natural rate of vibration. This will be, as proven in elementary mechanics, proportional to the square root of the density divided by the elasticity, or to the square root of the product of the inertia and compressibility.

Hence $T = a\sqrt{MK}$ where a is a constant, M mass, K compressibility, and T time of vibration. In this case if the plunger *A* (Fig. *J*) be moved with a sinusoidal velocity of period T, the fluid will be thrown into vibrations which require one complete traversal of the circuit to make a wave length. Hence if there is no power taken from the circuit there are nodes, or points of no motion, at a and a', and antinodes, or points of maximum motion, at the plungers. Since the direction of motion in the two halves of a wave are in opposite directions, the two plungers move in opposite directions in the tube. As the velocity of the fluid varies from node to antinode as a sinusoidal function, the loss of power by friction is reduced to one-half the value which it has for an equal plunger velocity in the inertialess, incompressible fluid.

Since the velocity of the fluid falls off from plungers to the nodes, the pressure upon the fluid exerted by the plungers must be proportionally multiplied at the nodes, in order that the same power may be transmitted there as was applied at the prime plunger *A*. The condition of pressure and velocity is diagrammatically represented in Fig. *M*. If power is transferred to an outside object by plunger *A'*, it is impossible for the velocity at the nodal points to be zero, but it must be sufficiently great to transfer the power through the nodal point with the pressure at that point. The relative motions of the plungers, under the conditions here cited, require that *the power be transferred from one to the other wholly through its absorption and redelivery by the fluid through the effects of inertia and elasticity.* The fluid must therefore have a sufficient mass so that, at the slow velocity of the nodes, its kinetic energy shall be sufficient to carry the energy in the circuit across the nodal points.

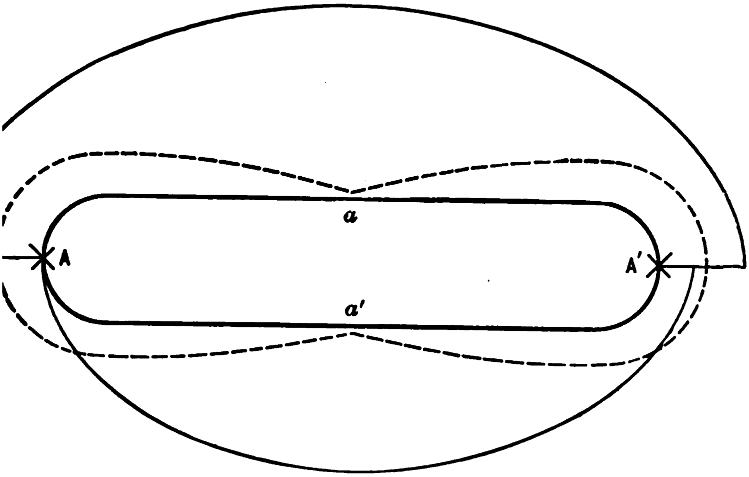

Fig. M

This analogue fully represents the conditions in the resonant electric circuit. Carrying the analogue, and the diagrammatic representation of current and pressure in Fig. *M*, in mind, it is easy to draw definite conclusions in regard to the effect of resonance on the operation of circuits for the transmission of power by currents of electricity.

The advantages of a resonant circuit for electrical transmission are then: (1) a gain of upwards of one-half of the C^2R loss that would be caused by the transmission of an equal amount of power at an equal receiving pressure over the same circuit when out of resonance; (2) more satisfactory regulation than would be found in a non-resonant but reactive line, since the difference in pressure between generator and receiver is equal to current times resistance instead of current times an impedance which is greater than the resistance.

The principal disadvantage of a resonant circuit for electrical transmission is: a very large excess of pressure on the line at certain points, or nodes of current, which excess decreases toward the antinodes. If satisfactory resonance is to be gained by adjusting the self-inductance and capacity of the circuit so that the pressure at the nodes is no greater than ten times that at the antinodes, the average pressure along the line must be caused to be seven times $\left(\frac{10}{\sqrt{2}}\right)$ that of the antinodes, using a sinusoidal function. In other words, if the pressure which is safe for use is limited by the insulation, we may say that the average thickness of insulation on the line must be seven times as great as would be necessary at the generators. This enormous increase of insulation must be made to save fifty per cent of the C^2R loss caused by the transmission of a certain amount of power over a given line. A much more reasonable plan would be to reduce the self-inductance and capacity of the line to a minimum, avoiding resonance and raising the generator pressure to 1.4 its previous value. Now the same power could be transferred over the line with the same resistance as before, the C^2R loss being the same as when the line was resonant, but the average strain on the insulation would be only one-fifth as great as in the resonant line.

The highest pressure which can be economically used on circuits for the electrical transmission of power over long distances is generally conceded to be set at the limit which may be properly insulated. If this is true, the preceding paragraph shows that, with equal insulation, the generator pressure may be safely made $\frac{X}{\sqrt{2}}$ times greater on a non-resonant, long-distance transmission line than that which is safe on a resonant line, where X is the ratio of the maximum pressure to the generator pressure on the resonant line. This shows that the non-resonant line would be by far the most economical for long-distance transmission of power, even if it were commercially possible to maintain resonance on service circuits. For the distribution of power over short distances, the pressure is usually quite low, and the pressure limit is not approached, so that resonance might be introduced without adding to the insulation; but the reactions of transformers and motors on the line make it practically impossible to keep the line in resonance. Similar defects are seen in the propositions for using resonant lines for various other classes of electrical transmission.

These deductions in regard to resonance have been made upon the assumption of exactly sinusoidal currents. In practice these are now seldom met, since iron-cored transformers and motors, and tooth-cored alternators, introduce distortions, and a circuit which is resonant for the fundamental wave is not resonant for its harmonics. As the question of resonance now rests, it does not present any opportunities for application in practice, nor does it enter into problems relating to ordinary electric circuits in such a way as to modify practice. In some cases of long-distance transmission of power by alternating currents with a distorted wave of pressure, the harmonics may accidently come into resonance with the line and cause an undue strain on the insulation; but this is readily guarded against by using a generator which generates an approximate sine pressure curve.

Many articles have been written upon resonance and its effects in electric circuits, but the following will serve to give a general view of the subject: —

April, 1891. Lodge, The Effect of a Condenser Introduced into an Alternate-Current Circuit, *London Electrician*, Vol. 26, p. 762.

May, 1891. Fleming, On Some Effects of Alternating-Current Flow in Circuits having Capacity and Self-Induction, *Jour. Inst. E. E.*, Vol. 20, p. 362.

May, 1893. Pupin, Practical Aspects of Low Frequency Electrical Resonance, *Trans. Amer. Inst. E. E.*, Vol. 10, p. 370.

June, 1894. Anthony, Electrical Resonance as Related to the Transmission of Energy, *Electrical Engineer* (N.Y.), Vol. 17, p. 545.

October, 1894. Blondel, Inductance des Lignes Aériennes pour Courants Alternatifs, *L'Éclairage Électrique*, Vol. I, p. 241.

April, 1895. Houston and Kennelly, Resonance in Alternating-Current Lines, *Trans. Amer. Inst. E. E.*, Vol. 12, p. 133.

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